UNIT I BASIC CONCEPTS AND ISENTROPIC FLOWS

Introduction

The purpose of this applet is to simulate the operation of a converging-diverging nozzle, perhaps the most important and basic piece of engineering hardware associated with propulsion and the high speed flow of gases. This device was invented by Carl de Laval toward the end of the l9th century and is thus often referred to as the 'de Laval' nozzle. This applet is intended to help students of compressible aerodynamics visualize the flow through this type of nozzle at a range of conditions.

Technical Background

The usual configuration for a converging diverging (CD) nozzle is shown in the figure. Gas flows through the nozzle from a region of high pressure (usually referred to as the chamber) to one of low pressure (referred to as the ambient or tank). The chamber is usually big enough so that any flow velocities here are negligible. The pressure here is denoted by the symbol p_c. Gas flows from the chamber into the converging portion of the nozzle, past the throat, through the diverging portion and then exhausts into the ambient as a jet. The pressure of the ambient is referred to as the 'back pressure' and given the symbol p_b.

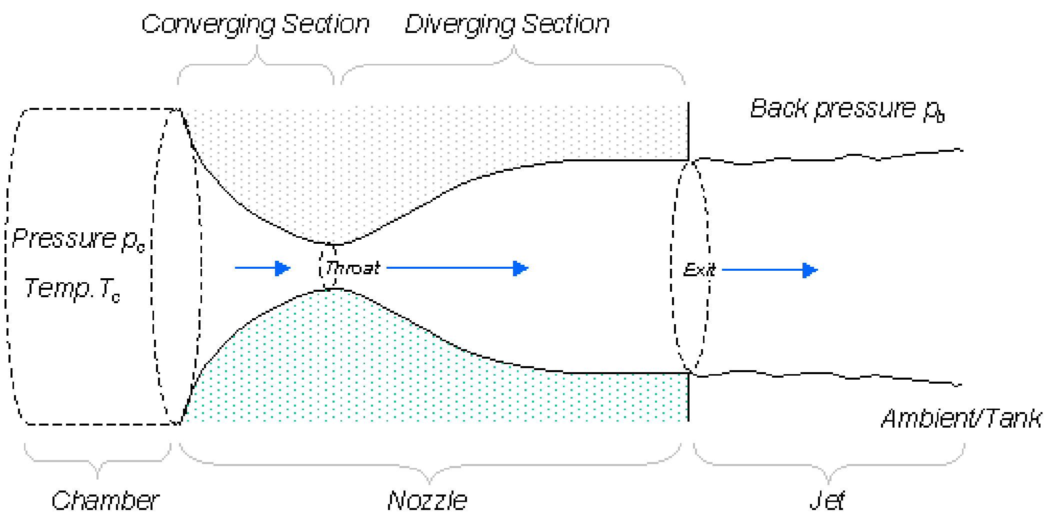


Figure 1. Converging Diverging Nozzle Configuration

A **diffuser** is the mechanical device that is designed to control the characteristics of a fluid at the entrance to a thermodynamic open system. Diffusers are used to slow the fluid's velocity and to enhance its mixing into the surrounding fluid. In contrast, a nozzle is often intended to increase the discharge velocity and to direct the flow in one particular direction.

Frictional effects may sometimes be important, but usually they are neglected. However, the external work transfer is always assumed to be zero. It is also assumed that changes in thermal energy are significantly greater than changes in potential energy and therefore the latter can usually be neglected for the purpose of analysis.

Mach Cone

a conical surface that bounds the region in a supersonic flow of gas in which the sound waves (perturbations) emanating from a point source A of the perturbations are

concentrated (see Figure 1). In a homogeneous supersonic flow of gas, the angle a between the generatrices of the Mach cone and its axis is called the Mach angle; it is related to the Mach number by the equation sin a = 1/M

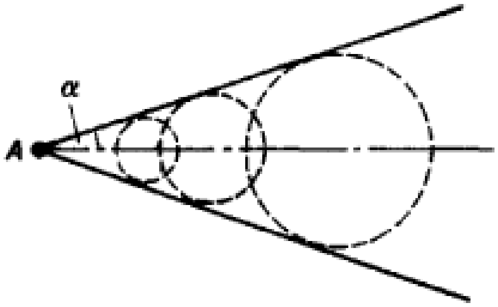
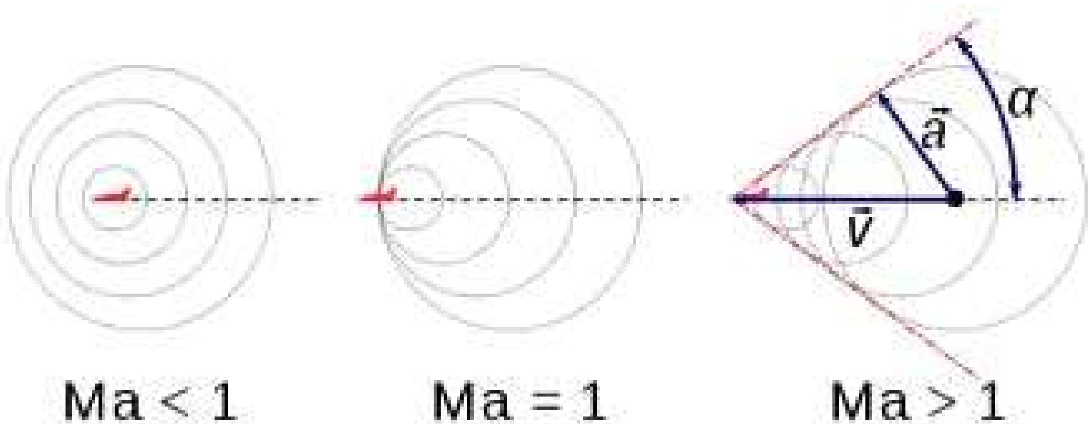


Figure 1. Mach cone that arises upon supersonic flow around an object



Mach number

In fluid mechanics, **Mach number** (**M** or **Ma**) /ˈmɑːx/ is a dimensionless quantity

representing the ratio of speed of an object moving through a fluid and the local speed of sound.[1][2]

$$\mathrm{M} = \frac{v}{v_{\mathrm{sound}}}$$

Isentropic Flow

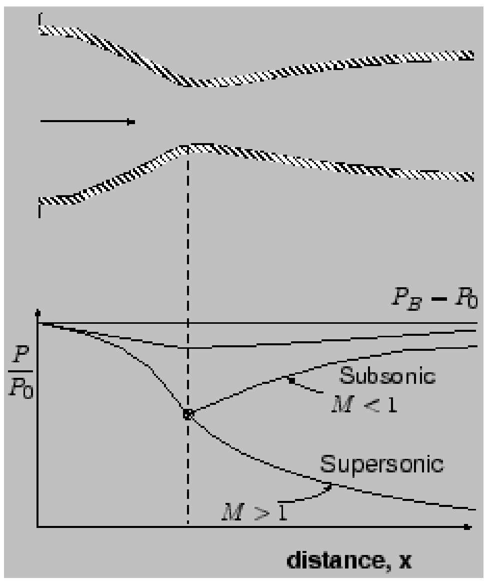


Figure 4.1: Flow thorough a converging diverging nozzle

In this chapter a discussion on a steady state flow through a smooth and continuous area flow rate is presented. A discussion about the flow through a converging-diverging nozzle is also part of this chapter. The isentropic flow models are important because of two main reasons: One, it provides the information about the trends and important parameters. Two, the correction factors can be introduced later to account for deviations from the ideal state.

Relationships for Small Mach Number

Even with today's computers a simplified method can reduce the tedious work involved in computational work. In particular, the trends can be examined with analytical methods. It further will be used in the book to examine trends in derived models. It can be noticed that the Mach number involved in the above equations is in a square power. Hence, if an acceptable error is of about %1 then $M < 0.1$ provides the desired range. Further, if a higher power is used, much smaller error results. First it can be noticed that the ratio of temperature to stagnation temperature, $\frac{T}{T_0}$ is provided in power series. Expanding of the equations according to the binomial expansion of

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \cdots \tag{4.16}$$

will result in the same fashion

$$\frac{P_0}{P} = 1 + \frac{(k-1)M^2}{4} + \frac{kM^4}{8} + \frac{2(2-k)M^6}{48} \cdots \tag{4.17}$$

$$\frac{\rho_0}{\rho} = 1 + \frac{(k-1)M^2}{4} + \frac{kM^4}{8} + \frac{2(2-k)M^6}{48} \cdots \tag{4.18}$$

The pressure difference normalized by the velocity (kinetic energy) as correction factor is

$$\frac{P_0 - P}{\rho U^2} = 1 + \overbrace{\frac{M^2}{4} + \frac{(2-k)M^4}{24} + \cdots}^{compressibility\ correction} \tag{4.19}$$

From the above equation, it can be observed that the correction factor approaches zero when $M \longrightarrow 0$ and then equation (4.19) approaches the standard equation for incompressible flow.

The definition of the star Mach is ratio of the velocity and star speed of sound at M=1.

$$M^* = \frac{U}{c^*} = \sqrt{\frac{k+1}{2}} M \left(1 - \frac{k-1}{4} M^2 + \cdots\right) \tag{4.20}$$

$$\frac{P_0 - P}{P} = \frac{kM^2}{2} \left(1 + \frac{M^2}{4} + \cdots\right) \tag{4.21}$$

$$\frac{\rho_0 - \rho}{\rho} = \frac{M^2}{2} \left(1 - \frac{kM^2}{4} + \cdots\right) \tag{4.22}$$

The normalized mass rate becomes

$$\frac{\dot{m}}{A} = \sqrt{\frac{k{P_0}^2 M^2}{RT_0}} \left(1 + \frac{k-1}{4} M^2 + \cdots\right) \tag{4.23}$$

The ratio of the area to star area is

$$\frac{A}{A^*} = \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} \left(\frac{1}{M} + \frac{k+1}{4} M + \frac{(3-k)(k+1)}{32} M^3 + \cdots\right) \tag{4.24}$$

Isentropic Converging-Diverging Flow in Cross Section

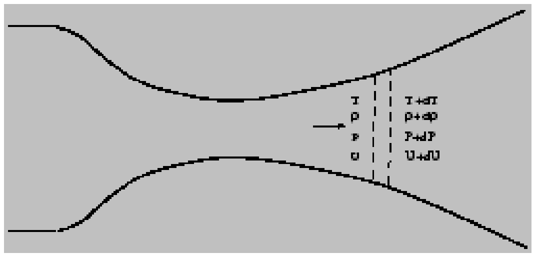


Figure: Control volume inside a converging-diverging nozzle.

The important sub case in this chapter is the flow in a converging-diverging nozzle. The control volume is shown in Figure (4.4). There are two models that assume variable area flow: First is isentropic and adiabatic model. Second is isentropic and isothermal model. Clearly, the stagnation temperature, T_0, is constant through the adiabatic flow because there isn't heat transfer. Therefore, the stagnation pressure is also constant through the flow because the flow isentropic. Conversely, in mathematical terms, equation (4.9) and equation (4.11) are the same. If the right hand side is constant for one variable, it is constant for the other. In the same argument, the stagnation density is constant through the flow. Thus, knowing the Mach number or the temperature will provide all that is needed to find the other properties. The only properties that need to be connected are the cross section area and the Mach number. Examination of the relation between properties can then be carried out.

The Properties in the Adiabatic Nozzle

When there is no external work and heat transfer, the energy equation, reads

$$dh + UdU = 0 \tag{4.25}$$

Differentiation of continuity equation, $\rho A U = \dot{m} = constant$, and dividing by the continuity equation reads

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dU}{U} = 0 \tag{4.26}$$

The thermodynamic relationship between the properties can be expressed as

$$Tds = dh - \frac{dP}{\rho} \tag{4.27}$$

For isentropic process $ds \equiv 0$ and combining equations (4.25) with (4.27) yields

$$\frac{dP}{\rho} + UdU = 0 \tag{4.28}$$

Differentiation of the equation state (perfect gas), $P = \rho RT$, and dividing the results by the equation of state (ρRT) yields

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \tag{4.29}$$

Obtaining an expression for dU/U from the mass balance equation (4.26) and using it in equation (4.28) reads

$$\frac{dP}{\rho} - U^2 \overbrace{\left[\frac{dA}{A} + \frac{d\rho}{\rho}\right]}^{\frac{dU}{U}} = 0 \tag{4.30}$$

Rearranging equation (4.30) so that the density, dρ, can be replaced by the static pressure, dP/ρ yields

$$\frac{dP}{\rho} = U^2\left(\frac{dA}{A} + \frac{d\rho}{\rho}\frac{dP}{dP}\right) = U^2\left(\frac{dA}{A} + \overbrace{\frac{d\rho}{dP}}^{\frac{1}{c^2}}\frac{dP}{\rho}\right) \tag{4.31}$$

Recalling that $dP/d\rho = c^2$ and substitute the speed of sound into equation (4.31) to obtain

$$\frac{dP}{\rho}\left[1 - \left(\frac{U}{c}\right)^2\right] = U^2\frac{dA}{A} \tag{4.32}$$

Or in a dimensionless form

$$\frac{dP}{\rho}\left(1 - M^2\right) = U^2\frac{dA}{A} \tag{4.33}$$

Equation (4.33) is a differential equation for the pressure as a function of the cross section area. It is convenient to rearrange equation (4.33) to obtain a variables separation form of

$$dP = \frac{\rho U^2}{A}\frac{dA}{1 - M^2} \tag{4.34}$$

The pressure Mach number relationship

Before going further in the mathematical derivation it is worth looking at the physical meaning of equation (4.34). The term $\rho U^2/A$ is always positive (because all the three terms can be only positive). Now, it can be observed that dP can be positive or negative depending on the dA and Mach number. The meaning of the sign change for the pressure

differential is that the pressure can increase or decrease. It can be observed that the critical Mach number is one. If the Mach number is larger than one than dP has opposite sign of dA. If Mach number is smaller than one dP and dA have the same sign. For the subsonic branch $M < 1$ the term $1/(1-M^2)$ is positive hence

$$dA > 0 \Longrightarrow dP > 0$$

$$dA < 0 \Longrightarrow dP < 0$$

From these observations the trends are similar to those in incompressible fluid. An increase in area results in an increase of the static pressure (converting the dynamic pressure to a static pressure). Conversely, if the area decreases (as a function of x) the pressure decreases. Note that the pressure decrease is larger in compressible flow compared to incompressible flow.

For the supersonic branch $M > 1$, the phenomenon is different. For $M > 1$ the term $1/1 - M^2$ is negative and change the character of the equation.

$$dA > 0 \Rightarrow dP < 0$$

$$dA < 0 \Rightarrow dP > 0$$

This behavior is opposite to incompressible flow behavior.

For the special case of M=1 (sonic flow) the value of the term $1 - M^2 = 0$ thus mathematically $dP \rightarrow \infty$ or $dA = 0$. Since physically dP can increase only in a finite amount it must that $dA = 0$. It must also be noted that when M=1 occurs only when $dA = 0$. However, the opposite, not necessarily means that when $dA = 0$ that M =1 . In that case, it is possible that $dM = 0$ thus the diverging side is in the subsonic branch and

the flow isn't choked.

The relationship between the velocity and the pressure can be observed from equation (4.28) by solving it for dU.

$$dU = -\frac{dP}{PU} \tag{4.35}$$

From equation (4.35) it is obvious that dU has an opposite sign to dP (since the term PU is positive). Hence the pressure increases when the velocity decreases and vice versa.

From the speed of sound, one can observe that the density, ρ, increases with pressure and vice versa (see equation 4.36).

$$d\rho = \frac{1}{c^2} dP \tag{4.36}$$

It can be noted that in the derivations of the above equations (4.35 - 4.36), the equation of state was not used. Thus, the equations are applicable for any gas (perfect or imperfect gas).

The second law (isentropic relationship) dictates that $ds = 0$ and from thermodynamics

$$ds = 0 = C_p \frac{dT}{T} - R\frac{dP}{P}$$

and for perfect gas

$$\frac{dT}{T} = \frac{k-1}{k} \frac{dP}{P} \tag{4.37}$$

Thus, the temperature varies according to the same way that pressure does.

The relationship between the Mach number and the temperature can be obtained by utilizing the fact that the process is assumed to be adiabatic $dT_0 = 0$. Differentiation of

equation (4.9), the relationship between the temperature and the stagnation temperature becomes

$$dT_0 = 0 = dT\left(1 + \frac{k-1}{2}M^2\right) + T(k-1)MdM \tag{4.38}$$

and simplifying equation (4.38) yields

$$\frac{dT}{T} = -\frac{(k-1)MdM}{1+\frac{k-1}{2}M^2} \tag{4.39}$$

Relationship Between the Mach Number and Cross Section Area

The equations used in the solution are energy (4.39), second law (4.37), state (4.29), mass (4.26)[4.1]. Note, equation (4.33) isn't the solution but demonstration of certain properties on the pressure.

The relationship between temperature and the cross section area can be obtained by utilizing the relationship between the pressure and temperature (4.37) and the relationship of pressure and cross section area (4.33). First stage equation (4.39) is combined with equation (4.37) and becomes

$$\frac{(k-1)}{k}\frac{dP}{P} = -\frac{(k-1)MdM}{1+\frac{k-1}{2}M^2} \tag{4.40}$$

Combining equation (4.40) with equation (4.33) yields

$$\frac{1}{k}\frac{\frac{\rho U^2}{A}\frac{dA}{1-M^2}}{P} = -\frac{MdM}{1+\frac{k-1}{2}M^2} \tag{4.41}$$

The following identify, $\rho U^2 = kMP$ can be proved as

$$kM^2P = k\overbrace{\frac{U^2}{c^2}}^{M^2}\overbrace{\rho RT}^{P} = k\frac{U^2}{kRT}\overbrace{\rho RT}^{P} = \rho U^2 \tag{4.42}$$

Using the identity in equation (4.42) changes equation (4.41) into

$$\frac{dA}{A} = \frac{M^2 - 1}{M\left(1 + \frac{k-1}{2}M^2\right)}dM \tag{4.43}$$

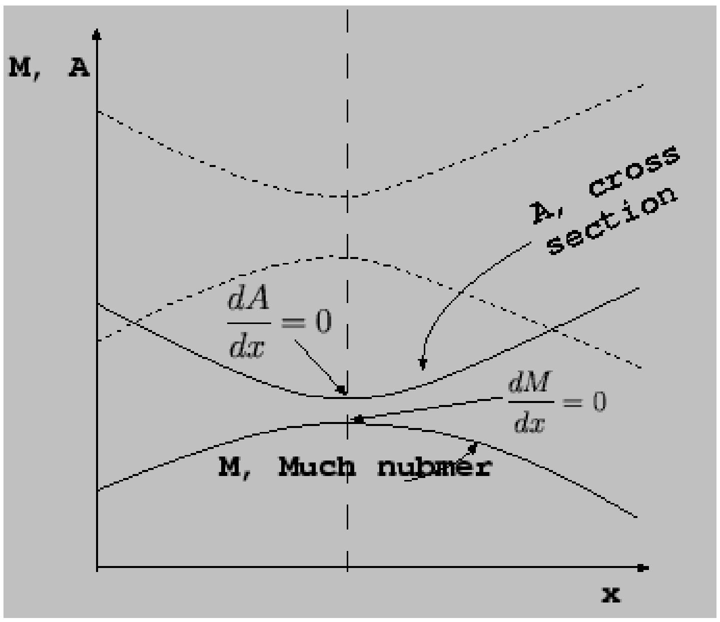


Figure: The relationship between the cross section and the Mach number on the subsonic branch

Equation (4.43) is very important because it relates the geometry (area) with the relative velocity (Mach number). In equation (4.43), the factors $M\left(1+\frac{k-1}{2}M^2\right)$ and A are positive regardless of the values of M or A. Therefore, the only factor that affects relationship between the cross area and the Mach number is M^2-1. For $M<1$ the Mach number is varied opposite to the cross section area. In the case of $M>1$ the Mach number increases with the cross section area and vice versa. The special case is when M= 1 which requires that $dA=0$. This condition imposes that internal[4.2] flow has to pass a converting-diverging device to obtain supersonic velocity. This minimum area is referred to as ``throat."

Again, the opposite conclusion that when $dA=0$ implies that M=1 is not correct because possibility of $dM=0$. In subsonic flow branch, from the mathematical point of view: on one hand, a decrease of the cross section increases the velocity and the Mach number, on the other hand, an increase of the cross section decreases the velocity and Mach number (see Figure (4.5)).

Isentropic Flow Examples

Air is allowed to flow from a reservoir with temperature of $21°C$ and with pressure of 5[MPa] through a tube. It was measured that air mass flow rate is 1[kg/sec]. At some point on the tube static pressure was measured to be 3[MPa]. Assume that process is isentropic and neglect the velocity at the reservoir, calculate the Mach number, velocity, and the cross section area at that point where the static pressure was measured. Assume that the ratio of specific heat is $k = C_p/C_v =$ 1.4.

Solution

The stagnation conditions at the reservoir will be maintained through out tube because the process is isentropic. Hence the stagnation temperature can be written $T_0 = constant$ and $P_0 = constant$ and both of them are known (the condition at the reservoir). For the point where the static pressure is known, the Mach number can be calculated utilizing the pressure ratio. With known Mach number, the temperature, and velocity can be calculated. Finally, the cross section can be calculated with all these information.

In the point where the static pressure is known

$$\bar{P} = \frac{P}{P_0} = \frac{3[MPa]}{5[MPa]} = 0.6$$

From Table (4.2) or from Figure (4.3) or by utilizing the enclosed program, Potto-GDC, or simply by using the equations that

Isentropic Flow		**Input: Pbar**			**k = 1.4**	
M	**T/T0**	**ρ/ρ0**	**A/A***	**P/P0**	**PAR**	**F/F***
0.886393	0.864201	0.694283	1.01155	0.6	0.606928	0.531054

With these values the static temperature and the density can be calculated.

$$T = 0.86420338 \times (273 + 21) = 254.076K$$

$$\rho = \frac{\rho}{\rho_0} \overbrace{\frac{P_0}{RT_0}}^{\rho_0} = 0.69428839 \times \frac{5 \times 10^6 [Pa]}{287.0 \left[\frac{J}{kgK}\right] \times 294 [K]}$$

$$= 41.1416 \left[\frac{kg}{m^3}\right]$$

The velocity at that point is

$$U = M \overbrace{\sqrt{kRT}}^{c} = 0.88638317 \times \sqrt{1.4 \times 287 \times 294} = 304 [m/sec]$$

The tube area can be obtained from the mass conservation as

The Mach number at point A on tube is measured to be $M = 2$[1] and the static pressure is $2[Bar]$[2]. Downstream at point B the pressure was measured to be 1.5[Bar]. Calculate the Mach number at point B under the isentropic flow assumption. Also, estimate the temperature at point B. Assume that the specific heat ratio $k = 1.4$ and assume a perfect gas model. 4.3 4.4

Solution

With known Mach number at point A all the ratios of the static properties to total (stagnation) properties can be calculated. Therefore, the stagnation pressure at point A is known and stagnation temperature can be calculated.

At $M = 2$ (supersonic flow) the ratios are

Isentropic Flow		Input: M			k = 1.4	
M	T/T0	ρ/ρ0	A/A*	P/P0	PAR	F/F*
2	0.555556	0.230048	1.6875	0.127805	0.21567	0.593093

With this information the pressure at Point B expressed

$$\frac{P_A}{P_0} = \overbrace{\frac{P_B}{P_0}}^{\substack{\text{from the table} \\ 4.2\ @\ M=2}} \times \frac{P_A}{P_B} = 0.12780453 \times \frac{2.0}{1.5} = 0.17040604$$

The corresponding Mach number for this pressure ratio is 1.8137788 and $T_B = 0.60315132$

$\frac{P_B}{P_0} = 0.17040879$. The stagnation temperature can be ``bypassed'' to calculated the temperature at point **B**

$$T_B = T_A \times \overbrace{\frac{T_0}{T_A}}^{M=2} \times \overbrace{\frac{T_B}{T_0}}^{M=1.81..} = 250[K] \times \frac{1}{0.55555556} \times 0.60315132 \simeq 271.42[K]$$

Gas flows through a converging–diverging duct. At point "A" the cross section area is 50 $[cm^2]$ and the Mach number was measured to be 0.4. At point B in the duct the cross section area is 40 $[cm^2]$. Find the Mach number at point B. Assume that the flow is isentropic and the gas specific heat ratio is 1.4.

Solution

To obtain the Mach number at point B by finding the ratio of the area to the critical area. This relationship can be obtained by

$$\frac{A_B}{A*} = \frac{A_B}{A_A} \times \frac{A_A}{A^*} = \frac{40}{50} \times \overbrace{1.59014}^{\text{from the Table (4.2)}} = 1.272112$$

With the value of $\frac{A_B}{A*}$ from the Table (4.2) or from Potto-GDC two solutions can be obtained. The two possible solutions: the first supersonic M = 1.6265306 and second subsonic M = 0.53884934. Both solution are possible and acceptable. The supersonic branch solution is possible only if there where a transition at throat where M=1.

Isentropic Flow		**Input: A/A***			**k = 1.4**	
M	**T/T0**	**ρ/ρ0**	**A/A***	**P/P0**	**PAR**	**F/F***
0.538865	0.945112	0.868378	1.27211	0.820715	1.04404	0.611863
1.62655	0.653965	0.345848	1.27211	0.226172	0.287717	0.563918

Mass Flow Rate (Number)

One of the important engineering parameters is the mass flow rate which for ideal gas is

$$\dot{m} = \rho U A = \frac{P}{RT} U A \tag{4.44}$$

This parameter is studied here, to examine the maximum flow rate and to see what is the effect of the compressibility on the flow rate. The area ratio as a function of the Mach number needed to be established, specifically and explicitly the relationship for the chocked flow. The area ratio is defined as the ratio of the cross section at any point to the throat area (the narrow area). It is convenient to rearrange the equation (4.44) to be expressed in terms of the stagnation properties as

$$\frac{\dot{m}}{A} = \frac{P}{P_0} \frac{P_0 U}{\sqrt{kRT}} \sqrt{\frac{k}{R}} \sqrt{\frac{T_0}{T}} \frac{1}{\sqrt{T_0}} = \frac{P_0}{\sqrt{T_0}} M \sqrt{\frac{k}{R}} \overbrace{\frac{P}{P_0} \sqrt{\frac{T_0}{T}}}^{f(M,k)} \tag{4.45}$$

Expressing the temperature in terms of Mach number in equation (4.45) results in

$$\frac{\dot{m}}{A} = \left(\frac{k M P_0}{\sqrt{kRT_0}} \right) \left(1 + \frac{k-1}{2} M^2 \right)^{-\frac{k+1}{2(k-1)}} \tag{4.46}$$

It can be noted that equation (4.46) holds everywhere in the converging-diverging duct and this statement also true for the throat. The throat area can be denoted as by A^*. It can be noticed that at the throat when the flow is chocked or in other words M=1 and that the stagnation conditions (i.e. temperature, pressure) do not change. Hence equation (4.46) obtained the form

$$\frac{\dot{m}}{A^*} = \left(\frac{\sqrt{k}P_0}{\sqrt{RT_0}}\right)\left(1 + \frac{k-1}{2}\right)^{-\frac{k+1}{2(k-1)}} \tag{4.47}$$

Since the mass flow rate is constant in the duct, dividing equations (4.47) by equation (4.46) yields

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2(k-1)}} \tag{4.48}$$

Equation (4.48) relates the Mach number at any point to the cross section area ratio.

The maximum flow rate can be expressed either by taking the derivative of equation (4.47) in with respect to M and equating to zero. Carrying this calculation results at M=1 .

$$\left(\frac{\dot{m}}{A^*}\right)_{max}\frac{P_0}{\sqrt{T_0}} = \sqrt{\frac{k}{R}}\left(\frac{k+1}{2}\right)^{-\frac{k+1}{2(k-1)}} \tag{4.49}$$

For specific heat ratio, $k = 1.4$

$$\left(\frac{\dot{m}}{A^*}\right)_{max}\frac{P_0}{\sqrt{T_0}} \sim \frac{0.68473}{\sqrt{R}} \tag{4.50}$$

The maximum flow rate for air ($R = 287j/kgK$) becomes,

$$\frac{\dot{m}\sqrt{T_0}}{A^* P_0} = 0.040418 \tag{4.51}$$

Equation (4.51) is known as Fliegner's Formula on the name of one of the first engineers who observed experimentally the choking phenomenon. It can be noticed that Fliengner's equation can lead to definition of the Fliengner's Number.

$$\frac{\dot{m}\sqrt{T_0}}{A^* P_0} = \frac{\dot{m}\overbrace{\sqrt{kRT_0}}^{c_0}}{\sqrt{kR}A^* P_0} = \overbrace{\frac{\dot{m}c_0}{\sqrt{R}A^* P_0}}^{Fn}\frac{1}{\sqrt{k}} \tag{4.52}$$

The definition of Fliengner's number (Fn) is

$$Fn \equiv \frac{\dot{m}c_0}{\sqrt{R}A^* P_0} \tag{4.53}$$

Utilizing Fliengner's number definition and substituting it into equation (4.47) results in

$$Fn = kM\left(1 + \frac{k-1}{2}M^2\right)^{-\frac{k+1}{2(k-1)}} \tag{4.54}$$

and the maximum point for Fn at M=1 is

$$Fn = k\left(\frac{k+1}{2}\right)^{-\frac{k+1}{2(k-1)}} \tag{4.55}$$

General Relationship

In this section, the other extreme case model where the heat transfer to the gas is perfect, (e.g. Eckert number is very small) is presented. Again in reality the heat transfer is somewhere in between the two extremes. So, knowing the two limits provides a tool to examine where the reality should be expected. The perfect gas model is again assumed (later more complex models can be assumed and constructed in a future versions). In isothermal process the perfect gas model reads

$$P = \rho RT \leadsto dP = d\rho RT \tag{4.76}$$

Substituting equation (4.76) into the momentum equation[4.6] yields

$$UdU + \frac{RTdP}{P} = 0 \tag{4.77}$$

Integration of equation (4.77) yields the Bernoulli's equation for ideal gas in isothermal process which reads

$$\leadsto \frac{{U_2}^2 - {U_1}^2}{2} + RT\ln\frac{P_2}{P_1} = 0 \tag{4.78}$$

Thus, the velocity at point 2 becomes

$$U_2 = \sqrt{2RT\ln\frac{P_2}{P_1} - {U_1}^2} \tag{4.79}$$

The velocity at point 2 for stagnation point, $U_1 \approx 0$ reads

$$U_2 = \sqrt{2RT \ln \frac{P_2}{P_1}} \tag{4.80}$$

Or in explicit terms of the stagnation properties the velocity is

$$U = \sqrt{2RT \ln \frac{P}{P_0}} \tag{4.81}$$

Transform from equation (4.78) to a dimensionless form becomes

$$\leadsto \frac{kRconstantT(M_2{}^2 - M_1{}^2)}{2} = RconstantT \ln \frac{P_2}{P_1} \tag{4.82}$$

Simplifying equation (4.82) yields

$$\leadsto \frac{k(M_2{}^2 - M_1{}^2)}{2} = \ln \frac{P_2}{P_1} \tag{4.83}$$

Or in terms of the pressure ratio equation (4.83) reads

$$\frac{P_2}{P_1} = e^{\frac{k(M_1{}^2 - M_2{}^2)}{2}} = \left(\frac{e^{M_1{}^2}}{e^{M_2{}^2}}\right)^{\frac{k}{2}} \tag{4.84}$$

As oppose to the adiabatic case ($T_0 = constant$) in the isothermal flow the stagnation temperature ratio can be expressed

$$\frac{T_{01}}{T_{02}} = 1\frac{T_1}{T_2}\frac{\left(1+\frac{k-1}{2}{M_1}^2\right)}{\left(1+\frac{k-1}{2}{M_2}^2\right)} = \frac{\left(1+\frac{k-1}{2}{M_1}^2\right)}{\left(1+\frac{k-1}{2}{M_2}^2\right)} \tag{4.85}$$

Utilizing conservation of the mass $A\rho M = constant$ to yield

$$\frac{A_1}{A_2} = \frac{M_2 P_2}{M_1 P_1} \tag{4.86}$$

Combining equation (4.86) and equation (4.84) yields

$$\frac{A_2}{A_1} = \frac{M_1}{M_2}\left(\frac{e^{{M_2}^2}}{e^{{M_1}^2}}\right)^{\frac{k}{2}} \tag{4.87}$$

The change in the stagnation pressure can be expressed as

$$\frac{P_{02}}{P_{01}} = \frac{P_2}{P_1}\left(\frac{1+\frac{k-1}{2}{M_2}^2}{1+\frac{k-1}{2}{M_1}^2}\right)^{\frac{k}{k-1}} = \left[\frac{e^{{M_1}^2}}{e^{{M_1}^2}}\right]^{\frac{k}{2}} \tag{4.88}$$

The critical point, at this stage, is unknown (at what Mach number the nozzle is choked is unknown) so there are two possibilities: the choking point or M=1 to normalize the equation. Here the critical point defined as the point where M=1 so results can be compared to the adiabatic case and denoted by star. Again it has to emphasis that this

critical point is not really related to physical critical point but it is arbitrary definition. The true critical point is when flow is choked and the relationship between two will be presented.

The critical pressure ratio can be obtained from (4.84) to read

$$\frac{P}{P^*} = \frac{\rho}{\rho^*} = e^{\frac{(1-M^2)k}{2}} \tag{4.89}$$

Equation (4.87) is reduced to obtained the critical area ratio writes

$$\frac{A}{A^*} = \frac{1}{M} e^{\frac{(1-M^2)k}{2}} \tag{4.90}$$

Similarly the stagnation temperature reads

$$\frac{T_0}{{T_0}^*} = \frac{2\left(1+\frac{k-1}{2}{M_1}^2\right)^{\frac{k}{k-1}}}{k+1} \tag{4.91}$$

Finally, the critical stagnation pressure reads

$$\frac{P_0}{{P_0}^*} = e^{\frac{(1-M^2)k}{2}} \left(\frac{2\left(1+\frac{k-1}{2}{M_1}^2\right)}{k+1}\right)^{\frac{k}{k-1}} \tag{4.92}$$

The maximum value of stagnation pressure ratio is obtained when $M = 0$ at which is

$$\left.\frac{P_0}{{P_0}^*}\right|_{M=0} = e^{\frac{k}{2}} \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \tag{4.93}$$

For specific heat ratio of $k = 1.4$, this maximum value is about two. It can be noted that the stagnation pressure is monotonically reduced during this process.

Of course in isothermal process $T = T^*$. All these equations are plotted in Figure (4.6).

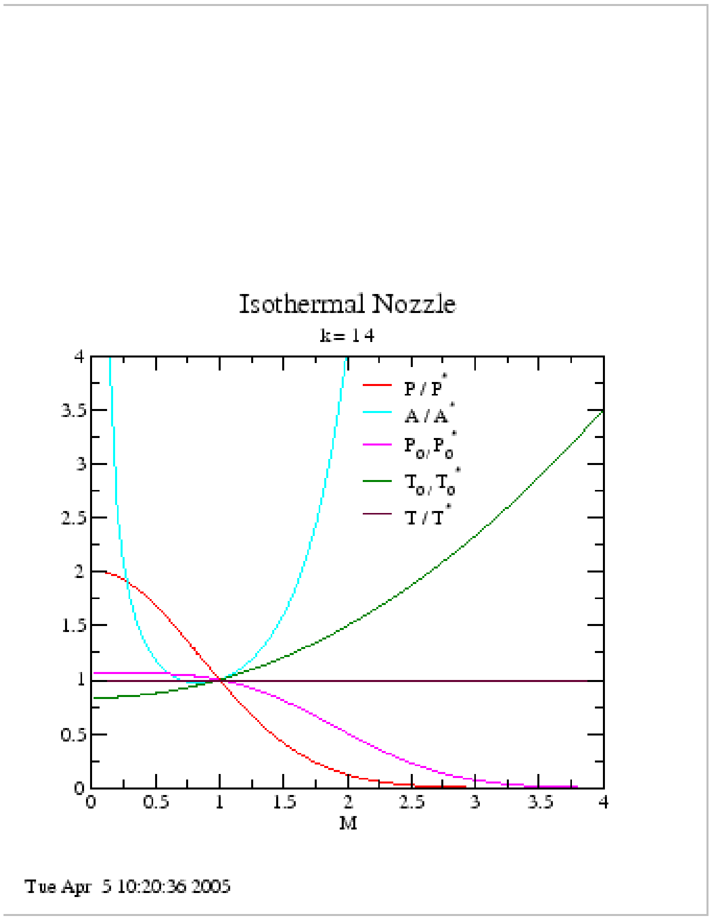


Figure 4.6: Various ratios as a function of Mach number for isothermal Nozzle

From the Figure 4.3 it can be observed that minimum of the curve A/A^* isn't on M=1 .

The minimum of the curve is when area is minimum and at the point where the flow is choked. It should be noted that the stagnation temperature is not constant as in the adiabatic case and the critical point is the only one constant.

The mathematical procedure to find the minimum is simply taking the derivative and equating to zero as following

$$\frac{d\left(\frac{A}{A^*}\right)}{dM} = \frac{kM^2 e^{\frac{k(M^2-1)}{2}} - e^{\frac{k(M^2-1)}{2}}}{M^2} = 0 \tag{4.94}$$

Equation (4.94) simplified to

$$kM^2 - 1 = 0 \leadsto M = \frac{1}{\sqrt{k}} \tag{4.95}$$

It can be noticed that a similar results are obtained for adiabatic flow. The velocity at the throat of isothermal model is smaller by a factor of $\sqrt{k}$. Thus, dividing the critical adiabatic velocity by $\sqrt{k}$ results in

$$U_{throat_{max}} = \sqrt{RT} \tag{4.96}$$

On the other hand, the pressure loss in adiabatic flow is milder as can be seen in Figure (4.7(a)).

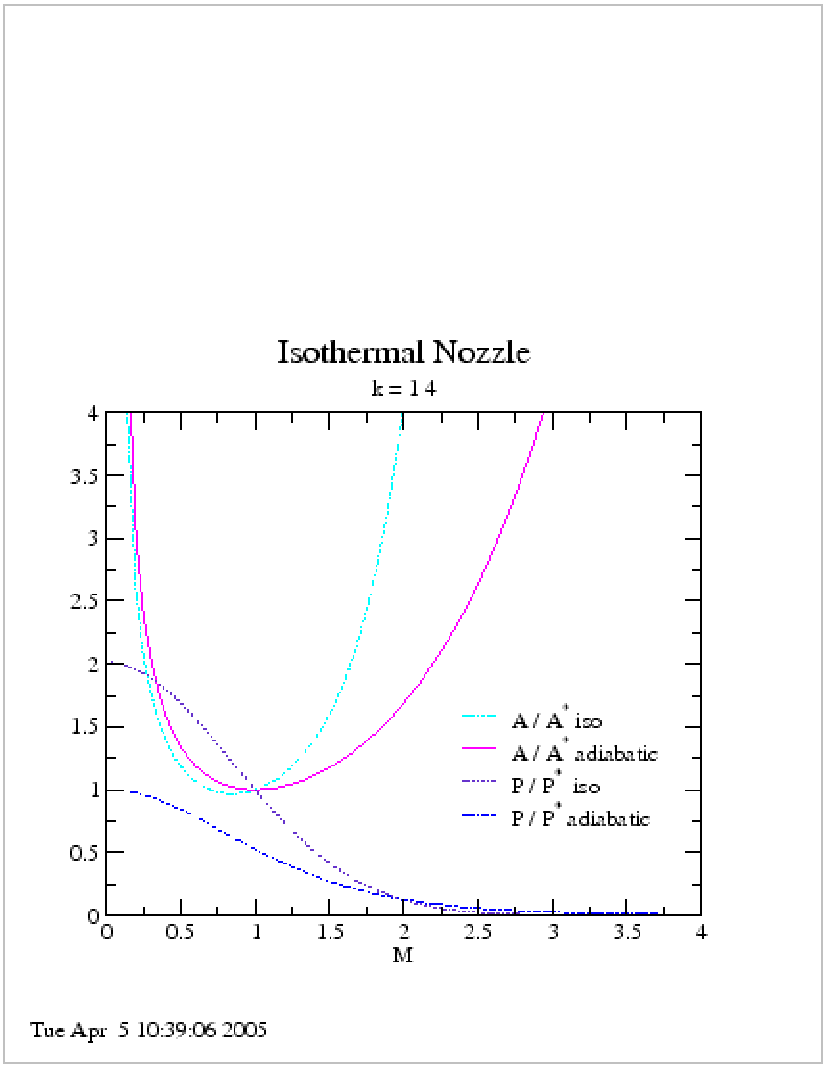

Isothermal Nozzle
k = 1.4
A / A* iso
A / A* adiabatic
P / P* iso
P / P* adiabatic
M
0
0.5
1
1.5
2
2.5
3
3.5
4
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Figure: Comparison between the isothermal nozzle and adiabatic nozzle in various variables

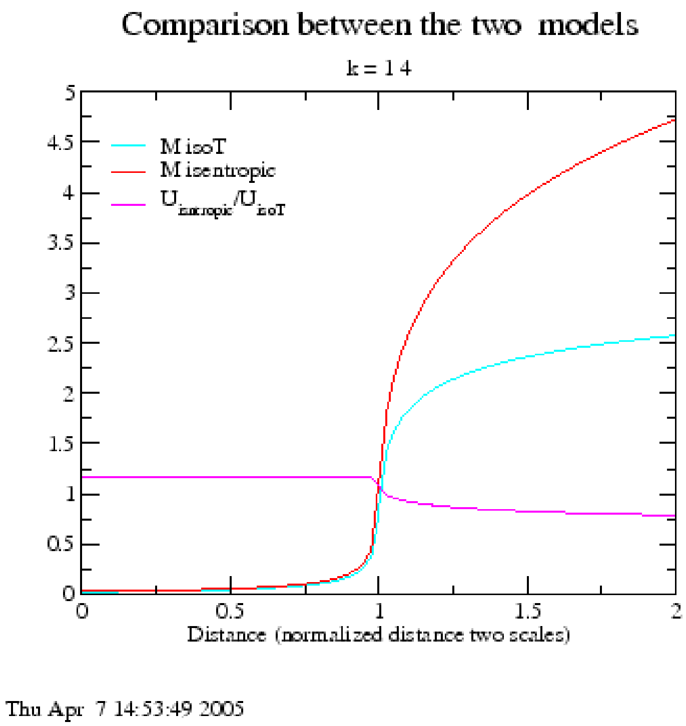


Figure: The comparison of the adiabatic model and isothermal model

It should be emphasized that the stagnation pressure decrees. It is convenient to find expression for the ratio of the initial stagnation pressure (the stagnation pressure before entering the nozzle) to the pressure at the throat. Utilizing equation (4.89) the following relationship can be obtained

$$\frac{P_{throat}}{P_{0_{initial}}} = \frac{P^*}{P_{0_{initial}}} \frac{P_{throat}}{P^*} =$$

$$\frac{1}{e^{\frac{(1-0^2)k}{2}}} e^{\left(1-\left(\frac{1}{\sqrt{k}}\right)^2\right)\frac{k}{2}} =$$

$$e^{-} = 0.60653 \tag{4.97}$$

Notice that the critical pressure is independent of the specific heat ratio, k, as opposed to the adiabatic case. It also has to be emphasized that the stagnation values of the isothermal model are not constant. Again, the heat transfer is expressed as

$$Q = C_p \left(T_{0_2} - T_{0_2}\right) \tag{4.98}$$

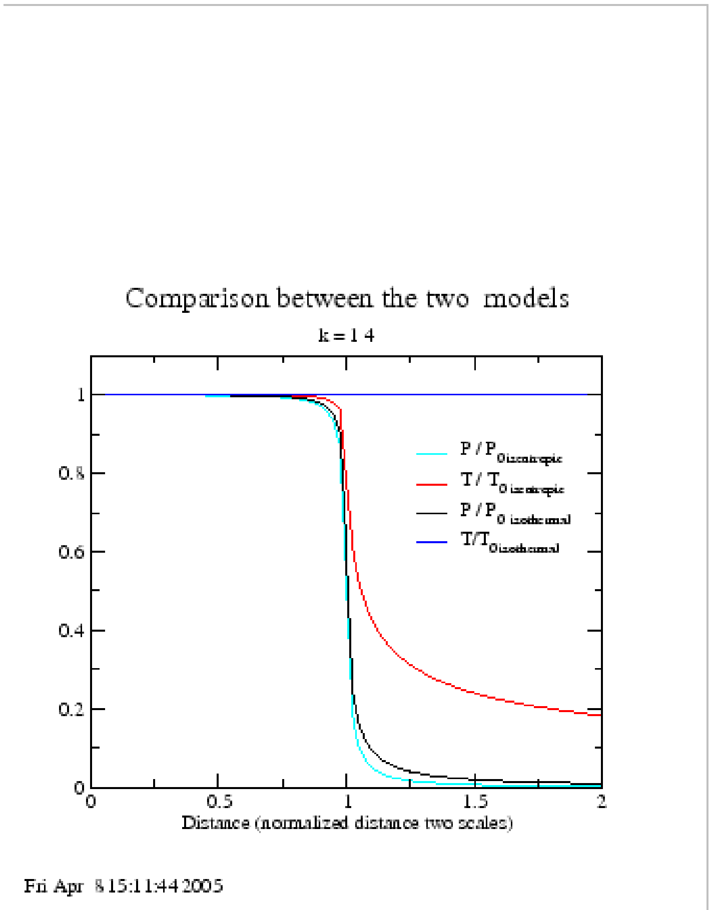


Figure: Comparison of the pressure and temperature drop as a function of the normalized length (two scales)

For comparison between the adiabatic model and the isothermal a simple profile of nozzle area as a function of the distance is assumed. This profile isn't an ideal profile but rather a simple sample just to examine the difference between the two models so in an

actual situation it can be bounded. To make sense and eliminate unnecessary details the distance from the entrance to the throat is normalized (to one (1)). In the same fashion the distance from the throat to the exit is normalized (to one (1)) (it doesn't mean that these distances are the same). In this comparison the entrance area ratio and the exit area ratio are the same and equal to 20. The Mach number was computed for the two models and plotted in Figure (4.7(b)). In this comparison it has to be remembered that critical area for the two models are different by about 3% (for $k = 1.4$). As can be observed from Figure (4.7(b)). The Mach number for the isentropic is larger for the supersonic branch but the velocity is lower. The ratio of the velocities can be expressed as

$$\frac{U_s}{U_T} = \frac{M_s\sqrt{kRT_s}}{M_T\sqrt{kRT_s}} \tag{4.99}$$

It can be noticed that temperature in the isothermal model is constant while temperature in the adiabatic model can be expressed as a function of the stagnation temperature. The initial stagnation temperatures are almost the same and can be canceled out to obtain

$$\frac{U_s}{U_T} \sim \frac{M_s}{M_T\sqrt{1+\frac{k-1}{2}{M_s}^2}} \tag{4.100}$$

By utilizing equation (4.100) the velocity ratio was obtained and is plotted in Figure (4.7(b)).

Thus, using the isentropic model results in under prediction of the actual results for the velocity in the supersonic branch. While, the isentropic for the subsonic branch will be over prediction. The prediction of the Mach number are similarly shown in Figure (4.7(b)).

Two other ratios need to be examined: temperature and pressure. The initial stagnation temperature is denoted as T_{0int}. The temperature ratio of T/T_{0int} can be obtained via the

isentropic model as

$$\frac{T}{T_{0int}} = \frac{1}{1 + \frac{k-1}{2} M^2} \tag{4.101}$$

While the temperature ratio of the isothermal model is constant and equal to one (1). The pressure ratio for the isentropic model is

$$\frac{P}{P_{0int}} = \frac{1}{\left(1 + \frac{k-1}{2} M^2\right)^{\frac{k-1}{k}}} \tag{4.102}$$

and for the isothermal process the stagnation pressure varies and has to be taken into account as the following:

$$\frac{P_z}{P_{0int}} = \frac{P_0{}^*}{P_{0int}} \frac{P_{0z}}{P_0{}^*} \overbrace{\frac{P_z}{P_{0z}}}^{isentropic} \tag{4.103}$$

where z is an arbitrary point on the nozzle. Using equations (4.88) and the isentropic relationship, the sought ratio is provided.

Figure (4.8) shows that the range between the predicted temperatures of the two models is very large, while the range between the predicted pressure by the two models is relatively small. The meaning of this analysis is that transferred heat affects the temperature to a larger degree but the effect on the pressure is much less significant.

The Impulse Function

Impulse in Isentropic Adiabatic Nozzle

One of the functions that is used in calculating the forces is the Impulse function. The

Impulse function is denoted here as $\underline{F}$, but in the literature some denote this function as $\underline{I}$. To explain the motivation for using this definition consider the calculation of the net forces that acting on section shown in Figure (4.9). To calculate the net forces acting in the x-direction the momentum equation has to be applied

$$F_{net} = \dot{m}(U_2 - U_1) + P_2 A_2 - P_1 A_1 \tag{4.104}$$

The net force is denoted here as F_{net}. The mass conservation also can be applied to our control volume

$$\dot{m} = \rho_1 A_1 U_1 = \rho_2 A_2 U_2 \tag{4.105}$$

Combining equation (4.104) with equation (4.105) and by utilizing the identity in equation (4.42) results in

$$\underline{F_{net} = kP_2 A_2 {M_2}^2 - kP_1 A_1 {M_1}^2 - P_2 A_2 - P_1 A_1} \tag{4.106}$$

Rearranging equation (4.106) and dividing it by $P_0 A^*$ results in

$$\frac{F_{net}}{P_0 A^*} = \overbrace{\frac{P_2 A_2}{P_0 A^*}}^{f(M_2)} \overbrace{\left(1 + k{M_2}^2\right)}^{f(M_2)} - \overbrace{\frac{P_1 A_1}{P_0 A^*}}^{f(M_1)} \overbrace{\left(1 + k{M_1}^2\right)}^{f(M_1)} \tag{4.107}$$

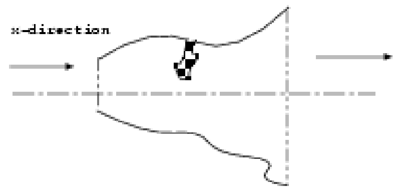


Figure 4.9: Schematic to explain the significances of the Impulse function

Examining equation ([4.107]) shows that the right hand side is only a function of Mach number and specific heat ratio, k. Hence, if the right hand side is only a function of the Mach number and k than the left hand side must be function of only the same parameters, M and k. Defining a function that depends only on the Mach number creates the convenience for calculating the net forces acting on any device. Thus, defining the Impulse function as

$$F = PA\left(1 + kM_2{}^2\right) \tag{4.108}$$

In the Impulse function when F (M=1) is denoted as F^*

$$F^* = P^* A^* \left(1 + k\right) \tag{4.109}$$

The ratio of the Impulse function is defined as

$$\frac{F}{F^*} = \frac{P_1 A_1}{P^* A^*} \frac{\left(1 + kM_1{}^2\right)}{\left(1+k\right)} = \frac{1}{\underbrace{\frac{P^*}{P_0}}_{\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}}} \overbrace{\frac{P_1 A_1}{P_0 A^*}\left(1 + kM_1{}^2\right)}^{\text{see function (4.107)}} \frac{1}{\left(1+k\right)} \tag{4.110}$$

This ratio is different only in a coefficient from the ratio defined in equation (4.107) which makes the ratio a function of k and the Mach number. Hence, the net force is

$$F_{net} = P_0 A^* (1+k) \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} \left(\frac{F_2}{F^*} - \frac{F_1}{F^*}\right) \tag{4.111}$$

To demonstrate the usefulness of the this function consider a simple situation of the flow through a converging nozzle

Figure: Schematic of a flow of a compressible substance (gas) thorough a converging nozzle for example (4.7)

The Impulse Function in Isothermal Nozzle

Previously Impulse function was developed in the isentropic adiabatic flow. The same is done here for the isothermal nozzle flow model. As previously, the definition of the Impulse function is reused. The ratio of the impulse function for two points on the nozzle is

$$\frac{F_2}{F_1} = \frac{P_2 A_2 + \rho_2 {U_2}^2 A_2}{P_1 A_1 + \rho_1 {U_1}^2 A_1} \tag{4.112}$$

Utilizing the ideal gas model for density and some rearrangement results in

$$\frac{F_2}{F_1} = \frac{P_2 A_2}{P_1 A_1} \frac{1 + \frac{{U_2}^2}{RT}}{1 + \frac{{U_1}^2}{RT}} \tag{4.113}$$

Since $U^2/RT = kM^2$ and the ratio of equation (4.86) transformed equation into (4.113)

$$\frac{F_2}{F_1} = \frac{M_1}{M_2}\frac{1+kM_2{}^2}{1+kM_1{}^2} \tag{4.114}$$

At the star condition (M=1) (not the minimum point) results in

$$\frac{F_2}{F*} = \frac{1}{M_2}\frac{1+kM_2{}^2}{1+k} \tag{4.115}$$

The effects of Real Gases

To obtained expressions for non-ideal gas it is communally done by reusing the ideal gas model and introducing a new variable which is a function of the gas properties like the critical pressure and critical temperature. Thus, a real gas equation can be expressed in equation (3.19). Differentiating equation (3.19) and dividing by equation (3.19) yields

$$\frac{dP}{P} = \frac{dz}{z} + \frac{d\rho}{\rho} + \frac{dT}{T} \tag{4.116}$$

Again, Gibb's equation (4.27) is reused to related the entropy change to the change in thermodynamics properties and applied on non-ideal gas. Since $ds = 0$ and utilizing the equation of the state $dh = dP/\rho$. The enthalpy is a function of the temperature and pressure thus, $h = h(T,P)$ and full differential is

$$dh = \left(\frac{\partial h}{\partial T}\right)_P dT + \left(\frac{\partial h}{\partial P}\right)_T dP \tag{4.117}$$

The definition of pressure specific heat is $C_p \equiv \frac{\partial h}{\partial T}$ and second derivative is Maxwell relation hence,

$$\left(\frac{\partial h}{\partial P}\right)_T = v - T\left(\frac{\partial s}{\partial T}\right)_P \tag{4.118}$$

First, the differential of enthalpy is calculated for real gas equation of state as

$$dh = C_p dT - \left(\frac{T}{Z}\right)\left(\frac{\partial z}{\partial T}\right)_P \frac{dP}{\rho} \tag{4.119}$$

Equations (4.27) and (3.19) are combined to form

$$\frac{ds}{R} = \frac{C_p}{R}\frac{dT}{T} - z\left[1 + \left(\frac{T}{Z}\right)\left(\frac{\partial z}{\partial T}\right)_P\right]\frac{dP}{P} \tag{4.120}$$

The mechanical energy equation can be expressed as

$$\int d\left(\frac{U^2}{2}\right) = -\int \frac{dP}{\rho} \tag{4.121}$$

At the stagnation the definition requires that the velocity is zero. To carry the integration of the right hand side the relationship between the pressure and the density has to be defined. The following power relationship is assumed

$$\frac{\rho}{\rho_0} = \left(\frac{P}{P_0}\right)^{\frac{1}{n}} \tag{4.122}$$

Notice, that for perfect gas the n is substituted by k. With integration of equation (4.121) when using relationship which is defined in equation (4.122) results

$$\frac{U^2}{2} = \int_{P_0}^{P_1} \frac{dP}{\rho} = \int_{P_0}^{P} \frac{1}{\rho_0} \left(\frac{P_0}{P}\right)^{\frac{1}{n}} dP \tag{4.123}$$

Substituting relation for stagnation density (3.19) results

$$\frac{U^2}{2} = \int_{P_0}^{P} \frac{z_0 R T_0}{P_0} \left(\frac{P_0}{P}\right)^{\frac{1}{n}} dP \tag{4.124}$$

For $n > 1$ the integration results in

$$U = \sqrt{z_0 R T_0 \frac{2n}{n-1} \left[1 - \left(\frac{P}{P_0}\right)^{\left(\frac{n-1}{n}\right)}\right]} \tag{4.125}$$

For $n = 1$ the integration becomes

$$U = \sqrt{2 z_0 R T_0 \ln\left(\frac{P_0}{P}\right)} \tag{4.126}$$

It must be noted that n is a function of the critical temperature and critical pressure. The mass flow rate is regardless to equation of state as following

$$\dot{m} = \rho^* A^* U^* \tag{4.127}$$

Where ρ^* is the density at the throat (assuming the chocking condition) and A^* is the cross area of the throat. Thus, the mass flow rate in our properties

$$\dot{m} = A^* \overbrace{\frac{P_0}{z_0 R T_0} \left(\frac{P}{P_0}\right)^{\frac{1}{n}}}^{\rho^*} \overbrace{\sqrt{z_0 R T_0 \frac{2n}{n-1} \left[1 - \left(\frac{P}{P_0}\right)^{\left(\frac{n-1}{n}\right)}\right]}}^{U^*} \tag{4.128}$$

For the case of $n = 1$

$$\dot{m} = A^* \overbrace{\frac{P_0}{z_0 R T_0} \left(\frac{P}{P_0}\right)^{\frac{1}{n}}}^{\rho^*} \overbrace{\sqrt{2 z_0 R T_0 \ln\left(\frac{P_0}{P}\right)}}^{U**} \tag{4.129}$$

The Mach number can be obtained by utilizing equation (3.34) to defined the Mach number as

$$M = \frac{U}{\sqrt{znRT}} \tag{4.130}$$

Integrating equation (4.120) when $ds = 0$ results

$$\int_{T_1}^{T_2} \frac{C_p}{R} \frac{dT}{T} = \int_{P_1}^{P_2} z \left(1 + \left(\frac{T}{Z}\right) \left(\frac{\partial z}{\partial T}\right)_P \frac{dP}{P}\right) \tag{4.131}$$

To carryout the integration of equation (4.131) looks at Bernnolli's equation which is

$$\int \frac{dU^2}{2} = - \int \frac{dP}{\rho} \tag{4.132}$$

After integration of the velocity

$$\frac{dU^2}{2} = - \int_1^{P/P_0} \frac{\rho_0}{\rho} d\left(\frac{P}{P_0}\right) \tag{4.133}$$

It was shown in Chapter (3) that (3.33) is applicable for some ranges of relative temperature and pressure (relative to critical temperature and pressure and not the stagnation conditions).

$$U = \sqrt{z_0 R T_0 \left(\frac{2n}{n-1}\right) \left[1 - \left(\frac{P}{P_0}\right)^{\frac{n-1}{n}}\right]} \tag{4.134}$$

When $n = 1$ or when $n \to 1$

$$U = \sqrt{2 z_0 R T_0 \ln\left(\frac{P_0}{P}\right)} \tag{4.135}$$

The mass flow rate for the real gas $\dot{m} = \rho^* U^* A^*$

$$\dot{m} = \frac{A^* P_0}{\sqrt{z_0 R T_0}} \sqrt{\frac{2n}{n-1}} \left(\frac{P^*}{P_0}\right)^{\frac{1}{n}} \left[1 - \frac{P^*}{P_0}\right] \tag{4.136}$$

And for $n = 1$

$$\dot{m} = \frac{A^* P_0}{\sqrt{z_0 R T_0}} \sqrt{\frac{2n}{n-1}} \sqrt{2 z_0 R T_0 \ln\left(\frac{P_0}{P}\right)} \tag{4.137}$$

Fliegner's number in this case is

$$Fn = \frac{\dot{m} c_0}{A^* P_0} \sqrt{\frac{2n}{n-1}} \left(\frac{P^*}{P_0}\right)^{\frac{1}{n}} \left[1 - \frac{P^*}{P_0}\right] \tag{4.138}$$

Fliegner's number for $n = 1$ is

$$Fn = \frac{\dot{m} c_0}{A^* P_0} = 2\left(\frac{P^*}{P_0}\right)^2 - \ln\left(\frac{P^*}{P_0}\right) \tag{4.139}$$

The critical ratio of the pressure is

$$\frac{P^*}{P_0} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \tag{4.140}$$

When $n = 1$ or more generally when $n \to 1$ this is a ratio approach

$$\frac{P^*}{P_0} = \sqrt{e} \tag{4.141}$$

To obtain the relationship between the temperature and pressure, equation (4.131) can be integrated

$$\frac{T_0}{T} = \left(\frac{P_0}{P}\right)^{\frac{R}{C_p}\left[z + T\left(\frac{\partial z}{\partial T}\right)_P\right]} \tag{4.142}$$

The power of the pressure ratio is approaching $\frac{k-1}{k}$ when z approaches 1. Note that

$$\frac{T_0}{T} = \left(\frac{z_0}{z}\right)\left(\frac{P_0}{P}\right)^{\frac{1-n}{n}} \tag{4.143}$$

The Mach number at every point at the nozzle can be expressed as

$$M = \sqrt{\left(\frac{2}{n-1}\right)\frac{z_0}{z}\frac{T_0}{T}\left[1-\left(\frac{P-0}{P}\right)^{\frac{1-n}{n}}\right]} \tag{4.144}$$

For $n=1$ the Mach number is

$$M = \sqrt{2\frac{z_0}{z}\frac{T_0}{T}\ln\frac{P_0}{P}} \tag{4.145}$$

The pressure ratio at any point can be expressed as a function of the Mach number as

$$\frac{T_0}{T} = \left[1+\frac{n-1}{2}M^2\right]^{\left(\frac{n-1}{n}\right)\left[z+T\left(\frac{\partial z}{\partial T}\right)_P\right]} \tag{4.146}$$

for $n=1$

$$\frac{T_0}{T} = e^{M^2\left[z+T\left(\frac{\partial z}{\partial T}\right)_P\right]} \tag{4.147}$$

The critical temperature is given by

$$\frac{T^*}{T_0} = \left(\frac{1+n}{2}\right)^{\left(\frac{n}{1-n}\right)\left[z+T\left(\frac{\partial z}{\partial T}\right)_P\right]} \tag{4.148}$$

and for $n=1$

$$\frac{T^*}{T_0} = \sqrt{e^{-\left[z+T\left(\frac{\partial z}{\partial T}\right)_P\right]}} \qquad (4.149)$$

The mass flow rate as a function of the Mach number is

$$\dot{m} = \frac{P_0 n}{c_0} M \sqrt{\left(1 + \frac{n-1}{2} M^2\right)^{\frac{n+1}{n-1}}} \qquad (4.150)$$

For the case of $n = 1$ the mass flow rate is

$$\dot{m} = \frac{P_0 A^* n}{c_0} \sqrt{e^{M^2}} \sqrt{\left(1 + \frac{n-1}{2} M^2\right)^{\frac{n+1}{n-1}}} \qquad (4.151)$$

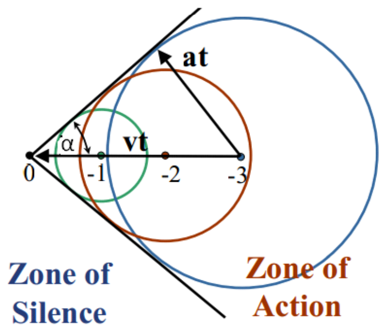
PART-A

UNIT-I BASIC CONCEPTS AND ISENTROPIC FLOWS

1	State the difference between compressible fluid and incompressible fluid ?

	Compressible flow is that type of flow in which the density of the fluid changes from point to point, i.e., density is not constant for the fluid . Examples: Gases, vapor Incompressible flow is that type of flow in which the density of the fluid constant . Examples: Liquids
2	What do you understand by adiabatic energy equations? Give the equations.
	The adiabatic energy equation is the equation derived from the energy equation for a flow process with q = 0. The heat transfer during the process such as expansion in gases and vapours in turbines are negligibly small. h1 + ½ c12 = h2 + ½ c22
3	Explain Mach cone and Mach angle?
	Mach cone: Tangents drawn from the source point on the spheres define a conical surface referred to as Mach cone. Mach angle: The angle between the Mach line and the direction of motion of the body (flow direction) is known as Mach angle
4	Derive the maximum fluid velocity.
	The fluid velocity (cmax) corresponding the condition of h = o, C = Cmax, is the maximum velocity that would be achieved by the fluid when it is accelerated to absolute zero temperature.
5	Define Mach number?
	The Mach number is an index of the ratio between inertia force and elastic force. M^2 = Inertia force / Elastic force It is also defined as the ratio of the fluid velocity (c) to the velocity of sound (a). M = c/a
6	Define the critical velocity of sound and prove that c* = {2 Cp (To – T*)1/2

	The critical velocity of a fluid is its velocity at a Mach number of unity. Mcritical = c*/a*; c* = a* = 1 CpT o = CpT* + ½ c*2 Therefore, c* = {2 Cp (To – T*)1/2
7	Why is it more convenient to use M* instead of M ?
	It is convenient since at high fluid velocities M approaches unity and M is not proportional to the fluid velocity alone
8	Differentiate nozzle and diffuser?
	Nozzle: It is a device which is used to increase the velocity and decrease the pressure of fluids. Diffuser: It is a device which is used to increase the pressure and decrease the velocity of fluids
9	What is chocked flow through a nozzle?
	The mass flow rate of nozzle is increased by decreasing the back pressure. The maximum mass flow conditions are reached when the throat pressure ratio achieves critical value. After that there is no further increase in mass flow with decrease in back pressure .This condition is called chocking. At chocking condition M=1.
10	Compare the isentropic and the adiabatic processes.
	The processes are compared with the following TS plot.
11	Define Mach cone and Mach angle?

	Velocity u of the source higher than the velocity of sound (M>1). The wave generated the positions 3, 2, 1 and S is shown. The point source is always ahead of the wave fronts. Tangent drawn from the point S on the sphere define a conical surface referred to as "Mach Cone"  Angle between Mach line and body motion is called as Mach angle. α=
12	What is the effect of Mach No on compressiblity?
	If the flow is assumed to be incompressible, the value of pressure co-efficient (or) compressibility factor obtained by Bernoulli equation is unity. +. . .
13	Define compressible flow.
	Compressible flow is that type of flow in which the density of the fluid changes from point to point, i.e., density is not constant for the fluid. Examples: Gases, vapor
14	What is Mach number?

	Mach number is a non-dimensional number and is used for the analysis of compressible fluid flows. Where and
15	What is stagnation stae?
	The state of a fluid attained by isentropically decelerating it to zero velocity at zero elevation is referred as stagnation state.
16	Define stagnation temperature. It is the temperature at which the fluid attained by isentropically decelerating it to zero velocity at zero elevation is referred as stagnation state. (SImiliary this can be mified for stagnation pressure and stagnation enthalpy)
17	An air jet (γ=1.4, R=287 J/kgK) at 420 K has sonic velocity. Determine its velocity.
18	What is the basic difference between compressible and incompressible fluid flow?
	(see table below)
19	Write the steady flow energy equation for an adiabatic flow of air.
	In an adiabatic flow q = 0. Therefore energy equation becomes $h1 + \frac{c_1^2}{2} + gZ_1 = h_2 + \frac{c_2^2}{2} + gZ_2 + W_s$ Adiabatic energy equation is $h_o = h + ½ c^2$
20	Define the mach number in terms of bulk modulus of elasticity.

Compressible	Incompressible
1. Fluid velocities are appreciable compared with the velocity of sound	1. Fluid velocities are small compared with the velocity of sound
2. Density is not constant	2. Density is constant
3. Compressibility factor is greater than one.	3. Compressibility factor is one.

	Mach number is a non-dimensional number and is used for the analysis of compressible fluid flows. $M = \sqrt{\frac{\text{int ertia force}}{\text{elastic force}}}$ $= \sqrt{\frac{\rho A c^2}{KA}}$ where K = Bulk modulus of elasticity $K = \rho a^2$ $\therefore M = \sqrt{\frac{\rho A c^2}{\rho A a^2}} = \frac{c}{a}$
21	Explain the meaning of stagnation state with example.
	The state of a fluid attained by isentropically decelerating it to zero velocity at zero elevation is referred as stagnation state. (e.g.) Fluid in a reservoir (or) in a settling chamber.
22	Distinguish between static and stagnation pressures.
	In stagnation pressure state, the velocity of the flowing fluid is zero whereas in the static pressure state, the fluid velocity is not equal to zero.
23	Differentiate between the static and stagnation temperatures.
	The actual temperature of the fluid in a particular state is known as "static temperature" whereas the temperature of the fluid when the fluid velocity is zero at zero elevation is known as "stagnation temperature". $T_0 = T + \frac{c^2}{2C_p}$ where T = static temperature T_0 = stagnation temperature $\frac{c^2}{2C_p}$ = velocity temperature

PART - B

1) Derive the energy equations

$a^2/\gamma -1 + \frac{1}{2} c^2 = \frac{1}{2} c^2 \max = a_0^2/\gamma -1 = h_0$

Stating the assumptions used.An air jet (γ =1.4, R=287 J/Kg K) at 400K has sonic velocity .Determine:

1. velocity of sound at 400 K (2)
2. Velocity of sound at the stagnation conditions. (4)
3. Maximum velocity of the jet. (4)
4. Staganation enthalpy. (4)
5. crocco number. (2)

2) The pressure, temperature and Mach number at the entry of a flow passage are 2.45 bar, 26.5° C and 1.4 respectively. If the exit Mach number is 2.5 determine for adiabatic flow of perfect gas (γ =1.3, R=0.469 KJ/Kg K). (16)

3) Air (γ =1.4,R=287.43 J/Kg K) enters a straight axis symmetric duct at 300 K,3.45 bar and 150

m/s and leaves it at 277 k,500cm² . Assuming adiabatic flow determines:

1. stagnation temperature, (4)
2. maximum velocity, (4)
3. mass flow rate, and, (4)
4. area of cross-section at exit. (4)

4) An aircraft flies at 800 Km/hr at an altitude of 10,000 meters (T=223.15 K,P=0.264 bar). The

air is reversibly compressed in an inlet diffuser. If the Mach number at the exit of the diffuser is 0.36 determine (a) entry Mach number and (b) velocity, pressure and temperature of air at diffuser exit. (16)

5) Air (Cp =1.05 KJ/Kg K,γ =1.38) at p1 =3*10 ^5 N/m² and T1 =500 k flows with a velocity of

200 m/s in a 30 cm diameter duct. Calculate mass flow rate, stagnation temperature, Mach number, and, Stagnation pressure values assuming the flow as compressible and incompressible. (16)

6) (a) What is the effect of Mach number on compressibility prove for

γ=1.4, Þo –þ / ½ þ c² = 1 +¼ M² + 1/40 M 4 + ……. (8)

(b) Show that for sonic flow the deviation between the compressible and incompressible flow

values of the pressure coefficient of a percent gas (γ =1.4) is about 27.5 per cent. (8)

7) Air flowing in a duct has a velocity of 300 m/s ,pressure 1.0 bar and temperature 290 k. Taking γ=1.4 and R =287J/Kg K determine:

1) Stagnation pressure and temperature, (4)

2) Velocity of sound in the dynamic and stagnation conditions, (6)

3) Stagnation pressure assuming constant density. (6)

8) A conical diffuser has entry and exit diameters of 15 cm and 30cm respectively . The

Pressure ,temperature and velocity of air at entry are 0.69bar,340 k and 180 m/s respectively . Determine

1) The exit pressure, (4)

2) The exit velocity and (6)

3) The force exerted on the diffuser walls. (6)

Assume isentropic flow,γ =1.4,Cp =1.00 KJ Kg-K.

9) A nozzle in a wind tunnel gives a test –section Mach number of 2.0 .Air enters the nozzle from a large reservoir at 0.69 bar and 310 k .The cross –sectional area of the throat is1000cm^2.Determine the following quantites for the tunnel for one dimensional isentropic flow

1) Pressures,temperature and velocities at the throat and test sections, (4)

2) Area of cross- sectional of the test section , (4)

3) Mass flow rate, (4)

4) Power rate required to drive the compressor. (4)

10) Air is discharged from a reservoir at Po =6.91bar and To =325°c through a nozzle to an exit pressure of 0.98 bar .If the flow rate is 3600Kg/hr determine for isentropic flow:

1)Throat area, pressure,and velocity, (6)

2)Exit area,Mach number ,and (6)

3)Maximum velocity. (4)

11) A super sonic wind tunnel settling chamber expands air or Freon-21 through a nozzle from a

nozzle from a pressure of 10 bar to 4bar in the test section . calculate the stagnation temperature to the maintained in the setting chamber to obtain a velocity of 500 m/s in

the test section for Air ,Cp =1.025 KJ/Kg K, Cv =0.735 KJ/Kg K, Freon -21 ,Cp =0.785 KJ/Kg K, ,Cv= 0.675 KJ/Kg K.

What is the test section Mach number in each case ? (16)

12) Derive the following relations for one dimensional isentropic flow:

 dA/A =dP/þ c²(1 -M²) (8)

 p*/p =(2/γ+1 +γ-1 /γ+1M²) (8)

13) Air flowing in a duct has a velocity of 300 m/s ,pressure 1.0 bar and temperature 290 K. Taking γ=1.4 and R =287J/Kg K determine:

1)Stagnation pressure and temperature, (4)

2)Velocity of sound in the dynamic and stagnation conditions (6)

3)Stagnation pressure assuming constant density. (6)

14) A conical diffuser has entry and exit diameters of 15 cm and 30cm respectively . The

pressure ,temperature and velocity of air at entry are 0.69bar,340 k and 180 m/s respectively . Determine

1) The exit pressure, (4)

2)The exit velocity and (6)

3)The force exerted on the diffuser walls. (6)

Assume isentropic flow,γ =1.4,Cp =1.00 KJ Kg-K.

15) A nozzle in a wind tunnel gives a test –section Mach number of 2.0 .Air enters the nozzle from

a large reservoir at 0.69 bar and 310 k .The cross –sectional area of the throat is 1000cm².Determine the following quantites for the tunnel for one dimensional isentropic flow:

1)Pressures,temperature and velocities at the throat and test sections, (4)

2)Area of cross- sectional of the test section , (4)

3)Mass flow rate, (4)

4)Power rate required to drive the compressor. (4)

16) Air is discharged from a reservoir at Po =6.91bar and To =325°c through a nozzle to an exit

pressure of 0.98 bar .If the flow rate is 3600Kg/hr determine for isentropic flow:

1)Throat area, pressure,and velocity, (6)

2)Exit area,Mach number ,and (6)

3)Maximum velocity. (4)

17) A super sonic wind tunnel settling chamber expands air or Freon-21 through a nozzle from a

nozzle from a pressure of 10 bar to 4bar in the test section . calculate the stagnation temperature to the maintained in the setting chamber to obtain a velocity of 500 m/s in the test section for,

Air ,Cp =1.025 KJ/Kg K, Cv =0.735 KJ/Kg K, Freon -21 ,Cp =0.785 KJ/Kg K Cv= 0.675 KJ/Kg K.

What is the test section Mach number is each case ? (16)

18) Derive the following relations for one dimensional isentropic flow:

- dA/A =dP/þ $c^2(1 - M^2)$ (8)
- $p^*/p = (2/\gamma+1 + \gamma-1 / \gamma+1 M^2)$ (8)

UNIT II FLOW THROUGH DUCTS

Fanno Flow

Introduction

Consider a gas flowing through a conduit with a friction (see Figure (9.1)). It is advantages to examine the simplest situation and yet without losing the core properties of the process. Later, more general cases will be examined[9.2].

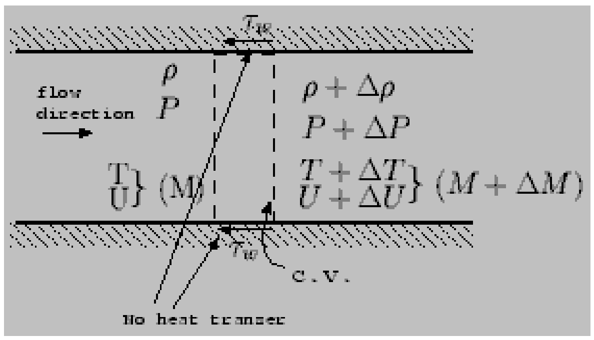


Figure: Control volume of the gas flow in a constant cross section

An adiabatic flow with friction is named after Ginno Fanno a Jewish engineer. This model is the second pipe flow model described here. The main restriction for this model is that heat transfer is negligible and can be ignored [9.1]. This model is applicable to flow processes which are very fast compared to heat transfer mechanisms with small Eckert number.

This model explains many industrial flow processes which includes emptying of pressured container through a relatively short tube, exhaust system of an internal combustion engine, compressed air systems, etc. As this model raised from need to explain the steam flow in turbines.

Model

The mass (continuity equation) balance can be written as

$$\dot{m} = \rho A U = constant \tag{9.1}$$

$$\hookrightarrow \rho_1 U_1 = \rho_2 U_2$$

The energy conservation (under the assumption that this model is adiabatic flow and the friction is not transformed into thermal energy) reads

$$T_{01} = T_{02}$$

$$\hookrightarrow T_1 + \frac{{U_1}^2}{2c_p} = T_2 + \frac{{U_2}^2}{2c_p} \tag{9.2}$$

Or in a derivative form

$$C_p dT + d\left(\frac{U^2}{2}\right) = 0 \tag{9.3}$$

Again for simplicity, the perfect gas model is assumed[9.3].

$$P = \rho R T \tag{9.4}$$

$$\hookrightarrow \frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$$

It is assumed that the flow can be approximated as one-dimensional. The force acting on the gas is the friction at the wall and the momentum conservation reads

$$-AdP - \tau_w dA_w = \dot{m}dU \tag{9.5}$$

It is convenient to define a hydraulic diameter as

$$D_H = \frac{4 \times \text{Cross Section Area}}{\text{wetted perimeter}} \tag{9.6}$$

Or in other words

$$A = \frac{\pi {D_H}^2}{4} \tag{9.7}$$

It is convenient to substitute $\underline{D}$ for D_H and yet it still will be referred to the same name as the hydraulic diameter. The infinitesimal area that shear stress is acting on is

$$dA_w = \pi D dx \tag{9.8}$$

Introducing the Fanning friction factor as a dimensionless friction factor which is some times referred to as the friction coefficient and reads as the following:

$$f = \frac{\tau_w}{\rho U^2} \tag{9.9}$$

By utilizing equation (9.2) and substituting equation (9.10) into momentum equation (9.6) yields

$$-\overbrace{\frac{\pi D^2}{4}}^{A} dP - \pi D dx \overbrace{f\left(\rho U^2\right)}^{\tau_w} = A \overbrace{\rho U}^{\frac{\dot{m}}{A}} dU$$ (9.10)

Dividing equation (9.11) by the cross section area, $\mathbf{A}$ and rearranging yields

$$-dP + \frac{4f\,dx}{D}\left(\rho U^2\right) = \rho U dU$$ (9.11)

The second law is the last equation to be utilized to determine the flow direction.

$$s_2 \geq s_1$$ (9.12

Non-Dimensionalization of the Equations

Before solving the above equation a dimensionless process is applied. By utilizing the definition of the sound speed to produce the following identities for perfect gas

$$M^2 = \left(\frac{U}{c}\right)^2 = \frac{U^2}{k \underbrace{RT}_{\frac{P}{\rho}}}$$ (9.13)

Utilizing the definition of the perfect gas results in

$$M^2 = \frac{\rho U^2}{kP}$$ (9.14)

Using the identity in equation (9.14) and substituting it into equation (9.11) and after some rearrangement yields

$$-dP+\frac{4f\,dx}{D_H}\left(kPM^2\right)=\frac{\rho U^2}{U}dU=\overbrace{kPM^2}^{\rho U^2}\frac{dU}{U} \tag{9.15}$$

By further rearranging equation (9.16) results in

$$-\frac{dP}{P}-\frac{4f\,dx}{D}\left(\frac{kM^2}{2}\right)=kM^2\frac{dU}{U} \tag{9.16}$$

It is convenient to relate expressions of (dP/P) and dU/U in terms of the Mach number and substituting it into equation (9.17). Derivative of mass conservation ((9.2)) results in

$$\frac{d\rho}{\rho}+\overbrace{\frac{dU^2}{U^2}}^{\frac{dU}{U}}=0 \tag{9.17}$$

The derivation of the equation of state (9.5) and dividing the results by equation of state (9.5) results

$$\frac{dP}{P}=\frac{d\rho}{\rho}+\frac{dT}{dT} \tag{9.18}$$

Derivation of the Mach identity equation (9.14) and dividing by equation (9.14) yields

$$\frac{d(M^2)}{M^2} = \frac{d(U^2)}{U^2} - \frac{dT}{T} \tag{9.19}$$

Dividing the energy equation (9.4) by C_p and by utilizing the definition Mach number yields

$$\frac{dT}{T} + \underbrace{\frac{1}{\left(\frac{kR}{(k-1)}\right)}}_{C_p} \frac{1}{T}\frac{U^2}{U^2} d\left(\frac{U^2}{2}\right) =$$

$$\hookrightarrow \frac{dT}{T} + \frac{(k-1)}{\underbrace{kRT}_{c^2}} \frac{U^2}{U^2} d\left(\frac{U^2}{2}\right) =$$

$$\hookrightarrow \frac{dT}{T} + \frac{k-1}{2} M^2 \frac{dU^2}{U^2} = 0 \tag{9.20}$$

Equations (9.17), (9.18), (9.19), (9.20), and (9.21) need to be solved. These equations are separable so one variable is a function of only single variable (the chosen as the independent variable). Explicit explanation is provided for only two variables, the rest variables can be done in a similar fashion. The dimensionless friction, $\frac{4fL}{D}$, is chosen as the independent variable since the change in the dimensionless resistance, $\frac{4fL}{D}$, causes

the change in the other variables.

Combining equations (9.19) and (9.21) when eliminating dT/T results

$$\frac{dP}{P} = \frac{d\rho}{\rho} - \frac{(k-1)M^2}{2}\frac{dU^2}{U^2} \tag{9.21}$$

The term $\frac{d\rho}{\rho}$ can be eliminated by utilizing equation (9.18) and substituting it into equation (9.22) and rearrangement yields

$$\frac{dP}{P} = -\frac{1+(k-1)M^2}{2}\frac{dU^2}{U^2} \tag{9.22}$$

The term dU^2/U^2 can be eliminated by using (9.23)

$$\frac{dP}{P} = -\frac{kM^2\left(1+(k-1)M^2\right)}{2(1-M^2)}\frac{4f\,dx}{D} \tag{9.23}$$

The second equation for Mach number, $\underline{M}$ variable is obtained by combining equation (9.20) and (9.21) by eliminating dT/T. Then $d\rho/\rho$ and $\underline{U}$ are eliminated by utilizing equation (9.18) and equation (9.22). The only variable that is left is $\underline{P}$ (or dP/P) which can be eliminated by utilizing equation (9.24) and results in

$$\frac{4f\,dx}{D} = \frac{\left(1-M^2\right)dM^2}{kM^4(1+\frac{k-1}{2}M^2)} \tag{9.24}$$

Rearranging equation (9.25) results in

$$\frac{dM^2}{M^2} = \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{4f\,dx}{D} \tag{9.25}$$

After similar mathematical manipulation one can get the relationship for the velocity to read

$$\frac{dU}{U} = \frac{kM^2}{2\left(1-M^2\right)}\frac{4f\,dx}{D} \tag{9.26}$$

and the relationship for the temperature is

$$\frac{dT}{T} = \frac{dc}{c} = -\frac{k(k-1)M^4}{2(1-M^2)}\frac{4f\,dx}{D} \tag{9.27}$$

density is obtained by utilizing equations (9.27) and (9.18) to obtain

$$\frac{d\rho}{\rho} = -\frac{kM^2}{2\left(1-M^2\right)}\frac{4f\,dx}{D} \tag{9.28}$$

The stagnation pressure is similarly obtained as

$$\frac{dP_0}{P_0} = -\frac{kM^2}{2}\frac{4f\,dx}{D} \tag{9.29}$$

The second law reads

$$ds = C_p \ln \frac{dT}{T} - R \ln \frac{dP}{P} \tag{9.30}$$

The stagnation temperature expresses as $T_0 = T(1 + (1-k)/2M^2)$. Taking derivative of this expression when M remains constant yields $dT_0 = dT(1 + (1-k)/2M^2)$ and thus when these equations are divided they yield

$$dT/T = dT_0/T_0 \tag{9.31}$$

In similar fashion the relationship between the stagnation pressure and the pressure can be substituted into the entropy equation and result in

$$ds = C_p \ln \frac{dT_0}{T_0} - R \ln \frac{dP_0}{P_0} \tag{9.32}$$

The first law requires that the stagnation temperature remains constant, $(dT_0 = 0)$. Therefore the entropy change is

$$\frac{ds}{C_p} = -\frac{(k-1)}{k} \frac{dP_0}{P_0} \tag{9.33}$$

Using the equation for stagnation pressure the entropy equation yields

$$\frac{ds}{C_p} = \frac{(k-1)M^2}{2} \frac{4f\,dx}{D} \tag{9.34}$$

The Mechanics and Why the Flow is Choked?

The trends of the properties can be examined by looking in equations (9.24) through (9.34). For example, from equation (9.24) it can be observed that the critical point is when $M = 1$. When $M < 1$ the pressure decreases downstream as can be seen from equation (9.24) because fdx and M are positive. For the same reasons, in the supersonic branch, $M > 1$, the pressure increases downstream. This pressure increase is what makes compressible flow so different from ``conventional'' flow. Thus the discussion will be divided into two cases: One, flow above speed of sound. Two, flow with speed below the speed of sound.

Why the flow is choked?

Here, the explanation is based on the equations developed earlier and there is no known explanation that is based on the physics. First, it has to be recognized that the critical point is when $M = 1$. It will be shown that a change in location relative to this point change the trend and it is singular point by itself. For example, $dP(@M = 1) = \infty$ and mathematically it is a singular point (see equation (9.24)). Observing from equation (9.24) that increase or decrease from subsonic just below one $M = (1 - \epsilon)$ to above just above one $M = (1 + \epsilon)$ requires a change in a sign pressure direction. However, the pressure has to be a monotonic function which means that flow cannot crosses over the point of $M = 1$. This constrain means that because the flow cannot ``crossover'' $M = 1$ the gas has to reach to this speed, $M = 1$ at the last point. This situation is called choked flow.

The Trends

The trends or whether the variables are increasing or decreasing can be observed from looking at the equation developed. For example, the pressure can be examined by looking

at equation (9.26). It demonstrates that the Mach number increases downstream when the flow is subsonic. On the other hand, when the flow is supersonic, the pressure decreases. The summary of the properties changes on the sides of the branch

	Subsonic	Supersonic
Pressure, P	decrease	increase
Mach number, M	increase	decrease
Velocity, U	increase	decrease
Temperature, T	decrease	increase
Density, ρ	decrease	increase
Stagnation Temperature, T_0	decrease	increase

The working equations

Integration of equation (9.25) yields

$$\frac{4}{D}\int_{L}^{L_{max}} f\,dx = \frac{1}{k}\frac{1-M^2}{M^2} + \frac{k+1}{2k}\ln\frac{\frac{k+1}{2}M^2}{1+\frac{k-1}{2}M^2} \tag{9.35}$$

A representative friction factor is defined as

$$\bar{f} = \frac{1}{L_{max}}\int_{0}^{L_{max}} f\,dx \tag{9.36}$$

By utilizing the mean average theorem equation (9.36) yields

$$\frac{4\bar{f}L_{max}}{D} = \frac{1}{k}\frac{1-M^2}{M^2} + \frac{k+1}{2k}\ln\frac{\frac{k+1}{2}M^2}{1+\frac{k-1}{2}M^2} \tag{9.37}$$

It is common to replace the $\bar{f}$ with f which is adopted in this book.

Equations (9.24), (9.27), (9.28), (9.29), (9.29), and (9.30) can be solved. For example, the pressure as written in equation (9.23) is represented by , and Mach number. Now equation (9.24) can eliminate term and describe the pressure on the Mach number. Dividing equation (9.24) in equation (9.26) yields

$$\frac{\frac{dP}{P}}{\frac{dM^2}{M^2}} = -\frac{1+(k-1M^2}{2M^2\left(1+\frac{k-1}{2}M^2\right)}dM^2 \tag{9.38}$$

The symbol ``*" denotes the state when the flow is choked and Mach number is equal to 1. Thus, $M=1$ when $P=P^*$ Equation (9.39) can be integrated to yield:

$$\frac{P}{P^*} = \frac{1}{M}\sqrt{\frac{\frac{k+1}{2}}{1+\frac{k-1}{2}M^2}} \tag{9.39}$$

In the same fashion the variables ratio can be obtained

$$\frac{T}{T^*} = \frac{c^2}{c^{*2}} = \frac{\frac{k+1}{2}}{1+\frac{k-1}{2}M^2} \tag{9.40}$$

$$\frac{\rho}{\rho^*} = \frac{1}{M}\sqrt{\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}} \tag{9.41}$$

$$\frac{U}{U^*} = \left(\frac{\rho}{\rho^*}\right)^{-1} = M\sqrt{\frac{\frac{k+1}{2}}{1+\frac{k-1}{2}M^2}} \tag{9.42}$$

The stagnation pressure decreases and can be expressed by

$$\frac{P_0}{{P_0}^*} = \frac{\overbrace{\frac{P_0}{P}}^{\left(1+\frac{1-k}{2}M^2\right)^{\frac{k}{k-1}}} \; P}{\underbrace{\frac{{P_0}^*}{P^*}}_{\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}} \; P^*} \tag{9.43}$$

Using the pressure ratio in equation (9.40) and substituting it into equation (9.44) yields

$$\frac{P_0}{{P_0}^*} = \left(\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{\frac{k}{k-1}} \frac{1}{M}\sqrt{\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}} \tag{9.44}$$

And further rearranging equation (9.45) provides

$$\frac{P_0}{{P_0}^*} = \frac{1}{M}\left(\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2(k-1)}} \tag{9.45}$$

The integration of equation (9.34) yields

$$\frac{s-s^*}{c_p} = \ln M^2 \sqrt{\left(\frac{k+1}{2M^2\left(1+\frac{k-1}{2}M^2\right)}\right)^{\frac{k+1}{k}}} \tag{9.46}$$

The results of these equations are plotted in Figure (9.2)

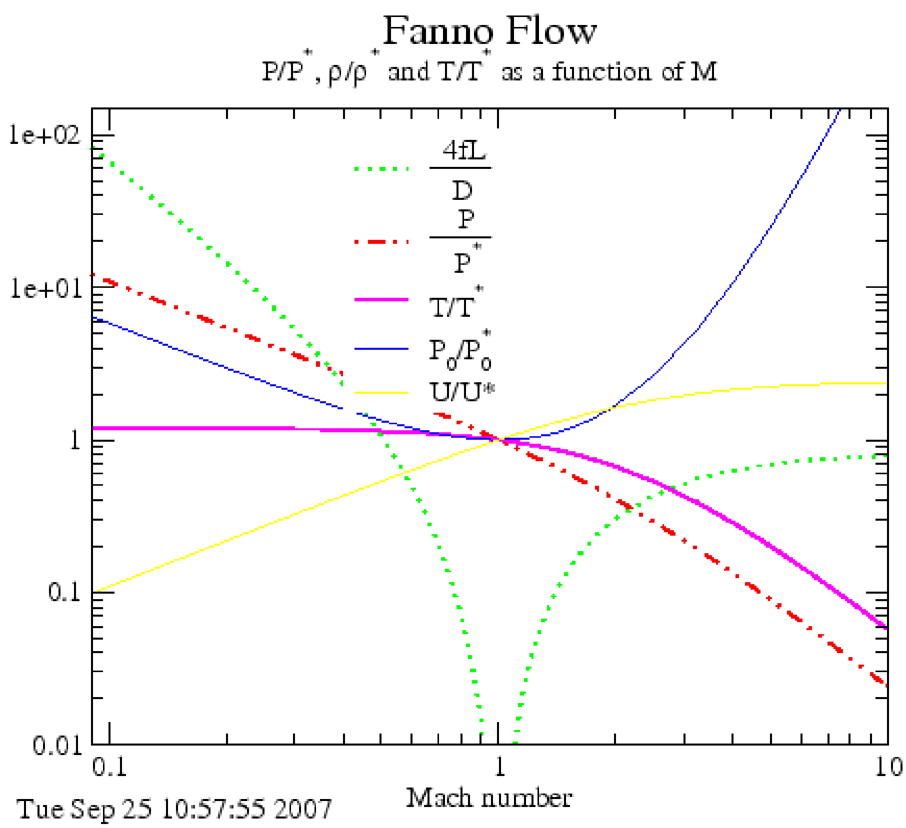


Figure: Various parameters in Fanno flow as a function of Mach number

The Fanno flow is in many cases shockless and therefore a relationship between two points should be derived. In most times, the ``star" values are imaginary values that represent the value at choking. The real ratio can be obtained by two star ratios as an example

$$\frac{T_2}{T_1} = \frac{\left.\frac{T}{T^*}\right|_{M_2}}{\left.\frac{T}{T^*}\right|_{M_1}} \tag{9.47}$$

A special interest is the equation for the dimensionless friction as following

$$\int_{L_1}^{L_2} \frac{4fL}{D} dx = \int_{L_1}^{L_{max}} \frac{4fL}{D} dx - \int_{L_2}^{L_{max}} \frac{4fL}{D} dx \tag{9.48}$$

Hence,

$$\left(\frac{4fL_{max}}{D}\right)_2 = \left(\frac{4fL_{max}}{D}\right)_1 - \frac{4fL}{D} \tag{9.49}$$

Supersonic Branch

In Chapter (8) it was shown that the isothermal model cannot describe adequately the situation because the thermal entry length is relatively large compared to the pipe length and the heat transfer is not sufficient to maintain constant temperature. In the Fanno model there is no heat transfer, and, furthermore, because the very limited amount of heat transformed it is closer to an adiabatic flow. The only limitation of the model is its uniform velocity (assuming parabolic flow for laminar and different profile for turbulent flow.). The information from the wall to the tube center[9.6] is slower in reality. However, experiments from many starting with 1938 work by Frossel[9.7]has shown that the error is not significant. Nevertheless, the comparison with reality shows that heat transfer cause changes to the flow and they need/should to be expected. These changes include the choking point at lower Mach number.

Maximum Length for the Supersonic Flow

It has to be noted and recognized that as opposed to subsonic branch the supersonic branch has a limited length. It also must be recognized that there is a maximum length for which only supersonic flow can exist[9.8]. These results were obtained from the mathematical derivations but were verified by numerous experiments[9.9]. The maximum length of the supersonic can be evaluated when M = ∞ as follows:

$$\frac{4fL_{max}}{D} = \frac{1-M^2}{kM^2} + \frac{k+1}{2k} \ln \frac{\frac{k+1}{2}M^2}{2\left(1+\frac{k-1}{2}M^2\right)}$$

$$(M \to \infty) \sim \frac{-\infty}{k \times \infty} + \frac{k+1}{2k} \ln \frac{(k+1)\infty}{(k-1)\infty}$$

$$= \frac{-1}{k} + \frac{k+1}{2k} \ln \frac{(k+1)}{(k-1)}$$

$$\frac{4fL_{max}}{D} = (M \to \infty, k = 1.4) = 0.8215 \tag{9.50}$$

The maximum length of the supersonic flow is limited by the above number. From the above analysis, it can be observed that no matter how high the entrance Mach number will be the tube length is limited and depends only on specific heat ratio, k as shown in Figure (9.5).

Figure 9.5: The maximum length as a function of specific heat, k

Variations of The Tube Length (4fL/D) Effects

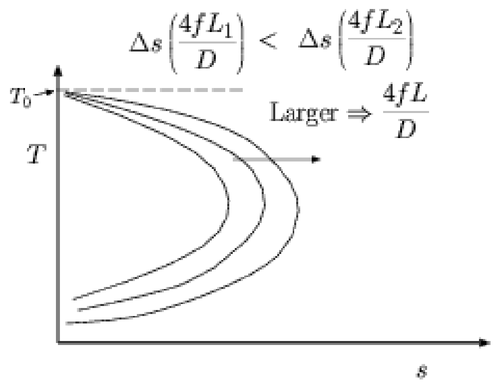


Figure 9.6: The effects of increase of $\frac{4fL}{D}$ on the Fanno line

In the analysis of this effect, it should be assumed that back pressure is constant and/or low as possible as needed to maintain a choked flow. First, the treatment of the two branches are separated.

Fanno Flow Subsonic branch

For converging nozzle feeding, increasing the tube length results in increasing the exit Mach number (normally denoted herein as M_2). Once the Mach number reaches maximum ($M = 1$), no further increase of the exit Mach number can be achieved. In this process, the mass flow rate decreases.

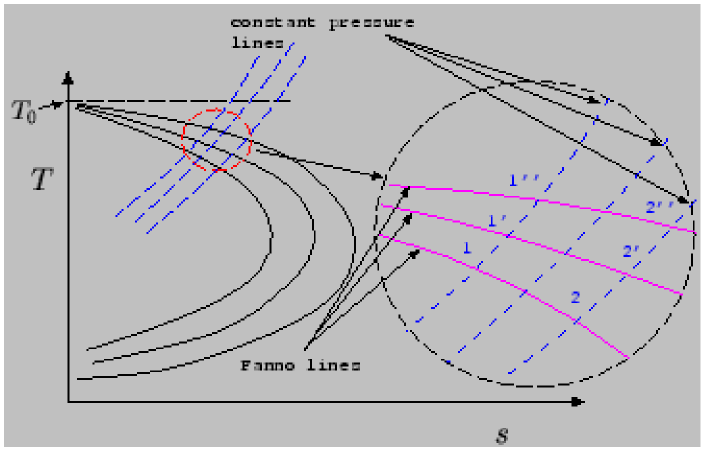


Figure 9.7: The development properties in of converging nozzle

It is worth noting that entrance Mach number is reduced (as some might explain it to reduce the flow rate). The entrance temperature increases as can be seen from Figure (9.7). The velocity therefore must decrease because the loss of the enthalpy (stagnation temperature) is ``used." The density decrease because $\rho = \frac{P}{RT}$ and when pressure is remains almost constant the density decreases. Thus, the mass flow rate must decrease. These results are applicable to the converging nozzle.

In the case of the converging-diverging feeding nozzle, increase of the dimensionless friction, -, results in a similar flow pattern as in the converging nozzle. Once the flow becomes choked a different flow pattern emerges

Fanno Flow Supersonic Branch

There are several transitional points that change the pattern of the flow. Point $\underline{a}$is the choking point (for the supersonic branch) in which the exit Mach number reaches to one. Point $\underline{b}$is the maximum possible flow for supersonic flow and is not dependent on the nozzle. The next point, referred here as the critical point $\underline{c}$, is the point in which no supersonic flow is possible in the tube i.e. the shock reaches to the nozzle. There is

another point **d**, in which no supersonic flow is possible in the entire nozzle-tube system. Between these transitional points the effect parameters such as mass flow rate, entrance and exit Mach number are discussed.

At the starting point the flow is choked in the nozzle, to achieve supersonic flow. The following ranges that has to be discussed includes (see Figure (9.8)):

0	$<$	$\frac{4fL}{D}$	$<$	$\left(\frac{4fL}{D}\right)_{choking}$	$0 \rightarrow \mathbf{a}$
$\left(\frac{4fL}{D}\right)_{choking}$	$<$	$\frac{4fL}{D}$	$<$	$\left(\frac{4fL}{D}\right)_{shockless}$	$\mathbf{a} \rightarrow \mathbf{b}$
$\left(\frac{4fL}{D}\right)_{shockless}$	$<$	$\frac{4fL}{D}$	$<$	$\left(\frac{4fL}{D}\right)_{chokeless}$	$\mathbf{b} \rightarrow \mathbf{c}$
$\left(\frac{4fL}{D}\right)_{chokeless}$	$<$	$\frac{4fL}{D}$	$<$	∞	$\mathbf{c} \rightarrow \infty$

◇

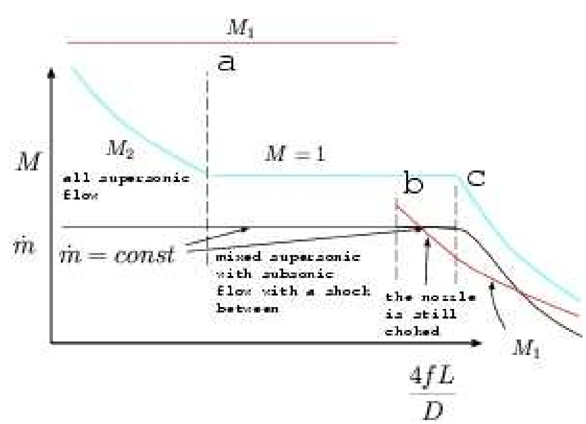


Figure 9.8: M_{in} and $\dot{m}$ as a function of the $\frac{4fL}{D}$

The 0- $\mathbf{\underline{a}}$ range, the mass flow rate is constant because the flow is choked at the nozzle. The entrance Mach number, M_1 is constant because it is a function of the nozzle design only. The exit Mach number, M_2 decreases (remember this flow is on the supersonic branch) and starts ($\frac{4fL}{D} = 0$) as $M_2 = M_1$. At the end of the range $\mathbf{\underline{a}}$, $M_2 = 1$. In the range of $\mathbf{a - b}$ the flow is all supersonic.

In the next range $\mathbf{a - -b}$ The flow is double choked and make the adjustment for the flow rate at different choking points by changing the shock location. The mass flow rate continues to be constant. The entrance Mach continues to be constant and exit Mach number is constant.

The total maximum available for supersonic flow $\mathbf{b - -b'}$, $\left(\frac{4fL}{D}\right)_{max}$, is only a theoretical length in which the supersonic flow can occur if nozzle is provided with a larger Mach number (a change to the nozzle area ratio which also reduces the mass flow rate). In the range $\mathbf{b - c}$, it is a more practical point.

In semi supersonic flow $\mathbf{b - c}$ (in which no supersonic is available in the tube but only in the nozzle) the flow is still double choked and the mass flow rate is constant. Notice that exit Mach number, M_2 is still one. However, the entrance Mach number, M_1, reduces with the increase of $\frac{4fL}{D}$.

It is worth noticing that in the $\mathbf{a - c}$ the mass flow rate nozzle entrance velocity and the

exit velocity remains constant![9.10]

In the last range $\mathbf{c} - \infty$ the end is really the pressure limit or the break of the model and the isothermal model is more appropriate to describe the flow. In this range, the flow rate decreases since ($\dot{m} \propto M_1$)[9.11].

To summarize the above discussion, Figures (9.8) exhibits the development of M_1, M_2 mass flow rate as a function of $\frac{4fL}{D}$. Somewhat different then the subsonic branch the mass flow rate is constant even if the flow in the tube is completely subsonic. This situation is because of the ``double'' choked condition in the nozzle. The exit Mach $M_2$ is a continuous monotonic function that decreases with $\frac{4fL}{D}$. The entrance Mach $M_1$ is a non continuous function with a jump at the point when shock occurs at the entrance ``moves'' into the nozzle.

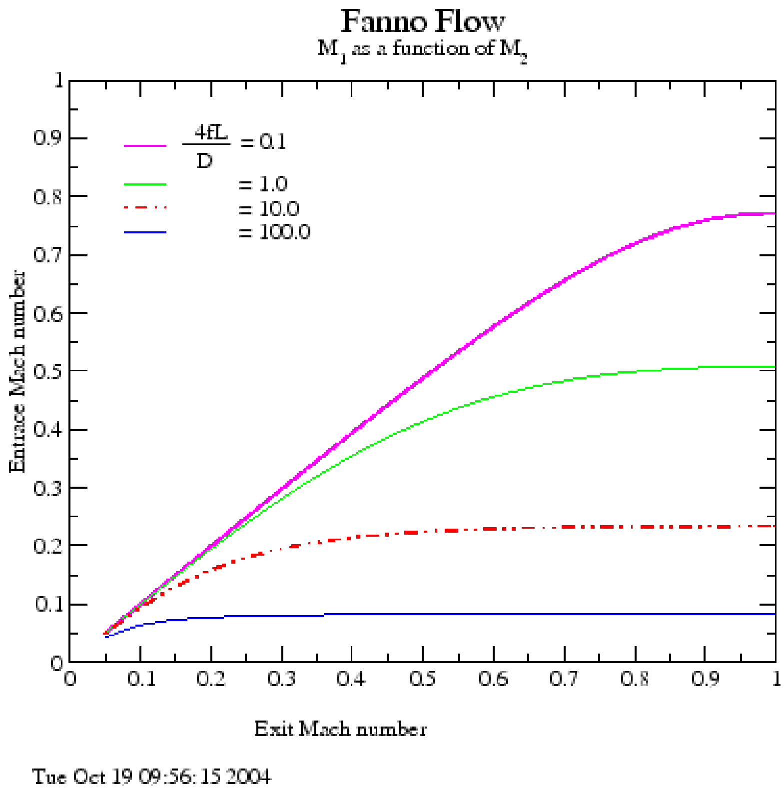


Figure: M1 as a function M2 for various 4fL/D

Figure (9.9) exhibits the M_1 as a function of M_2. The Figure was calculated by utilizing the data from Figure (9.2) by obtaining the $\left.\frac{4fL}{D}\right|_{max}$ for M_2 and subtracting the given $\frac{4fL}{D}$ and finding the corresponding M_1.

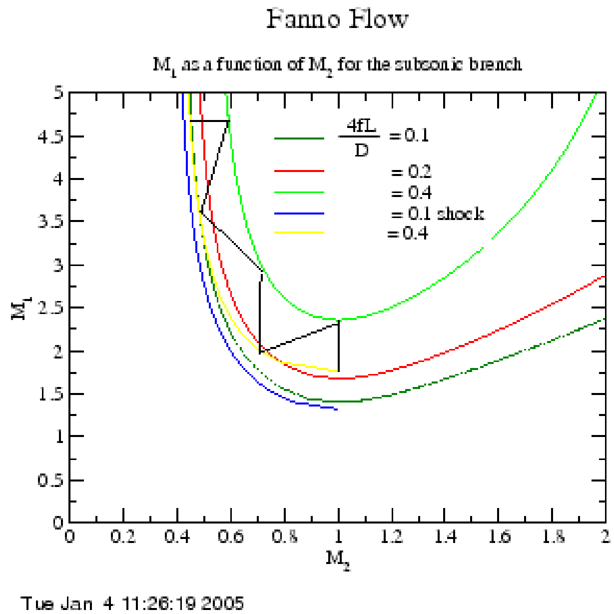


Figure: M1 as a function M2

The Figure (9.10) exhibits the entrance Mach number as a function of the M_2. Obviously there can be two extreme possibilities for the subsonic exit branch. Subsonic velocity occurs for supersonic entrance velocity, one, when the shock wave occurs at the tube exit and two, at the tube entrance. In Figure (9.10) only for $\frac{4fL}{D} = 0.1$ and $\frac{4fL}{D} = 0.4$ two extremes are shown. For $\frac{4fL}{D} = 0.2$ shown with only shock at the exit only. Obviously, and as can be observed, the larger $\frac{4fL}{D}$ creates larger differences between exit Mach

number for the different shock locations. The larger $\frac{4fL}{D}$ larger M_1 must occurs even for shock at the entrance.

For a given $\frac{4fL}{D}$, below the maximum critical length, the supersonic entrance flow has three different regimes which depends on the back pressure. One, shockless flow, tow, shock at the entrance, and three, shock at the exit. Below, the maximum critical length is mathematically

$$\frac{4fL}{D} > -\frac{1}{k} + \frac{1+k}{2k} \ln \frac{k+1}{k-1}$$

For cases of $\frac{4fL}{D}$ above the maximum critical length no supersonic flow can be over the whole tube and at some point a shock will occur and the flow becomes subsonic flow[9.12].

The Pressure Ratio, P2/P1, effects

In this section the studied parameter is the variation of the back pressure and thus, the pressure ratio $\frac{P_2}{P_1}$ variations. For very low pressure ratio the flow can be assumed as incompressible with exit Mach number smaller than < 0.3. As the pressure ratio increases (smaller back pressure, P_2), the exit and entrance Mach numbers increase. According to Fanno model the value of $\frac{4fL}{D}$ is constant (friction factor, f, is independent of the parameters such as, Mach number, Reynolds number et cetera) thus the flow remains on the same Fanno line. For cases where the supply come from a reservoir with a constant pressure, the entrance pressure decreases as well because of the increase in the entrance Mach number (velocity).

Again a differentiation of the feeding is important to point out. If the feeding nozzle is

converging than the flow will be only subsonic. If the nozzle is ``converging-diverging'' than in some part supersonic flow is possible. At first the converging nozzle is presented and later the converging-diverging nozzle is explained.

Figure: The pressure distribution as a function of 4fL/D for a short 4fL/D

Choking explanation for pressure variation/reduction

Decreasing the pressure ratio or in actuality the back pressure, results in increase of the entrance and the exit velocity until a maximum is reached for the exit velocity. The maximum velocity is when exit Mach number equals one. The Mach number, as it was shown in Chapter (4), can increases only if the area increase. In our model the tube area is postulated as a constant therefore the velocity cannot increase any further. However, for the flow to be continuous the pressure must decrease and for that the velocity must increase. Something must break since there are conflicting demands and it result in a ``jump'' in the flow. This jump is referred to as a choked flow. Any additional reduction in the back pressure will not change the situation in the tube. The only change will be at tube surroundings which are irrelevant to this discussion.

If the feeding nozzle is a ``converging-diverging'' then it has to be differentiated between two cases; One case is where the $\frac{4fL}{D}$ is short or equal to the critical length. The critical length is the maximum $\left.\frac{4fL}{D}\right|_{max}$ that associate with entrance Mach number.

Figure: The pressure distribution for a long 4fL/D

Fanno Flow Short 4fL/D

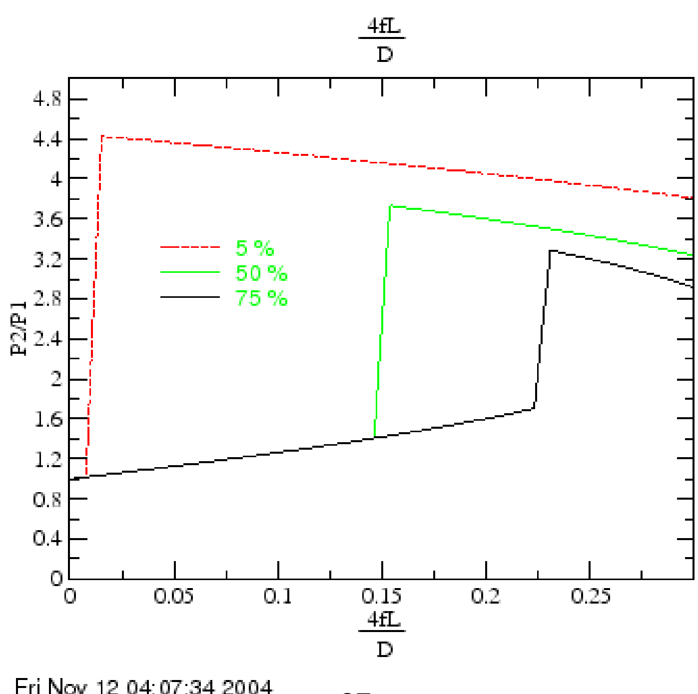
Figure (9.12) shows different pressure profiles for different back pressures. Before the flow reaches critical point a (in the Figure) the flow is subsonic. Up to this stage the nozzle feeding the tube increases the mass flow rate (with decreasing back pressure). Between point a and point b the shock is in the nozzle. In this range and further reduction of the pressure the mass flow rate is constant no matter how low the back pressure is

reduced. Once the back pressure is less than point b the supersonic reaches to the tube. Note however that exit Mach number, $M_2 < 1$ and is **not** 1. A back pressure that is at the critical point c results in a shock wave that is at the exit. When the back pressure is below point c, the tube is ``clean'' of any shock[9.13]. The back pressure below point c has some adjustment as it occurs with exceptions of point d

Long 4fL/D

In the case of $\frac{4fL}{D} > \left.\frac{4fL}{D}\right|_{max}$ reduction of the back pressure results in the same process as explained in the short $\frac{4fL}{D}$ up to point c. However, point c in this case is different from point c at the case of short tube $\frac{4fL}{D} < \left.\frac{4fL}{D}\right|_{max}$. In this point the exit Mach number is equal to 1 and the flow is double shock. Further reduction of the back pressure at this stage will not ``move'' the shock wave downstream the nozzle. At point c or location of the shock wave, is a function entrance Mach number, $M_1$ and the ``extra'' $\frac{4fL}{D}$. The is no analytical solution for the location of this point c. The procedure is (will be) presented in later stage.

P2/P1 Fanno Flow



Entrance Mach number, M1, effects

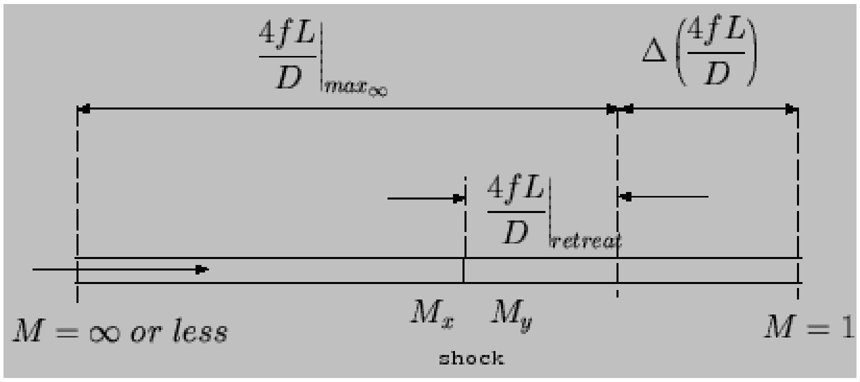


Figure 9.15: Schematic of a ``long" tube in supersonic branch

In this discussion, the effect of changing the throat area on the nozzle efficiency is neglected. In reality these effects have significance and needs to be accounted for some instances. This dissection deals only with the flow when it reaches the supersonic branch reached otherwise the flow is subsonic with regular effects. It is assumed that in this discussion that the pressure ratio $\frac{P_2}{P_1}$ is large enough to create a choked flow and $\frac{4fL}{D}$ is small enough to allow it to happen.

The entrance Mach number, M_1 is a function of the ratio of the nozzle's throat area to the nozzle exit area and its efficiency. This effect is the third parameter discussed here. Practically, the nozzle area ratio is changed by changing the throat area.

As was shown before, there are two different maximums for $\frac{4fL}{D}$; first is the total maximum $\frac{4fL}{D}$ of the supersonic which depends only on the specific heat, k, and second the maximum depends on the entrance Mach number, M_1. This analysis deals with the

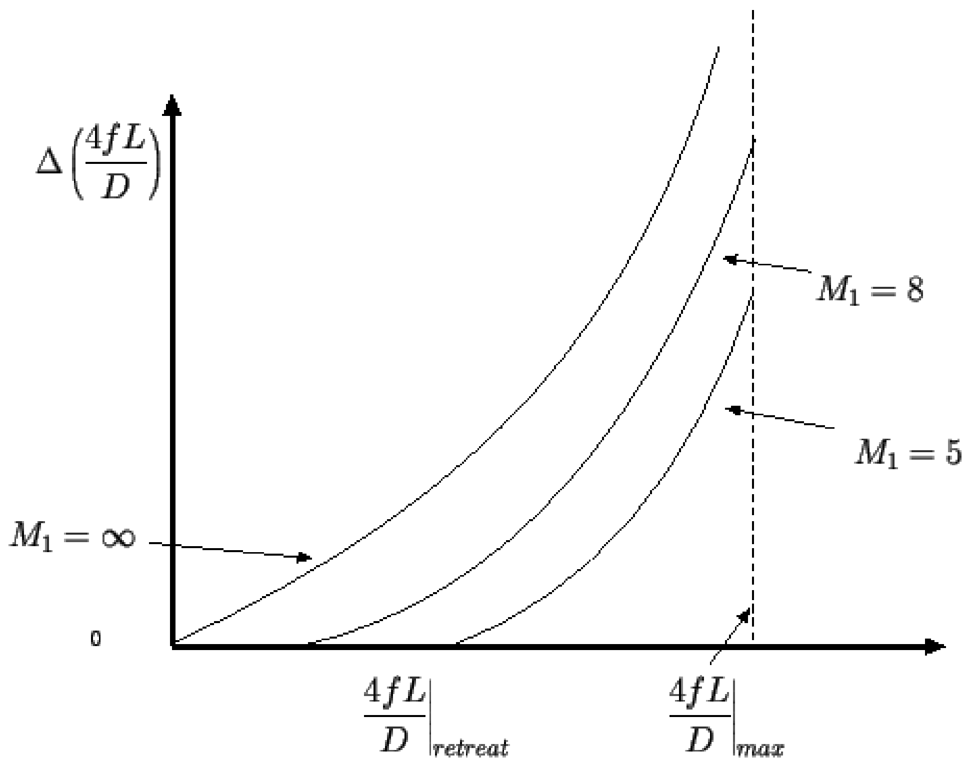
case where $\frac{4fL}{D}$ is shorter than total $\left.\frac{4fL}{D}\right|_{max}$.

Obviously, in this situation, the critical point is where $\frac{4fL}{D}$ is equal to $\left.\frac{4fL}{D}\right|_{max}$ as a result in the entrance Mach number.

The process of decreasing the converging-diverging nozzle's throat increases the entrance[9.14] Mach number. If the tube contains no supersonic flow then reducing the nozzle throat area wouldn't increase the entrance Mach number.

This part is for the case where some part of the tube is under supersonic regime and there is shock as a transition to subsonic branch. Decreasing the nozzle throat area moves the shock location downstream. The ``payment'' for increase in the supersonic length is by reducing the mass flow. Further, decrease of the throat area results in flushing the shock out of the tube. By doing so, the throat area decreases. The mass flow rate is proportionally linear to the throat area and therefore the mass flow rate reduces. The process of decreasing the throat area also results in increasing the pressure drop of the nozzle (larger resistance in the nozzle[9.15])[9.16].

In the case of large tube $\frac{4fL}{D} > \left.\frac{4fL}{D}\right|_{max}$ the exit Mach number increases with the decrease of the throat area. Once the exit Mach number reaches one no further increases is possible. However, the location of the shock wave approaches to the theoretical location if entrance Mach, $M_1 = \infty$.



The extra tube length as a function of the shock location, $\frac{4fL}{D}$ supersonic branch

Figure 9.16: The extra tube length as a function of the shock location

The maximum location of the shock

The main point in this discussion however, is to find the furthest shock location downstream. Figure (9.16) shows the possible $\Delta\left(\frac{4fL}{D}\right)$ as function of retreat of the location of the shock wave from the maximum location. When the entrance Mach number

is infinity, $M_1 = \infty$, if the shock location is at the maximum length, then shock at $M_x = 1$ results in $M_y = 1$.

The proposed procedure is based on Figure (9.16).

- Calculate the extra $\frac{4fL}{D}$ and subtract the actual extra $\frac{4fL}{D}$ assuming shock at the left side (at the max length).
- Calculate the extra $\frac{4fL}{D}$ and subtract the actual extra $\frac{4fL}{D}$ assuming shock at the right side (at the entrance).
- According to the positive or negative utilizes your root finding procedure.

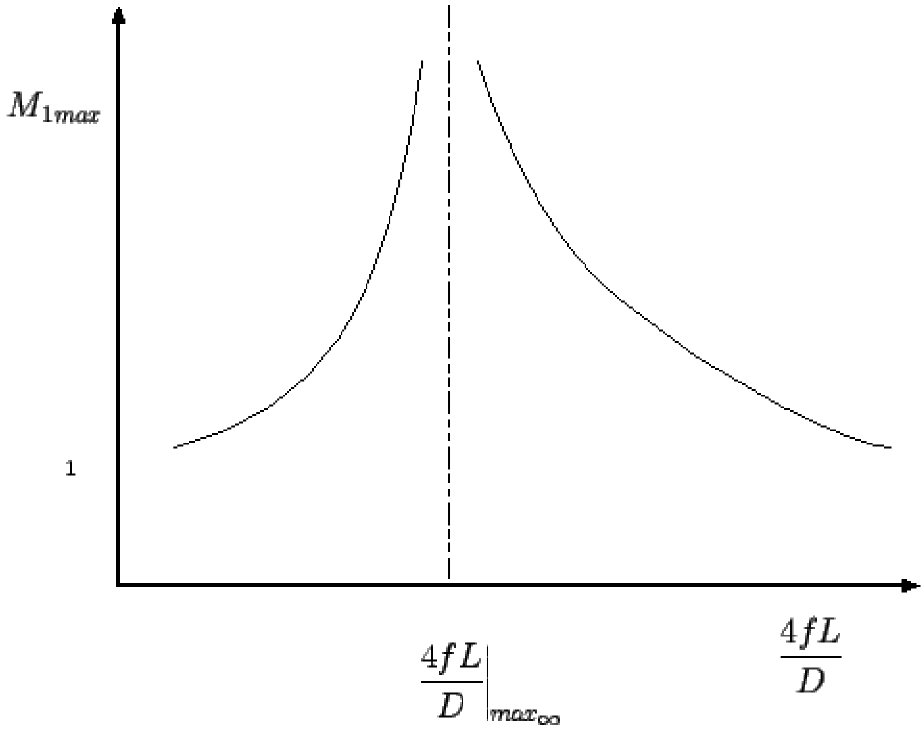


Figure: The maximum entrance Mach number as a function of 4fL/D

From numerical point of view, the Mach number equal infinity when left side assumes result in infinity length of possible extra (the whole flow in the tube is subsonic). To overcome this numerical problem it is suggested to start the calculation from ϵdistance from the right hand side.

Let denote

$$\Delta\left(\frac{4fL}{D}\right) = \frac{4\bar{f}L}{D}_{actual} - \left.\frac{4fL}{D}\right|_{sup} \tag{9.51}$$

Note that $\left.\frac{4fL}{D}\right|_{sup}$ is smaller than $\left.\frac{4fL}{D}\right|_{max_\infty}$. The requirement that has to be satisfied is that denote $\left.\frac{4fL}{D}\right|_{retreat}$ as difference between the maximum possible of length in which the supersonic flow is achieved and the actual length in which the flow is supersonic see Figure (9.15). The retreating length is expressed as subsonic but

$$\left.\frac{4fL}{D}\right|_{retreat} = \left.\frac{4fL}{D}\right|_{max_\infty} - \left.\frac{4fL}{D}\right|_{sup} \tag{9.52}$$

Figure (9.17) shows the entrance Mach number, M_1 reduces after the maximum length is exceeded.

The Approximation of the Fanno Flow by Isothermal Flow

The isothermal flow model has equations that theoreticians find easier to use and to compare to the Fanno flow model.

One must notice that the maximum temperature at the entrance is T_{01}. When the Mach number decreases the temperature approaches the stagnation temperature ($T \rightarrow T_0$). Hence, if one allows certain deviation of temperature, say about 1% that flow can be assumed to be isothermal. This tolerance requires that $(T_0 - T)/T_0 = 0.99$ which requires that enough for $M_1 < 0.15$ even for large $k = 1.67$. This requirement provides that somewhere (depend) in the vicinity of $\frac{4fL}{D} = 25$ the flow can be assumed isothermal.

Hence the mass flow rate is a function of $\frac{4fL}{D}$ because M_1 changes. Looking at the table or Figure (9.2) or the results from Potto-GDC attached to this book shows that reduction of the mass flow is very rapid.

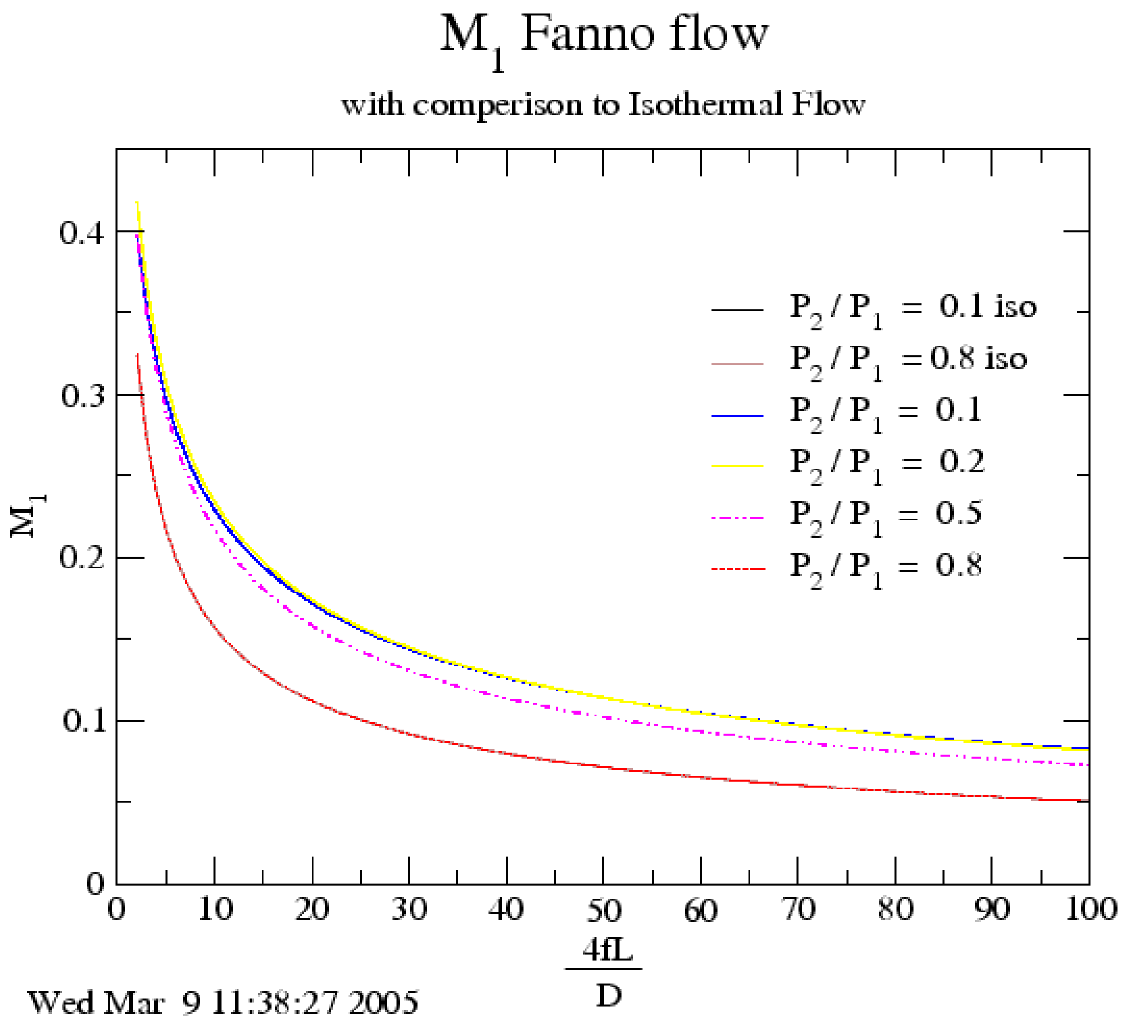


Figure: M1 as a function of 4fL/D comparison with Isothermal Flow

As it can be seen for the Figure (9.21) the dominating parameter is $\frac{4fL}{D}$. The results are very similar for isothermal flow. The only difference is in small dimensionless friction, $\frac{4fL}{D}$.

ubsonic Fanno Flow for Given 4fL/D and Pressure Ratio

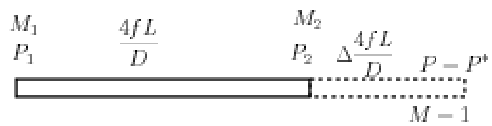


Figure: Unchoked flow calculations

This pair of parameters is the most natural to examine because, in most cases, this information is the only information that is provided. For a given pipe $\left(\frac{4fL}{D}\right)$, neither the entrance Mach number nor the exit Mach number are given (sometimes the entrance Mach number is give see the next section). There is no exact analytical solution. There are two possible approaches to solve this problem: one, by building a representative function and find a root (or roots) of this representative function. Two, the problem can be solved by an iterative procedure. The first approach require using root finding method and either method of spline method or the half method found to be good. However, this author experience show that these methods in this case were found to be relatively slow. The Newton-Rapson method is much faster but not were found to be unstable (at lease in the way that was implemented by this author). The iterative method used to solve constructed on the properties of several physical quantities must be in a certain range.. The first fact is that the pressure ratio P_2/P_1 is always between 0 and 1 (see Figure (9.18)). In the figure, a theoretical extra tube is added in such a length that cause the flow to choke (if it really was there). This length is always positive (at minimum is zero).

The procedure for the calculations is as the following:

- Calculate the entrance Mach number, $M_1{}'$ assuming the $\frac{4fL}{D} = \left.\frac{4fL}{D}\right|_{max}{}'$ (chocked flow);
- Calculate the minimum pressure ratio $(P_2/P_1)_{min}$ for $M_1{}'$ (look at table (9.1))
- Check if the flow is choked:

  There are two possibilities to check it.

  a)

  Check if the given $\frac{4fL}{D}$ is smaller than $\frac{4fL}{D}$ obtained from the given P_1/P_2, or

  b)

  check if the $(P_2/P_1)_{min}$ is larger than (P_2/P_1),

  continue if the criteria is satisfied. Or if not satisfied abort this procedure and continue to calculation for choked flow.

- Calculate the M_2 based on the $(P^*/P_2) = (P_1/P_2)$,
- calculate $\Delta\frac{4fL}{D}$ based on M_2,
- calculate the new (P_2/P_1), based on the new $f\left(\left(\frac{4fL}{D}\right)_1, \left(\frac{4fL}{D}\right)_2\right)$,

  (remember that $\Delta\frac{4fL}{D} = \left(\frac{4fL}{D}\right)_2$),

- calculate the corresponding M_1 and M_2,

- calculate the new and ``improve" the $\Delta\frac{4fL}{D}$ by

$$\left(\Delta\frac{4fL}{D}\right)_{new} = \left(\Delta\frac{4fL}{D}\right)_{old} * \frac{\left(\frac{P_2}{P_1}\right)_{given}}{\left(\frac{P_2}{P_1}\right)_{old}}$$

- Note, when the pressure ratios are matching also the $\Delta\frac{4fL}{D}$ will also match.
- Calculate the ``improved/new" M_2 based on the improve $\Delta\frac{4fL}{D}$ fanno flow! entrance Mach number calculations
- calculate the improved $\frac{4fL}{D}$ as $\frac{4fL}{D} = \left(\frac{4fL}{D}\right)_{given} + \Delta\left(\frac{4fL}{D}\right)_{new}$
- calculate the improved M_1 based on the improved $\frac{4fL}{D}$.
- Compare the abs ($(P_2/P_1)_{new} - (P_2/P_1)_{old}$) and if not satisfied returned to stage (6) until the solution is obtained.

To demonstrate how this procedure is working consider a typical example of $\frac{4fL}{D} = 1.7$ and $P_2/P_1 = 0.5$. Using the above algorithm the results are exhibited in the following figure.

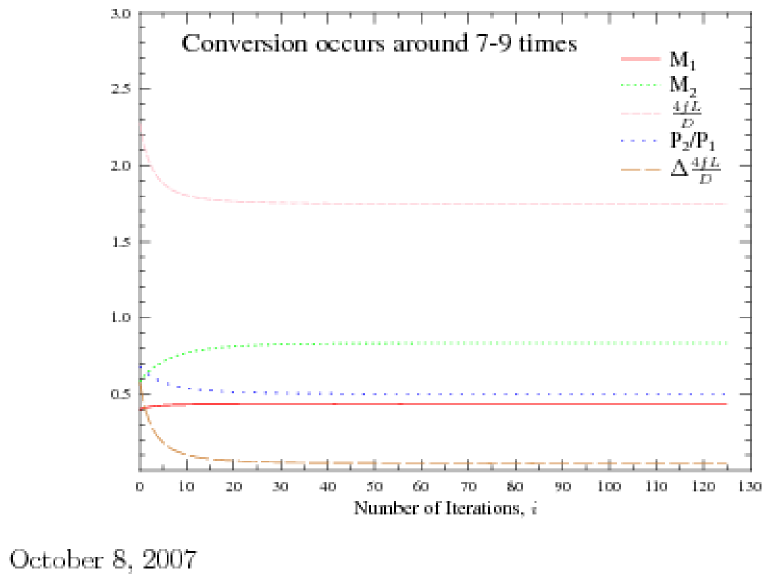


Figure 9.19: The results of the algorithm showing the conversion rate for unchoked Fanno flow model with a given $\frac{4fL}{D}$ and pressure ratio.

Figure (9.19) demonstrates that the conversion occur at about 7-8 iterations. With better first guess this conversion procedure will converts much faster (under construction).

Rayleigh Flow

Rayleigh flow is model describing a frictionless flow with heat transfer through a pipe of constant cross sectional area. In practice Rayleigh flow isn't a really good model for the real situation. Yet, Rayleigh flow is practical and useful concept in a obtaining trends and limits such as the density and pressure change due to external cooling or heating. As opposed to the two previous models, the heat transfer can be in two directions not like the

friction (there is no negative friction). This fact creates a situation different as compare to the previous two models. This model can be applied to cases where the heat transfer is significant and the friction can be ignored.

FLOW THROUGH CONSTANT AREA DUCT WITH HEAT TRANSFER

Fundamental equations

Continuity equation

$$m = \rho_1 A_1 c_1 = \rho_2 A_2 c_2$$

For constant area duct

$$m = \rho_1 c_1 = \rho_2 c_2$$

$$\frac{c_1}{c_2} = \frac{\rho_2}{\rho_1}$$

Momentum equations

$$P_1 A + m c_1 = P_2 A + m c_2$$

$$P_1 A - P_2 A = m c_2 - m c_1$$

$$(P_1 - P_2) A = m(c_2 - c_1)$$

$$(P_1 - P_2) A = \rho_2 A c_2 c_2 - \rho_1 A c_1 c_1$$

$$(P_1 - P_2) A = \rho_2 A c_2^2 - \rho_1 A c_1^2$$

$$(P_1 - P_2) = \rho_2 c_2^2 - \rho_1 c_1^2$$

$$(P_1 - P_2) = \frac{P_2}{RT_2} c_2^2 - \frac{P_1}{RT_1} c_1^2$$

$$(P_1 - P_2) = \frac{P_2}{RT_2} M_2^2 a_2^2 - \frac{P_1}{RT_1} M_1^2 a_1^2$$

$$(P_1 - P_2) = \frac{P_2}{RT_2} M_2^2 \gamma RT_2 - \frac{P_1}{RT_1} M_1^2 \gamma RT_1$$

$$(P_1 - P_2) = P_2 M_2^2 \gamma - P_1 M_1^2 \gamma$$

$$P_1 + P_1 M_1^2 \gamma = P_2 + P_2 M_2^2 \gamma$$

$$P_1 (1 + M_1^2 \gamma) = P_2 (1 + M_2^2 \gamma)$$

$$\frac{P_2}{P_1} = \frac{(1 + M_1^2 \gamma)}{(1 + M_2^2 \gamma)} \qquad (1)$$

Mach number

$$M_1 = \frac{c_1}{a_1} \quad \text{and} \quad M_2 = \frac{c_2}{a_2}$$

$$\frac{M_2}{M_1} = \frac{c_2 / a_2}{c_1 / a_1} = \frac{c_2}{c_1} * \frac{a_1}{a_2} = \frac{c_2}{c_1} * \frac{\sqrt{\gamma RT_1}}{\sqrt{\gamma RT_2}}$$

$$\frac{M_2}{M_1} = \frac{c_2}{c_1} * \left[\frac{T_1}{T_2} \right]^{1/2} \qquad (2)$$

Energy

$$Q = mC_p (T_{02} - T_{01}) \qquad (3)$$

Impulse function

$$F = (1 + \gamma M^2)$$

$$F_1 = (1 + \gamma M_1^2)$$

$$F_2 = (1 + \gamma M_2^2)$$

$$\frac{F_2}{F_1} = \frac{(1 + \gamma M_2^2)}{(1 + \gamma M_1^2)} * \frac{p_2}{p_1}$$

$$\frac{F_2}{F_1} = \frac{p_1}{p_2} * \frac{p_2}{p_1} = 1$$

(4) From eqn (1)

Stagnation pressure

WKT $\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$

$$\frac{p_{01}}{p_1} = \left[1 + \frac{\gamma - 1}{2} M_1^2\right]^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{p_{02}}{p_2} = \left[1 + \frac{\gamma - 1}{2} M_2^2\right]^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{p_{02} / p_2}{p_{01} / p_1} = \frac{\left[1 + \frac{\gamma - 1}{2} M_2^2\right]^{\frac{\gamma}{\gamma - 1}}}{\left[1 + \frac{\gamma - 1}{2} M_1^2\right]^{\frac{\gamma}{\gamma - 1}}}$$

$$\frac{p_{02}}{p_{01}} * \frac{p_1}{p_2} = \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{p_{02}}{p_{01}} = \frac{p_2}{p_1}\frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{p_{02}}{p_{01}} = \frac{(1+\gamma M_1^2)}{(1+\gamma M_2^2)}\frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}$$

Static temperature

WKT $$\frac{M_2}{M_1} = \frac{c_2}{c_1} * \left[\frac{T_1}{T_2}\right]^{1/2}$$

$$\left[\frac{T_1}{T_2}\right]^{1/2} = \frac{M_2}{M_1} * \frac{c_1}{c_2}$$

Isothermal Flow

Gas flow in long constant-area ducts, such as natural gas pipelines, is essentially

isothermal. Mach numbers in such flows are generally low, but significant pressure changes can occur as a result of frictional effects acting over long duct lengths. Hence, such flows cannot be treated as incompressible. The assumption of isothermal flow is much more appropriate.

For isothermal flow with friction (as opposed to the adiabatic flow with friction we previously discussed), the heat transfer d*Q/dm* is not zero. On the other hand, we have the simplification that the temperature is constant everywhere. As for adiabatic flow, we can start with our set of basic equations (Eqs. 13.1), describing onedimensional flow that is affected by area change, friction, heat transfer, and normal shocks,

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \rho V A = \dot{m} = \text{constant}$$

$$R_x + p_1 A_1 - p_2 A_2 = \dot{m} V_2 - \dot{m} V_1$$

$$\frac{\delta Q}{dm} + h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$\dot{m}(s_2 - s_1) \geq \int_{CS} \frac{1}{T}\left(\frac{\dot{Q}}{A}\right) dA$$

$$p = \rho R T$$

$$\Delta h = h_2 - h_1 = c_p \Delta T = c_p (T_2 - T_1)$$

$$\Delta s = s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

We can simplify these equations by setting D*T* _ 0, so *T*1 _ *T*2, and *A*1 _ *A*2 _ *A*. In addition we recall from Section 13-1 that the combination, *h* _ *V*2/2 is the *stagnation enthalpy*, *h*0. Using these, our final set of equations (renumbered for convenience) is

$$\rho_1 V_1 = \rho_2 V_2 = \rho V = G = \frac{\dot{m}}{A} = \text{constant}$$

$$R_x + p_1 A - p_2 A = \dot{m} V_2 - \dot{m} V_1$$

$$q = \frac{\delta Q}{dm} = h_{0_2} - h_{0_1} = \frac{V_2^2 - V_1^2}{2}$$

$$\dot{m}(s_2 - s_1) \geq \int_{CS} \frac{1}{T}\left(\frac{\dot{Q}}{A}\right) dA$$

$$p = \rho R T$$

$$\Delta s = s_2 - s_1 = -R \ln \frac{p_2}{p_1}$$

Equations 13.22 can be used to analyze frictional isothermal flow in a channel of constant area. For example, if we know conditions at section 1 (i.e., *p*1, r1, *T*1, *s*1, *h*1, and *V*1) we can use these equations to find conditions at some new section 2 after the fluid has experienced a total friction force *Rx*. We have five equations (not including the constraint of Eq. 13.22d) and five unknowns (*p*2, r2, *s*2, *V*2, and the heat transfer *q* that was necessary to maintain isothermal conditions). As we have seen before, in practice this procedure is unwieldy—we once again have a set of *nonlinear, coupled algebraic* equations to solve.

Before doing any calculations, we can see that the *Ts* diagram for this process will be simply a horizontal line passing through state 1 . To see in detail what happens to the flow, in addition to Eqs. 13.22, we can develop property relations as functions of the Mach number. For isothermal flow, *c* _ constant, so *V*2/*V*1 _ *M*2 /*M*1, and from Eq. 13.22a we have

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{M_1}{M_2}$$

Combining with the ideal gas equation, Eq. 13.22e, we obtain

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{M_1}{M_2}$$

At each state we can relate the local temperature to its stagnation temperature using Eq. 12.21b,

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

Applying this to states 1 and 2 , with the fact that *T*1 _ *T*2, we obtain

$$\frac{T_{0_2}}{T_{0_1}} = \frac{1 + \frac{k-1}{2}M_2^2}{1 + \frac{k-1}{2}M_1^2}$$

To determine the variation in Mach number along the duct length, it is necessary to consider the differential momentum equation for flow with friction. The analysis leading to Eq. 13.18 is valid for isothermal flow. Since *T* _ constant for isothermal flow, then from Eq. 13.18, with *dT* _ 0,

$$\frac{f}{D_h}\frac{kM^2}{2}dx = \left(\frac{1-kM^2}{2}\right)\frac{d(M^2)}{M^2}$$

$$\frac{f}{D_h}dx = \frac{(1-kM^2)d(M^2)}{kM^4}$$

Equation 13.25 shows (set *dx* _ 0) that the Mach number at which maximum length *L*max is reached is *M* _1/ ffiffiffi *k* p . Since *T* is constant, then the friction factor, *f* _ *f* (*Re*), is also constant. Integration of Eq. 13.25 between the limits of *M* _ *M* at *x* _ 0 and *M*_1/ ffiffiffi *k* p at *x* _ *L*max, where *L*max is the distance beyond which the isothermal flow may not proceed, gives

$$\frac{fL_{\max}}{D_h} = \frac{1 - kM^2}{kM^2} + \ln kM^2$$

The duct length, L, required for the flow Mach number to change from $M1$ to $M2$
can be obtained from

$$f\frac{L}{D_h} = f\,\frac{L_{\max_1} - L_{\max_2}}{D_h}$$
$$f\frac{L}{D_h} = \frac{1 - kM_1^2}{kM_1^2} - \frac{1 - kM_2^2}{kM_2^2} + \ln\frac{M_1^2}{M_2^2}$$

The distribution of heat exchange along the duct required to maintain isothermal flow can be determined from the differential form of Eq. 13.22c as

$$dq = dh_0 = c_p dT_0 = c_p d\left[T\left(1 + \frac{k-1}{2}M^2\right)\right]$$

$$dq = c_p T\left(\frac{k-1}{2}\right)dM^2 = \frac{c_p T_0 (k-1)}{2\left(1 + \frac{k-1}{2}M^2\right)}dM^2$$

$$dq = \frac{c_p T_0 (k-1) kM^4}{2\left(1 + \frac{k-1}{2}M^2\right)(1 - kM^2)}\frac{f}{D_h}dx$$

From Eq. 13.28 we note that as M ! 1/ ffiffiffi k

p

, then dq/dx ! _. Thus, an infinite rate of heat exchange is required to maintain isothermal flow as the Mach number approaches the limiting value. Hence, we conclude that isothermal acceleration of flow
in a constant-area duct is only physically possible for flow at low Mach number.

We summarize the set of Mach number–based equations (Eqs. 13.23, 13.24, and 13.27, respectively, renumbered) we can use for analysis of isothermal flow of an ideal gas in a duct with friction:

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{M_1}{M_2}$$

$$\frac{T_{0_2}}{T_{0_1}} = \frac{1 + \frac{k-1}{2} M_2^2}{1 + \frac{k-1}{2} M_1^2}$$

$$\frac{fL}{D_h} = \frac{1 - kM_1^2}{kM_1^2} - \frac{1 - kM_2^2}{kM_2^2} + \ln \frac{M_1^2}{M_2^2}$$

SUPERSONIC CHANNEL FLOW WITH SHOCKS (continued)

Supersonic Diffuser

Analysis of the effects of area change in isentropic flow (Section 13-2) showed that a converging channel reduces the speed of a supersonic stream; a converging channel is a *supersonic diffuser*. Because flow speed decreases, pressure rises in the flow direction, creating an adverse pressure gradient. Isentropic flow is not a completely accurate model for flow with an adverse pressure gradient,2 but the isentropic flow model with a normal shock may be used to demonstrate the basic features of supersonic diffusion.

For isentropic flow, a shock cannot stand in a stable position in a converging passage; a shock may stand stably only in a diverging passage. Real flow near *M*_1 is unstable, so it is not possible to reduce a supersonic flow exactly to sonic speed. The minimum Mach number that can be reached at a throat is 1.2 to 1.3.

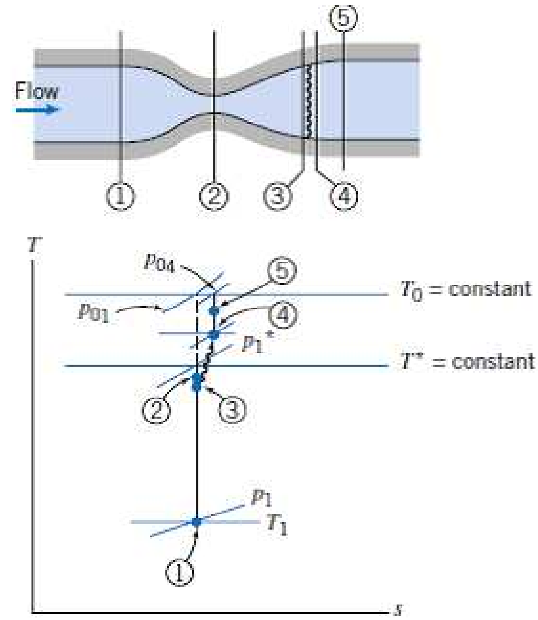


Fig. Schematic *Ts* diagram for flow in a supersonic diffuser with a normal shock.

Thus in real supersonic diffusers, flow is decelerated to *M*&1.3 in a converging passage. Downstream from the throat section of minimum area, the flow is allowed to accelerate to *M*&1.4, where a normal shock takes place. At this Mach number, the stagnation pressure loss (from Eq. 13.41b) is only about 4 percent. This small loss is an acceptable compromise in exchange for flow stability.

Figure 13.22 shows the idealized process of supersonic diffusion, in which flow is isentropic except across a normal shock. The slight reduction in stagnation pressure all takes place across the shock.

In the actual flow, additional losses in stagnation pressure occur during the supersonic and subsonic diffusion processes before and after the shock. Experimental data must be used to predict the actual losses in supersonic and subsonic diffusers.

Supersonic diffusion also is important for high-speed aircraft, where a supersonic

external free stream flow must be decelerated efficiently to subsonic speed. Some diffusion can occur outside the inlet by means of a weak oblique shock system. Variable geometry may be needed to accomplish efficient supersonic diffusion within the inlet as the flight Mach number varies. Multi-dimensional compressible flows are discussed in Section 13-7, and are treated in detail elsewhere.

Supersonic Wind Tunnel Operation

To build an efficient supersonic wind tunnel, it is necessary to understand shock behavior and to control shock location. The basic physical phenomena are described by Coles in the NCFMF video *Channel Flow of a Compressible Fluid*. In addition to *choking*—sonic flow at a throat, with upstream flow independent of downstream conditions—Coles discusses blocking and starting conditions for supersonic wind tunnels.

To accelerate flow to supersonic speed, followed by a test section of nearly constant area, and then a supersonic diffuser with a second throat. The circuit must be completed by compression machinery, coolers, and flow-control devices, as shown in Fig.

Consider the process of accelerating flow from rest to supersonic speed in the test section. Soon after flow at the nozzle throat becomes sonic, a shock wave forms in the divergence. The shock attains its maximum strength when it reaches the nozzle exit plane. Consequently, to *start* the tunnel and achieve steady supersonic flow in the test section, the shock must move through the second throat and into the subsonic diffuser. When this occurs, we say the shock has been *swallowed* by the second throat. Consequently, to start the tunnel, the supersonic diffuser throat must be larger than the nozzle throat. The second throat must be large enough to exceed the critical area for flow downstream from the strongest possible shock.

Blocking occurs when the second throat is not large enough to swallow the shock. When the channel is blocked, flow is sonic at both throats and flow in the test section is subsonic; flow in the test section cannot be controlled by varying conditions downstream from the supersonic diffuser.

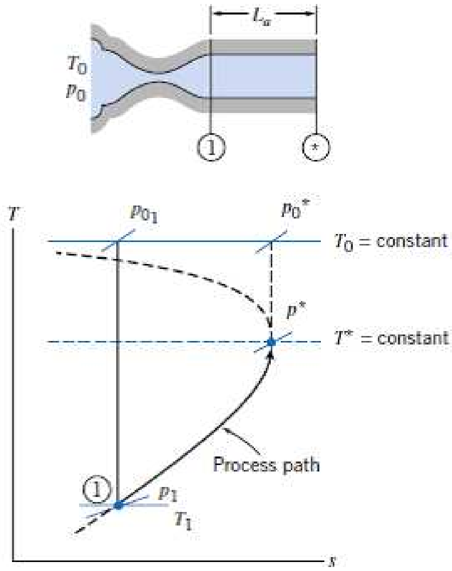
When the tunnel is *running* there is no shock in the nozzle or test section, so the energy

dissipation is much reduced. The second throat area may be reduced slightly during running to improve the diffuser efficiency. The compressor pressure ratio may be adjusted to move the shock in the subsonic diffuser to a lower Mach number. A combination of adjustable second throat and pressure ratio control may be used to achieve optimum running conditions for the tunnel. Small differences in efficiency are important when the tunnel drive system may consume more than half a million kilowatts.

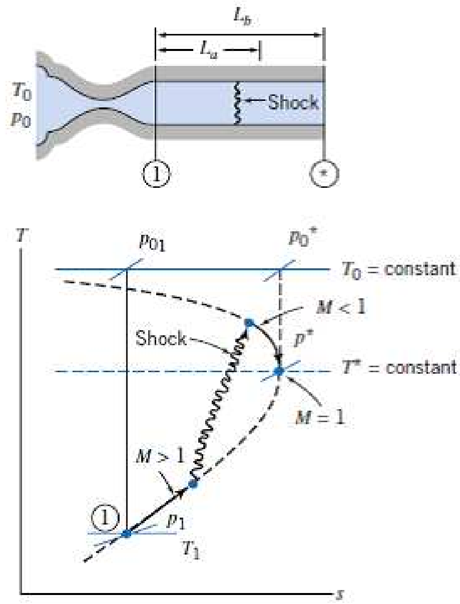
Supersonic Flow with Friction in a Constant-Area Channel

Flow in a constant-area channel with friction is dominated by viscous effects. Even when the main flow is supersonic, the no-slip condition at the channel wall guarantees subsonic flow near the wall. Consequently, supersonic flow in constant-area channels may form complicated systems of oblique and normal shocks. However, the basic behavior of adiabatic supersonic flow with friction in a constant-area channel is revealed by considering the simpler case of normal-shock formation in Fanno-line flow.

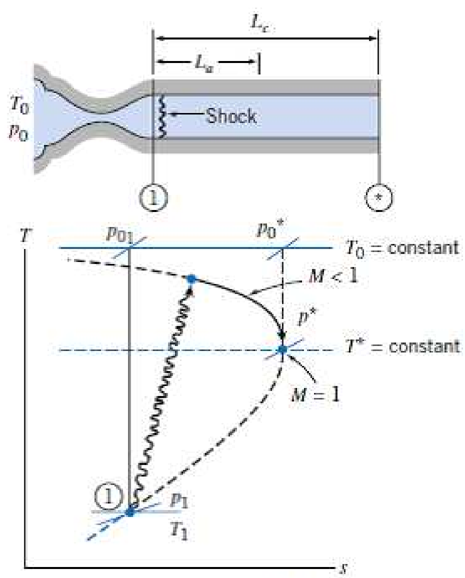
Supersonic flow along the Fanno line becomes choked after only a short length of duct, because at high speed the effects of friction are pronounced. Figure E.2 (Appendix E) shows that the limiting value of $f\,L_{max}/D_h$ is less than one; subsonic flows can have much longer runs. Thus when choking results from friction and duct length is increased further, the supersonic flow shocks down to subsonic to match downstream conditions. The *Ts* diagrams in Figs. 13.24*a* through 13.24*d* illustrate what happens when the length of constant-area duct, fed by a converging-diverging nozzle supplied from a reservoir with constant stagnation conditions, is increased. Supersonic flow on the Fanno line of Fig. 13.24*a* is choked by friction when the duct length is *La*. When additional duct is added to produce *Lb* _ *La*, Fig. 13.24*b*, a normal shock appears.Flow upstream from the shock does not change, because it is supersonic (no change in downstream condition can affect the supersonic flow before the shock).



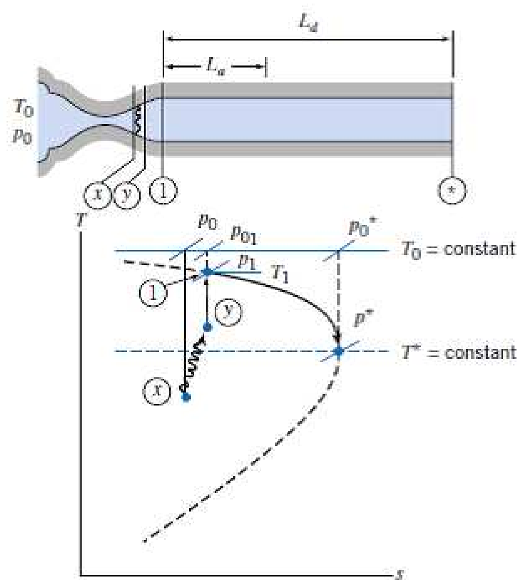
(*a*) Choked supersonic flow in channel.



(*b*) Choked flow in channel with shock.



(*c*) Choked flow with shock in nozzle exit plane.



(*d*) Choked flow with shock in nozzle; subsonic flow in channel.

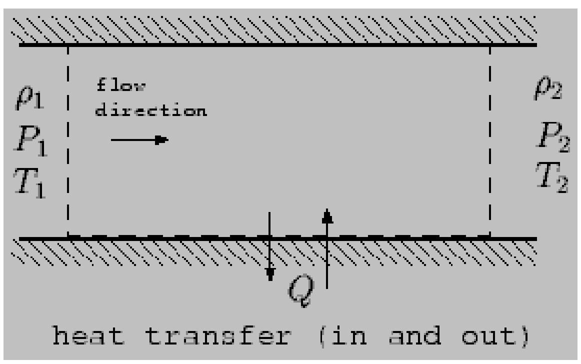
Fig. Schematic *Ts* diagrams for supersonic Fanno-line flows with normal shocks
In Fig. 13.24*b* the shock is shown in an arbitrary position. The shock moves toward the entrance of the constant-area channel (toward higher initial Mach number) as more duct is added.

Flow remains on the same Fanno line as the shock is driven upstream to state 1 by adding duct length; thus the mass flow rate remains unchanged. The duct length, *Lc*, which moves the shock into the channel entrance plane, Fig. 13.24*c*, may be calculated directly using the methods of Section 13-3. When duct length *Lc* is exceeded, the shock is driven back into the C-D nozzle, Fig. 13.24*d*. The mass flow rate remains constant until the shock reaches the nozzle throat. Only when more duct is added after the shock reaches the throat does the mass flow rate decrease, and the flow move to a new Fanno line. If the shock position is known, flow properties at each section and the duct length can be calculated directly. When length is specified and shock location is to be determined, iteration is necessary.

Figure 10.1: The control volume of Rayleigh Flow

Introduction

The third simple model for one-dimensional flow with constant heat transfer for frictionless flow. This flow is referred in the literature as Rayleigh Flow (see historical notes). This flow is another extreme case in which the friction effect is neglected because their relative effect is much smaller than the heat transfer effect. While the isothermal flow model has heat transfer and friction, the main assumption was that relative length is enables significant heat transfer to occur between the surroundings and tube. In contrast, the heat transfer in Rayleigh flow occurs between unknown temperature and the tube and the heat flux is maintained constant. As before, a simple model is built around the assumption of constant properties (poorer prediction to case where chemical reaction take a place).



This model is used to roughly predict the conditions which occur mostly in situations involving chemical reaction. In analysis of the flow, one has to be aware that properties do change significantly for a large range of temperatures. Yet, for smaller range of temperatures and lengths the calculations are more accurate. Nevertheless, the main characteristic of the flow such as a choking condition etc. is encapsulated in this model.

The basic physics of the flow revolves around the fact that the gas is highly compressible. The density changes through the heat transfer (temperature change). Contrary to Fanno flow in which the resistance always oppose the flow direction, Rayleigh flow, also, the cooling can be applied. The flow velocity acceleration change the direction when the cooling is applied.

Governing Equation

The energy balance on the control volume reads

$$Q = C_p\,(T_{02} - T_{01}) \tag{10.1}$$

the momentum balance reads

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1) \tag{10.2}$$

The mass conservation reads

$$\rho_1 U_1 A = \rho_2 U_2 A = \dot{m} \tag{10.3}$$

Equation of state

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \tag{10.4}$$

There are four equations with four unknowns, if the upstream conditions are known (or downstream conditions are known). Thus, a solution can be obtained. One can notice that equations (10.2), (10.3) and (10.4) are similar to the equations that were solved for the shock wave.

$$\frac{P_2}{P_1} = \frac{1 + k{M_1}^2}{1 + k{M_2}^2} \tag{10.5}$$

The equation of state (10.4) can further assist in obtaining the temperature ratio as

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} \qquad (10.6)$$

The density ratio can be expressed in terms of mass conservation as

$$\frac{\rho_1}{\rho_2} = \frac{U_2}{U_1} = \frac{\frac{U_2}{\sqrt{kRT_2}}\sqrt{kRT_2}}{\frac{U_1}{\sqrt{kRT_1}}\sqrt{kRT_1}} = \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}} \qquad (10.7)$$

Substituting equations (10.5) and (10.7) into equation (10.6) yields

$$\frac{T_2}{T_1} = \frac{1+kM_1{}^2}{1+kM_2{}^2}\frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}} \qquad (10.8)$$

Transferring the temperature ratio to the left hand side and squaring the results gives

$$\frac{T_2}{T_1} = \left[\frac{1+kM_1{}^2}{1+kM_2{}^2}\right]^2\left(\frac{M_2}{M_1}\right)^2 \qquad (10.9)$$

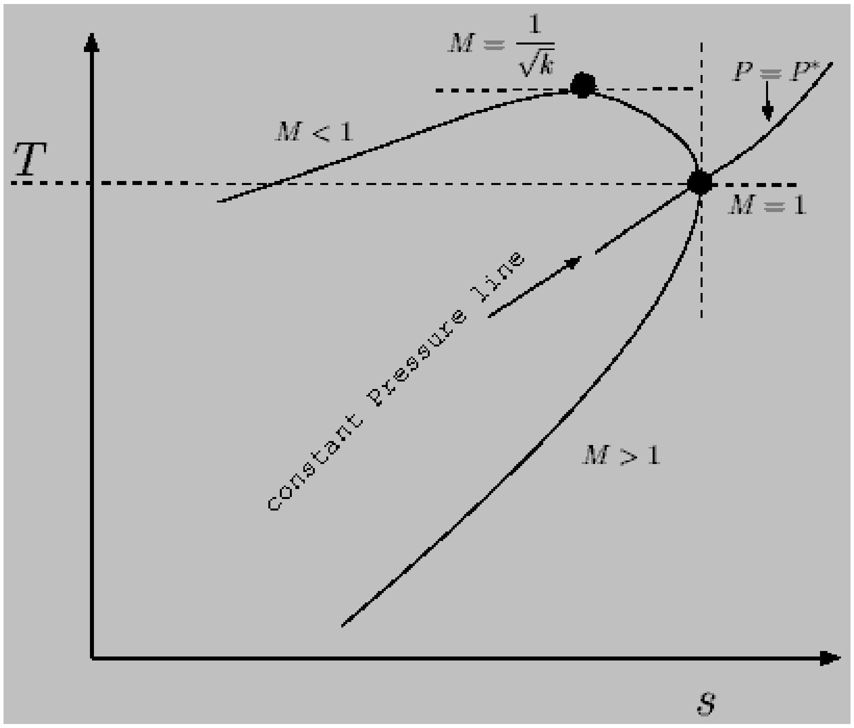


Figure 10.2: The temperature entropy diagram for Rayleigh line

The Rayleigh line exhibits two possible maximums one for $dT/ds = 0$ and for $ds/dT = 0$. The second maximum can be expressed as $dT/ds = \infty$. The second law is used to find the expression for the derivative.

$$\frac{s_1 - s_2}{C_p} = \ln\frac{T_2}{T_1} - \frac{k-1}{k}\ln\frac{P_2}{P_1} \tag{10.10}$$

$$\frac{s_1 - s_2}{C_p} = 2\ln\left[(\frac{1+kM_1{}^2)}{(1+kM_2{}^2)}\frac{M_2}{M_1}\right] + \frac{k-1}{k}\ln\left[\frac{1+kM21^2}{1+kM_1{}^2}\right] \tag{10.11}$$

Let the initial condition M_1, and s_1 be constant and the variable parameters are M_2, and s_2. A derivative of equation (10.11) results in

$$\frac{1}{C_p}\frac{ds}{dM} = \frac{2(1-M^2)}{M(1+kM^2)} \tag{10.12}$$

Taking the derivative of equation (10.12) and letting the variable parameters be T_2, and M_2 results in

$$\frac{dT}{dM} = constant \times \frac{1-kM^2}{(1+kM^2)^3} \tag{10.13}$$

Combining equations (10.12) and (10.13) by eliminating dM results in

$$\frac{dT}{ds} = constant \times \frac{M(1-kM^2)}{(1-M^2)(1+kM^2)^2} \tag{10.14}$$

On T-s diagram a family of curves can be drawn for a given constant. Yet for every curve, several observations can be generalized. The derivative is equal to zero when

$1 - kM^2 = 0$ or $M = 1/\sqrt{k}$ or when $M \to 0$. The derivative is equal to infinity, $dT/ds = \infty$ when $M = 1$. From thermodynamics, increase of heating results in increase of entropy. And cooling results in reduction of entropy. Hence, when cooling is applied to a tube the velocity decreases and when heating is applied the velocity increases. At peculiar point of $M = 1/\sqrt{k}$ when additional heat is applied the temperature decreases. The derivative is negative, $dT/ds < 0$, yet note this point is not the choking point. The choking occurs only when $M = 1$because it violates the second law. The transition to supersonic flow occurs when the area changes, somewhat similarly to Fanno flow. Yet, choking can be explained by the fact that increase of energy must be accompanied by increase of entropy. But the entropy of supersonic flow is lower (see Figure (10.2)) and therefore it is not possible (the maximum entropy at $M = 1$.).

It is convenient to refer to the value of $M = 1$. These values are referred to as the ``star"[10.1]values. The equation (10.5) can be written between choking point and any point on the curve.

$$\frac{P^*}{P_1} = \frac{1 + kM_1{}^2}{1 + k} \tag{10.15}$$

The temperature ratio is

$$\frac{T^*}{T_1} = \frac{1}{M^2}\left(\frac{1 + kM_1{}^2}{1 + k}\right)^2 \tag{10.16}$$

$$\frac{\rho_1}{\rho^*} = \frac{U^*}{U_1} = \frac{\frac{U^*}{\sqrt{kRT^*}}\sqrt{kRT^*}}{\frac{U_1}{\sqrt{kRT_1}}\sqrt{kRT_1}} = \frac{1}{M_1}\sqrt{\frac{T^*}{T_1}} \tag{10.17}$$

$$\frac{T_{01}}{T_0{}^*} = \frac{T_1\left(1+\frac{k-1}{2}M_1{}^2\right)}{T^*\left(\frac{1+k}{2}\right)} = \frac{2(1+k)M_1{}^2}{(1+kM^2)^2}\left(1+\frac{k-1}{2}M_1{}^2\right) \tag{10.18}$$

The stagnation pressure ratio reads

$$\frac{P_{01}}{P_0{}^*} = \frac{P_1\left(1+\frac{k-1}{2}M_1{}^2\right)}{P^*\left(\frac{1+k}{2}\right)} = \left(\frac{1+k}{1+kM_1{}^2}\right)\left(\frac{1+kM_1{}^2}{\frac{(1+k)}{2}}\right)^{\frac{k}{k-1}} \tag{10.19}$$

UNIT-2 **FLOW THROUGH DUCTS**

1 What are the assumptions that are considered for the derivation of the equation of the fanno flow?

The assumptions are

a. Free from work and heat transfer.

b. Free from area change

c. Free from gravitational effects

2 Differentiate Fanno flow and Rayleigh flow?

Rayleigh flow:

Flow in a constant area duct with heat transfer and without friction is known as Rayleighs flow.

Fanno Flow:

Flow in a constant area duct with friction and without heat transfer is known as Fanno flow

3 Explain chocking in Fanno flow?

In a fanno flow, subsonic flow region, the effect of friction will increase the velocity and Mach number and to decrease the enthalpy and pressure of the gas. In supersonic flow region, the effect of friction will decrease the velocity and Mach number and to increase the enthalpy and pressure of the gas. In both cases entropy increases up to limiting state where the Mach number is one(M=1) and it is constant afterwards. At this point flow is said to be chocked flow.

4 Give two practical examples where the Fanno flow occurs?

- Flow in air breathing engines
- Flow in refrigeration and air conditioning
- Flow of fluids in long pipes

5 What is Rayleigh line and Fanno line?

Rayleigh line:

Flow in a constant duct area with heat transfer and without friction is described by a curve is

known as Rayleigh line.

Fanno Line:

Flow in a constant duct area without heat transfer and with friction is described by a curve is

known as Fanno line.

6 Explain the difference between Fanno flow and Isothermal flow?

Fanno flow

• Flow in a constant area duct with friction and without heat transfer is known as fanno flow.

• Static temperature is not constant

I s o t h e r m a l flow

• Flow in a constant area duct with friction and the heat transfer is known as isothermal flow.

• Static temperature remains constant.

7 Define fanning's coefficient of skin friction

It is the ration between wall shear stress and dynamic head it is denoted by 'f'

8 What are the three equation governing Fanno flow?

• Energy equation

• Continuity equation

• Equation of state

9 Give two practical examples for the Rayleigh flow.

Subsonic and supersonic heating process for air through the control volume.

10 Under what conditions the assumption of Rayleigh flow is not valid in a heat exchanger.

Under stagnation conditions, the assumption of Rayleigh flow is not valid in a heat exchanger.

11 State the assumptions made fro Rayleigh flow.

i Perfect gas with constant specific heats and molecular weight.

ii Constant area duct,

iii One dimensional, steady frictionless flow with heat transfer.

iv Absence of body forces.

Note: Answer is available from HMT data book.

PART B

1) A circular duct passes 8.25Kg/s of air at an exit Mach number of 0.5. The entry pressure and

temperature are 3.45 bar and 38°C respectively and the coefficient of friction 0.005.If the Mach

number at entry is 0.15, determine :

I. The diameter of the duct , (2)

II. Length of the duct, (4)

III. Pressure and temperature at the exit, (4)

IV. Stagnation pressure loss, and (4)

V. Verify the exit Mach number through exit velocity and temperature. (2)

2) A gas (γ =1.3,R=0.287 KJ/KgK) at p1 =1bar, T1 =400 k enters a 30cm diameter duct at a Mach number of 2.0.A normal shock occurs at a Mach number of 1.5 and the exit Mach number is1.0, If the mean value of the friction factor is 0.003 determine:

1)Lengths of the duct upstream and downstream of the shock wave, (6)

2)Mass flow rate of the gas and (4)

downstream of the shock. (6)

3) Air enters a long circular duct (d =12.5cm,f=0.0045) at a Mach number 0.5, pressure 3.0 bar and temperature 312 K.If the flow is isothermal throughout the duct determine (a) the length of the duct required to change the Mach number to 0.7,(b) pressure and temperature of air at M =0.7 (c) the lengthof the duct required to attain limiting Mach number, and (d) state of air at the limiting Mach umber.compare these values with those obtained in adiabatic flow. (16)

4) A convergent –divergent nozzle is provided with a pipe of constant cross-section at its exit

;the exit diameter of the nozzle and that of the pipe is 40cm. The mean coefficient of friction for the pipe is 0.0025. Stagnation pressure and temperature of air at the nozzle entry are 12 bar and 600k. The flow is isentropic in the nozzle and adiabatic in the pipe.The Mach numbers at the entry and exit of the pipe are 1.8 and 1.0 respectively .Determine

a) The length of the pipe , (4)

b) Diameter of the nozzle throat,and (6)

c) Pressure and temperature at the pipe exit. (6)

5) Show that the upper and lower branches of a Fanno curve represent subsonic and supersonic flows respectively . prove that at the maximum entropy point Mach number is unity and all processes approach this point .How would the state of a gas in a flow change from the

supersonic to subsonic branch ? (16)

Flow in constant area ducts with heat transfer(Rayleigh flow)

6) The Mach number at the exit of a combustion chamber is 0.9. The ratio of stagnation temperature at exit and entry is 3.74. If the pressure and temperature of the gas at exit are 2.5 bar and 1000°C respectively determine (a) Mach number, pressure and temperature of the gas at entry, (b) the heat supplied per kg of the gas and (c) the maximum heat that can be supplied. Take γ= 1.3, Cp= 1.218 KJ/KgK (16)

7) The conditions of a gas in a combuster at entry are: $P1$=0.343 bar ,$T1$ = 310K ,$C1$= 60m/s.Detemine the Mach number ,pressure ,temperature and velocity at the exit if the increase in stagnation enthalpy of the gas between entry and exit is 1172.5KJ/Kg. Take Cp=1.005KJ/KgK, γ =1.4 (16)

8) A combustion chamber in a gas turbine plant receives air at 350 K ,0.55bar and 75 m/s .The air –fuel ratio is 29 and the calorific value of the fuel is 41.87 MJ/Kg .Taking γ=1.4 and R =0.287 KJ/kg K for the gas determine.

a) The initial and final Mach numbers, (4)

b) Final pressure, temperature and velocity of the gas, (4)

c) Percent stagnation pressure loss in the combustion chamber, and (4)

d) The maximum stagnation temperature attainable. (4)

9) Obtain an equation representing the Rayleigh line . Draw Rayleigh lines on the h-s and p-v planes for two different values of the mass flux. Show that the slope of the Rayleigh line on the p-v plane is {dp/dv} = þ^2 c^2 (16)

UNIT III NORMAL AND OBLIQUE SHOCKS

Normal Shock

In this chapter the relationships between the two sides of normal shock are presented. In this discussion, the flow is assumed to be in a steady state, and the thickness of the shock is assumed to be very small. A discussion on the shock thickness will be presented in a forthcoming section[5.1].

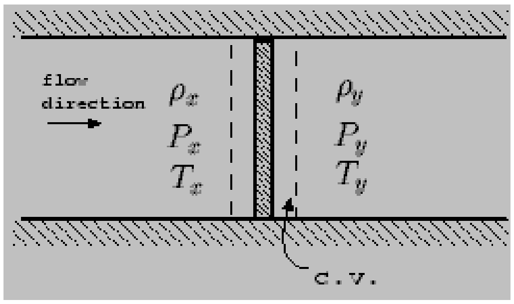


Figure: A shock wave inside a tube, but it can also be viewed as a one-dimensional shock wave.

A shock can occur in at least two different mechanisms. The first is when a large difference (above a small minimum value) between the two sides of a membrane, and when the membrane bursts (see the discussion about the shock tube). Of course, the shock travels from the high pressure to the low pressure side. The second is when many sound waves ``run into'' each other and accumulate (some refer to it as ``coalescing'') into a large difference, which is the shock wave. In fact, the sound wave can be viewed as an extremely weak shock. In the speed of sound analysis, it was assumed the medium is continuous, without any abrupt changes. This assumption is no longer valid in the case of a shock. Here, the relationship for a perfect gas is constructed.

In Figure (5.1) a control volume for this analysis is shown, and the gas flows from left to right. The conditions, to the left and to the right of the shock, are assumed to be uniform[5.2]. The conditions to the right of the shock wave are uniform, but different from the left side. The transition in the shock is abrupt and in a very narrow width.

The chemical reactions (even condensation) are neglected, and the shock occurs at a very narrow section. Clearly, the isentropic transition assumption is not appropriate in this case

because the shock wave is a discontinued area. Therefore, the increase of the entropy is fundamental to the phenomenon and the understanding of it.

It is further assumed that there is no friction or heat loss at the shock (because the heat transfer is negligible due to the fact that it occurs on a relatively small surface). It is customary in this field to denote x as the upstream condition and y as the downstream condition.

The mass flow rate is constant from the two sides of the shock and therefore the mass balance is reduced to

$$\rho_x U_x = \rho_y U_y \tag{5.1}$$

In a shock wave, the momentum is the quantity that remains constant because there are no external forces. Thus, it can be written that

$$P_x - P_y = \left(\rho_x {U_y}^2 - \rho_y {U_x}^2\right) \tag{5.2}$$

The process is adiabatic, or nearly adiabatic, and therefore the energy equation can be written as

$$C_p T_x + \frac{{U_x}^2}{2} = C_p T_y + \frac{{U_y}^2}{2} \tag{5.3}$$

The equation of state for perfect gas reads

$$P = \rho R T \tag{5.4}$$

If the conditions upstream are known, then there are four unknown conditions downstream. A system of four unknowns and four equations is solvable. Nevertheless,

one can note that there are two solutions because of the quadratic of equation (5.3). These two possible solutions refer to the direction of the flow. Physics dictates that there is only one possible solution. One cannot deduce the direction of the flow from the pressure on both sides of the shock wave. The only tool that brings us to the direction of the flow is the second law of thermodynamics. This law dictates the direction of the flow, and as it will be shown, the gas flows from a supersonic flow to a subsonic flow. Mathematically, the second law is expressed by the entropy. For the adiabatic process, the entropy must increase. In mathematical terms, it can be written as follows:

$$s_y - s_x > 0 \tag{5.5}$$

Note that the greater-equal signs were not used. The reason is that the process is irreversible, and therefore no equality can exist. Mathematically, the parameters are $P, T, U,$ and ρ, which are needed to be solved. For ideal gas, equation (5.5) is

$$\ln \frac{T_y}{T_x} - (k-1)\frac{P_y}{P_x} > 0 \tag{5.6}$$

It can also be noticed that entropy, s, can be expressed as a function of the other parameters. Now one can view these equations as two different subsets of equations. The first set is the energy, continuity, and state equations, and the second set is the momentum, continuity, and state equations. The solution of every set of these equations produces one additional degree of freedom, which will produce a range of possible solutions. Thus, one can have a whole range of solutions. In the first case, the energy equation is used, producing various resistance to the flow. This case is called Fanno flow, and Chapter (9) deals extensively with this topic. The mathematical explanation is given Chapter (9) in greater detail. Instead of solving all the equations that were presented, one can solve only four (4) equations (including the second law), which will require

additional parameters. If the energy, continuity, and state equations are solved for the arbitrary value of the T_y , a parabola in the T-s diagram will be obtained. On the other hand, when the momentum equation is solved instead of the energy equation, the degree of freedom is now energy, i.e., the energy amount ``added'' to the shock. This situation is similar to a frictionless flow with the addition of heat, and this flow is known as Rayleigh flow. This flow is dealt with in greater detail in Chapter (10).

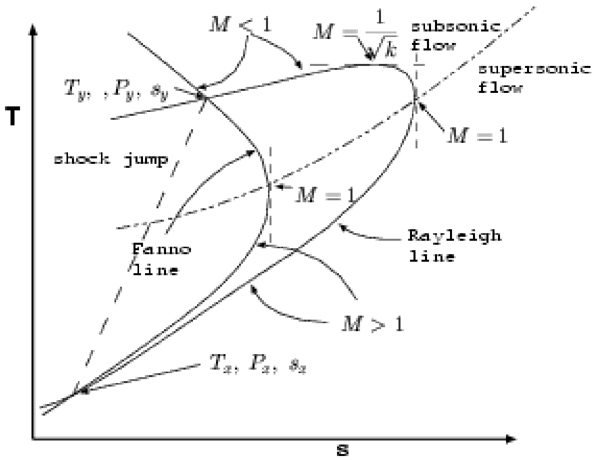


Figure: The intersection of Fanno flow and Rayleigh flow produces two solutions for the shock wave.

Since the shock has no heat transfer (a special case of Rayleigh flow) and there isn't essentially any momentum transfer (a special case of Fanno flow), the intersection of these two curves is what really happened in the shock. In Figure (5.2), the intersection is shown and two solutions are obtained. Clearly, the increase of the entropy determines the direction of the flow. The entropy increases from point x to point y. It is also worth noting that the temperature at M=1 on Rayleigh flow is larger than that on the Fanno line.

Informal Model

Accepting the fact that the shock is adiabatic or nearly adiabatic requires that total energy is conserved, $T_{0x} = T_{0y}$. The relationship between the temperature and the stagnation temperature provides the relationship of the temperature for both sides of the shock.

$$\frac{T_y}{T_x} = \frac{\frac{T_y}{T_{0_y}}}{\frac{T_x}{T_{0_x}}} = \frac{1+\frac{k-1}{2}{M_x}^2}{1+\frac{k-1}{2}{M_y}^2} \tag{5.7}$$

All the other relationships are essentially derived from this equation. The only issue left to derive is the relationship between M_x and M_y. Note that the Mach number is a function of temperature, and thus for known M_x all the other quantities can be determined, at least, numerically. The analytical solution is discussed in the next section.

Formal Model

Equations (5.1), (5.2), and (5.3) can be converted into a dimensionless form. The reason that dimensionless forms are heavily used in this book is because by doing so it simplifies and clarifies the solution. It can also be noted that in many cases the dimensionless equations set is more easily solved.

From the continuity equation (5.1) substituting for density, ρ, the equation of state yields

$$\frac{P_x}{RT_x}U_x = \frac{P_y}{RT_y}U_y \tag{5.8}$$

Squaring equation (5.8) results in

$$\frac{P_x^{\,2}}{R^2 T_x^{\,2}} U_x^{\,2} = \frac{P_y^{\,2}}{R^2 T_y^{\,2}} U_y^{\,2} \tag{5.9}$$

Multiplying the two sides by the ratio of the specific heat, k, provides a way to obtain the speed of sound definition/equation for perfect gas, $c^2=kRT$ to be used for the Mach number definition, as follows:

$$\frac{P_x^{\,2}}{T_x \underbrace{kRT_x}_{c_x^{\,2}}} U_x^{\,2} = \frac{P_y^{\,2}}{T_y \underbrace{kRT_y}_{c_y^{\,2}}} U_y^{\,2} \tag{5.10}$$

Note that the speed of sound on the different sides of the shock is different. Utilizing the definition of Mach number results in

$$\frac{P_x^{\,2}}{T_x} M_x^{\,2} = \frac{P_y^{\,2}}{T_y} M_y^{\,2} \tag{5.11}$$

Rearranging equation (5.11) results in

$$\frac{T_y}{T_x} = \left(\frac{P_y}{P_x}\right)^2 \left(\frac{M_y}{M_x}\right)^2 \tag{5.12}$$

Energy equation (5.3) can be converted to a dimensionless form which can be expressed as

$$T_y \left(1 + \frac{k-1}{2} M_y^{\,2}\right) = T_x \left(1 + \frac{k-1}{2} M_x^{\,2}\right) \tag{5.13}$$

It can also be observed that equation (5.13) means that the stagnation temperature is the same, $T_{0y}= T_{0x}$. Under the perfect gas model, ρU^2 is identical to k P M^2 because

$$\rho U^2 = \overbrace{\frac{P}{RT}}^{\rho} \overbrace{\left(\frac{U^2}{\underbrace{kRT}_{c^2}} \right)}^{M^2} kRT = kPM2 \tag{5.14}$$

Using the identity (5.14) transforms the momentum equation (5.2) into

$$P_x + kP_x{M_x}^2 = P_y + kP_y{M_y}^2 \tag{5.15}$$

Rearranging equation (5.15) yields

$$\frac{P_y}{P_x} = \frac{1 + k{M_x}^2}{1 + k{M_y}^2} \tag{5.16}$$

The pressure ratio in equation (5.16) can be interpreted as the loss of the static pressure. The loss of the total pressure ratio can be expressed by utilizing the relationship between the pressure and total pressure (see equation (4.11)) as

$$\frac{P_{0_y}}{P_{0_x}} = \frac{P_y \left(1 + \frac{k-1}{2}{M_y}^2\right)^{\frac{k}{k-1}}}{P_x \left(1 + \frac{k-1}{2}{M_x}^2\right)^{\frac{k}{k-1}}} \tag{5.17}$$

The relationship between M_x and M_y is needed to be solved from the above set of

equations. This relationship can be obtained from the combination of mass, momentum, and energy equations. From equation (5.13) (energy) and equation (5.12) (mass) the temperature ratio can be eliminated.

$$\left(\frac{P_y M_y}{P_x M_x}\right)^2 = \frac{1 + \frac{k-1}{2} {M_x}^2}{1 + \frac{k-1}{2} {M_y}^2} \tag{5.18}$$

Combining the results of (5.18) with equation (5.16) results in

$$\left(\frac{1 + k{M_x}^2}{1 + k{M_y}^2}\right)^2 = \left(\frac{M_x}{M_y}\right)^2 \frac{1 + \frac{k-1}{2} {M_x}^2}{1 + \frac{k-1}{2} {M_y}^2} \tag{5.19}$$

Equation (5.19) is a symmetrical equation in the sense that if M_y is substituted with M_x and M_x substituted with M_y the equation remains the same. Thus, one solution is

$$M_y = M_x \tag{5.20}$$

It can be observed that equation (5.19) is biquadratic. According to the Gauss Biquadratic Reciprocity Theorem this kind of equation has a real solution in a certain range[5.3]which will be discussed later. The solution can be obtained by rewriting equation (5.19) as a polynomial (fourth order). It is also possible to cross-multiply equation (5.19) and divide it by (M_x^2 - M_y^2) results in

$$1 + \frac{k-1}{2}\left({M_y}^2 + {M_y}^2\right) - k{M_y}^2{M_y}^2 = 0 \tag{5.21}$$

Equation (5.21) becomes

$$M_y{}^2 = \frac{M_x{}^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_x{}^2 - 1} \tag{5.22}$$

The first solution (5.20) is the trivial solution in which the two sides are identical and no shock wave occurs. Clearly, in this case, the pressure and the temperature from both sides of the nonexistent shock are the same, i.e. $T_x = T_x$; $P_x = P_x$. The second solution is where the shock wave occurs.

The pressure ratio between the two sides can now be as a function of only a single Mach number, for example, M_x. Utilizing equation (5.16) and equation (5.22) provides the pressure ratio as only a function of the upstream Mach number as

$$\frac{P_y}{P_x} = \frac{2k}{k+1}M_x{}^2 - \frac{k-1}{k+1}$$

$$\frac{P_y}{P_x} = 1 + \frac{2k}{k+1}\left(M_x{}^2 - 1\right) \tag{5.23}$$

The density and upstream Mach number relationship can be obtained in the same fashion to became

$$\frac{\rho_y}{\rho_x} = \frac{U_x}{U_y} = \frac{(k+1)M_x{}^2}{2+(k-1)M_x{}^2} \tag{5.24}$$

The fact that the pressure ratio is a function of the upstream Mach number, M_x, provides additional way of obtaining an additional useful relationship. And the temperature ratio,

as a function of pressure ratio, is transformed into

$$\frac{T_y}{T_x} = \left(\frac{P_y}{P_x}\right)\left(\frac{\frac{k+1}{k-1} + \frac{P_y}{P_x}}{1 + \frac{k+1}{k-1}\frac{P_y}{P_x}}\right) \tag{5.25}$$

In the same way, the relationship between the density ratio and pressure ratio is

$$\frac{\rho_x}{\rho_y} = \frac{1 + \left(\frac{k+1}{k-1}\right)\left(\frac{P_y}{P_x}\right)}{\left(\frac{k+1}{k-1}\right) + \left(\frac{P_y}{P_x}\right)} \tag{5.26}$$

which is associated with the shock wave.

Figure: The exit Mach number and the stagnation pressure ratio as a function of upstream Mach number.

The Maximum Conditions

The maximum speed of sound is when the highest temperature is achieved. The maximum temperature that can be achieved is the stagnation temperature

$$U_{max} = \sqrt{\frac{2k}{k-1}RT_0} \tag{5.27}$$

The stagnation speed of sound is

$$c_0 = \sqrt{kRT_0} \tag{5.28}$$

Based on this definition a new Mach number can be defined

$$M_0 = \frac{U}{c_0} \tag{5.29}$$

he Star Conditions

The speed of sound at the critical condition can also be a good reference velocity. The speed of sound at that velocity is

$$c^* = \sqrt{kRT^*} \tag{5.30}$$

In the same manner, an additional Mach number can be defined as

$$M^* = \frac{U}{c^*} \tag{5.31}$$

Prandtl's Condition

It can be easily observed that the temperature from both sides of the shock wave is discontinuous. Therefore, the speed of sound is different in these adjoining mediums. It is therefore convenient to define the star Mach number that will be independent of the specific Mach number (independent of the temperature).

$$M^* = \frac{U}{c^*} = \frac{c}{c^*}\frac{U}{c} = \frac{c}{c^*}M \tag{5.32}$$

The jump condition across the shock must satisfy the constant energy.

$$\frac{c^2}{k-1} + \frac{U^2}{2} = \frac{c^{*2}}{k-1} + \frac{c^{*2}}{2} = \frac{k+1}{2(k-1)}c^{*2} \tag{5.33}$$

Dividing the mass equation by the momentum equation and combining it with the perfect gas model yields

$$\frac{{c_1}^2}{kU_1} + U_1 = \frac{{c_2}^2}{kU_2} + U_2 \tag{5.34}$$

Combining equation (5.33) and (5.34) results in

$$\frac{1}{kU_1}\left[\frac{k+1}{2}{c^*}^2 - \frac{k-1}{2}U_1\right] + U_1 = \frac{1}{kU_2}\left[\frac{k+1}{2}{c^*}^2 - \frac{k-1}{2}U_2\right] + U_2 \tag{5.35}$$

After rearranging and diving equation (5.35) the following can be obtained:

$$U_1 U_2 = {c^*}^2 \tag{5.36}$$

or in a dimensionless form

$$M^*{}_1 M^*{}_2 = {c^*}^2 \tag{5.37}$$

Operating Equations and Analysis

In Figure (5.3), the Mach number after the shock, M_y, and the ratio of the total pressure, P_{0y}/P_{0x}, are plotted as a function of the entrance Mach number. The working equations were presented earlier. Note that the M_y has a minimum value which depends on the specific heat ratio. It can be noticed that the density ratio (velocity ratio) also has a finite value regardless of the upstream Mach number.

The typical situations in which these equations can be used also include the moving shocks. The equations should be used with the Mach number (upstream or downstream) for a given pressure ratio or density ratio (velocity ratio). This kind of equations requires examining Table (5.1) for $k = 1.4$ or utilizing Potto-GDC for for value of the specific heat ratio. Finding the Mach number for a pressure ratio of 8.30879 and k=1.32 and is only a few mouse clicks away from the following table.

This table was generated by Potto-GDC (in HTML)

Normal Shock		**Input: Py/Px**		**k = 1.32**	
Mx	**My**	**Ty/Tx**	**ρy/ρx**	**Py/Px**	**P0y/P0x**
2.7245	0.476422	2.111	3.93596	8.30879	0.381089

Figure: The ratios of the static properties of the two sides of the shock.

The Limitations of the Shock Wave

When the upstream Mach number becomes very large, the downstream Mach number (see equation (5.22)) is limited by

$$ {M_y}^2 = \frac{1 + \sim 0 \frac{2}{(k-1){M_x}^2}}{\frac{2k}{k-1} - \sim 0 \frac{1}{{M_x}^2}} \qquad = \frac{k-1}{2k} \tag{5.38} $$

This result is shown in Figure (5.3). The limits of the pressure ratio can be obtained by looking at equation (5.16) and by utilizing the limit that was obtained in equation (5.38).

Small Perturbation Solution

The small perturbation solution refers to an analytical solution where only a small change (or several small changes) occurs. In this case, it refers to a case where only a ``small shock" occurs, which is up to M_x=1.3. This approach had a major significance and usefulness at a time when personal computers were not available. Now, during the writing of this version of the book, this technique is used mostly in obtaining analytical expressions for simplified models. This technique also has an academic value and therefore will be described in the next version (0.5.x series).

The strength of the shock wave is defined as

$$ \hat{\mathcal{P}} = \frac{P_y - P_x}{P_x} = \frac{P_y}{P_x} - 1 \tag{5.39} $$

By using equation (5.23) transforms equation (5.39) into

$$\hat{\mathcal{P}} = \frac{2k}{k+1}\left(M_x{}^2 - 1\right) \tag{5.40}$$

or by utilizing equation (5.24) the following is obtained:

$$\hat{\mathcal{P}} = \frac{\frac{2k}{k-1}\left(\frac{\rho_y}{\rho_x} - 1\right)}{\frac{2}{k-1} - \left(\frac{\rho_y}{\rho_x} - 1\right)} \tag{5.41}$$

Shock Thickness

The issue of shock thickness (which will be presented in a later version) is presented here for completeness. This issue has a very limited practical application for most students; however, to convince the students that indeed the assumption of very thin shock is validated by analytical and experimental studies, the issue should be presented.

The shock thickness can be defined in several ways. The most common definition is by passing a tangent to the velocity at the center and finding out where the theoretical upstream and downstream conditions are meet.

Shock or Wave Drag

It is communally believed that regardless to the cause of the shock, the shock creates a drag (due to increase of entropy). In this section, the first touch of this phenomenon will be presented. The fact that it is assumed that the flow is frictionless does not change whether or not shock drag occur. This explanation is broken into two sections: one for stationary shock wave, two for moving shock shock wave. A better explanation should appear in the oblique shock chapter.

Consider a normal shock as shown in figure (5.5).

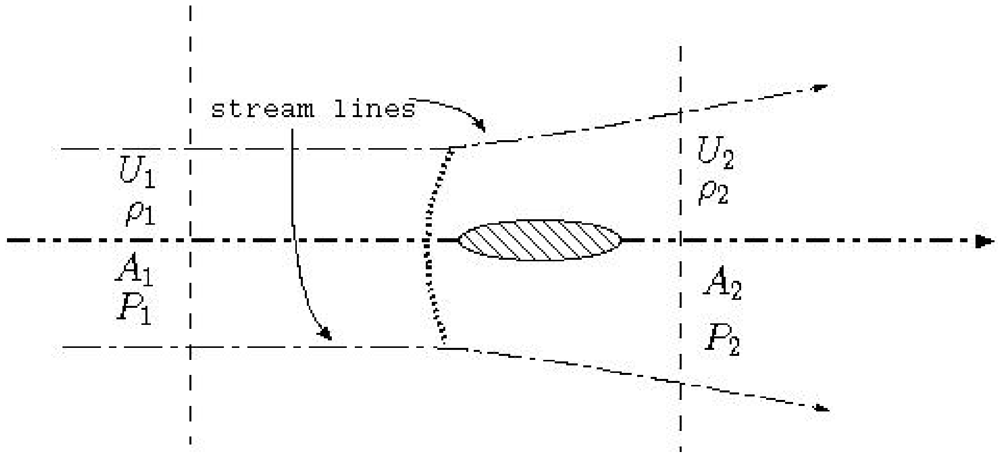


Figure: The diagram that reexplains the shock drag effect.

Gas flows in a supersonic velocity around a two-dimensional body and creates a shock. This shock is an oblique shock, however in this discussion, if the control volume is chosen close enough to the body is can be considered as almost a normal shock (in the oblique shock chapter a section on this issue will be presented that explains the fact that shock is oblique, to be irrelevant).

The control volume that is used here is along two stream lines. The other two boundaries are arbitrary but close enough to the body. Along the stream lines there is no mass exchange and therefore there is no momentum exchange. Moreover, it is assumed that the gas is frictionless, therefore no friction occurs along any stream line. The only change is two arbitrary surfaces since the pressure, velocity, and density are changing. The velocity is reduced $U_x > U_y$. However, the density is increasing, and in addition, the pressure is increasing. So what is the momentum net change in this situation? To answer this question, the momentum equation must be written and it will be similar to equation (❑). However, since $F_y/F^* = F_x/F^*$ there is no net force acting on the body. For example, consider upstream of M_x=3. and for which

Normal Shock		**Input: Mx**		**k = 1.4**	
Mx	**My**	**Ty/Tx**	**ρy/ρx**	**Py/Px**	**P0y/P0x**

3	0.475191	2.67901	3.85714	10.3333	0.328344

and the corespondent Isentropic information for the Mach numbers is

Isentropic Flow		**Input: M**			**k = 1.4**	
M	**T/T0**	**ρ/ρ0**	**A/A***	**P/P0**	**PAR**	**F/F***
3	0.357143	0.0762263	4.23457	0.0272237	0.115281	0.653256
0.47519	0.95679	0.895451	1.3904	0.856759	1.19123	0.653257

Now, after it was established, it is not a surprising result. After all, the shock analysis started with the assumption that no momentum is change. As conclusion there is no shock drag at stationary shock. This is not true for moving shock as it will be discussed in section (5.3.1).

The Moving Shocks

In some situations, the shock wave is not stationary. This kind of situation arises in many industrial applications. For example, when a valve is suddenly [5.4] closed and a shock propagates upstream. On the other extreme, when a valve is suddenly opened or a membrane is ruptured, a shock occurs and propagates downstream (the opposite direction of the previous case). In some industrial applications, a liquid (metal) is pushed in two rapid stages to a cavity through a pipe system. This liquid (metal) is pushing gas (mostly) air, which creates two shock stages. As a general rule, the shock can move downstream or upstream. The last situation is the most general case, which this section will be dealing with. There are more genera cases where the moving shock is created which include a change in the physical properties, but this book will not deal with them at this stage. The reluctance to deal with the most general case is due to fact it is highly specialized and complicated even beyond early graduate students level. In these changes (of opening a valve and closing a valve on the other side) create situations in which different shocks are moving in the tube. The general case is where two shocks collide into one shock and moves upstream or downstream is the general case. A specific example is common in die-casting: after the first shock moves a second shock is created in which its velocity is dictated by the upstream and downstream velocities.

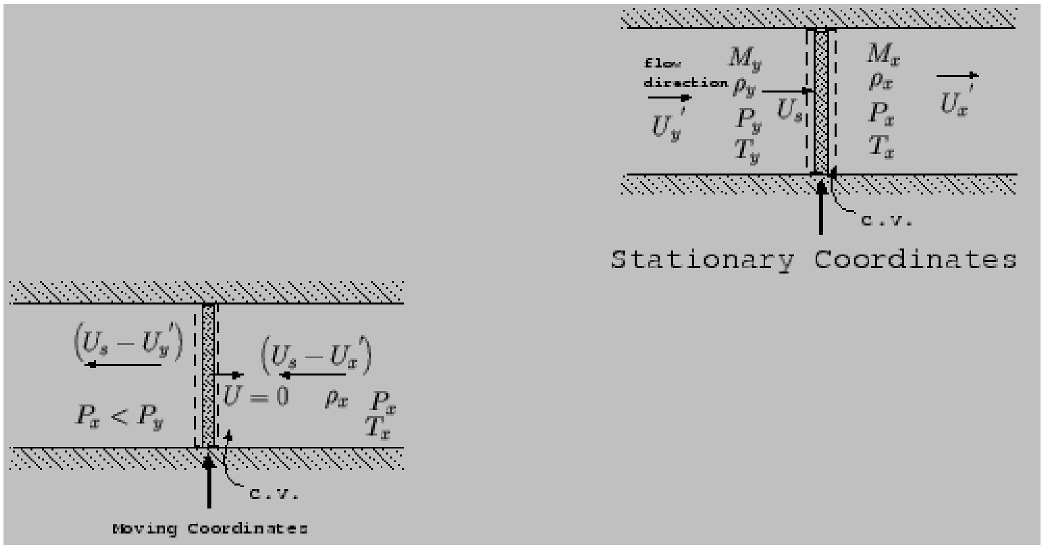


Figure: Comparison between stationary shock and moving shock in ducts

In cases where the shock velocity can be approximated as a constant (in the majority of cases) or as near constant, the previous analysis, equations, and the tools developed in this chapter can be employed. The problem can be reduced to the previously studied shock, i.e., to the stationary case when the coordinates are attached to the shock front. In such a case, the steady state is obtained in the moving control value.

For this analysis, the coordinates move with the shock. Here, the prime ' denote the values of the static coordinates. Note that this notation is contrary to the conventional notation found in the literature. The reason for the deviation is that this choice reduces the programing work (especially for object-oriented programing like C++). An observer moving with the shock will notice that the pressure in the shock is

$$P_x{'} = P_x \qquad P_y{'} = P_y \tag{5.42}$$

The temperature measured by the observer is

$$T_x{}' = T_x \qquad T_y{}' = T_y \tag{5.43}$$

Assuming that the shock is moving to the right, (refer to Figure (5.6)) the velocity measured by the observer is

$$U_x = U_s - U_x{}' \tag{5.44}$$

Where U_s is the shock velocity which is moving to the right. The ``downstream'' velocity is

$$U_y{}' = U_s - U_y \tag{5.45}$$

The speed of sound on both sides of the shock depends only on the temperature and it is assumed to be constant. The upstream prime Mach number can be defined as

$$M_x{}' = \frac{U_s - U_x}{c_x} = \frac{U_s}{c_x} - M_x = M_{sx} - M_x \tag{5.46}$$

It can be noted that the additional definition was introduced for the shock upstream Mach number, $M_{sx} = U_s / c_x$. The downstream prime Mach number can be expressed as

$$M_y{}' = \frac{U_s - U_y}{c_y} = \frac{U_s}{c_y} - M_y = M_{sy} - M_y \tag{5.47}$$

Similar to the previous case, an additional definition was introduced for the shock downstream Mach number, M_{sy}. The relationship between the two new shock Mach numbers is

$$\frac{U_s}{c_x} = \frac{c_y}{c_x}\frac{U_s}{c_y}$$

$$M_{sx} = \sqrt{\frac{T_y}{T_x}} M_{sy} \tag{5.48}$$

The ``upstream" stagnation temperature of the fluid is

$$T_{0x} = T_x \left(1 + \frac{k-1}{2} {M_x}^2\right) \tag{5.49}$$

and the ``upstream" prime stagnation pressure is

$$P_{0x} = P_x \left(1 + \frac{k-1}{2} {M_x}^2\right)^{\frac{k}{k-1}} \tag{5.50}$$

The same can be said for the ``downstream" side of the shock. The difference between the stagnation temperature is in the moving coordinates

$$T_{0y} - T_{0x} = 0 \tag{5.51}$$

It should be noted that the stagnation temperature (in the stationary coordinates) rises as opposed to the stationary normal shock. The rise in the total temperature is due to the fact that a new material has entered the c.v. at a very high velocity, and is ``converted" or added into the total temperature,

$$T_{0y} - T_{0x} = T_y \left(1 + \frac{k-1}{2} \left(M_{sy} - {M_y}'\right)^2\right) - T_x \left(1 + \frac{k-1}{2} \left(M_{sx} - {M_x}'\right)^2\right)$$

$$0 = \overbrace{T_y \left(1 + \frac{k-1}{2} {M_y}'^2\right)}^{{T_{0y}}'} + T_y M_{sy} \frac{k-1}{2} \left(M_{sy} - 2M_y\right)$$

$$- \overbrace{T_x \left(1 + \frac{k-1}{2} {M_x}'^2\right)}^{{T_{0x}}'} - T_x M_{sx} \frac{k-1}{2} \left(M_{sx} - 2M_x\right) \tag{5.52}$$

and according to equation (5.51) leads to

$${T_{0y}}' - {T_{0x}}' = U_s \left(\frac{T_x}{c_x} \frac{k-1}{2} \left(M_{sx} - 2M_x\right) - \frac{T_y}{c_y} \frac{k-1}{2} \left(M_{sy} - 2M_y\right)\right) \tag{5.53}$$

Again, this difference in the moving shock is expected because moving material velocity (kinetic energy) is converted into internal energy. This difference can also be viewed as a result of the unsteady state of the shock.

Shock or Wave Drag Result from a Moving Shock

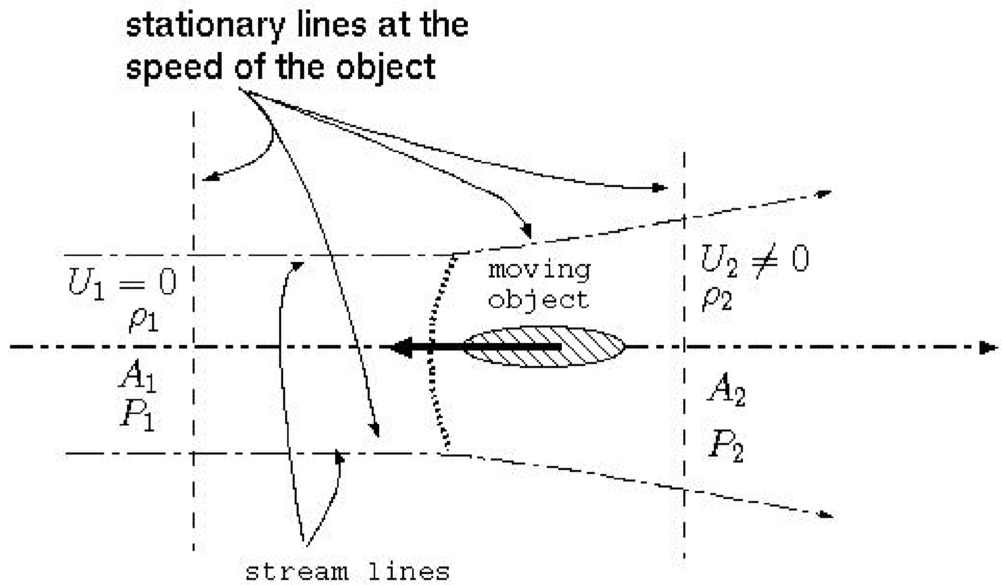


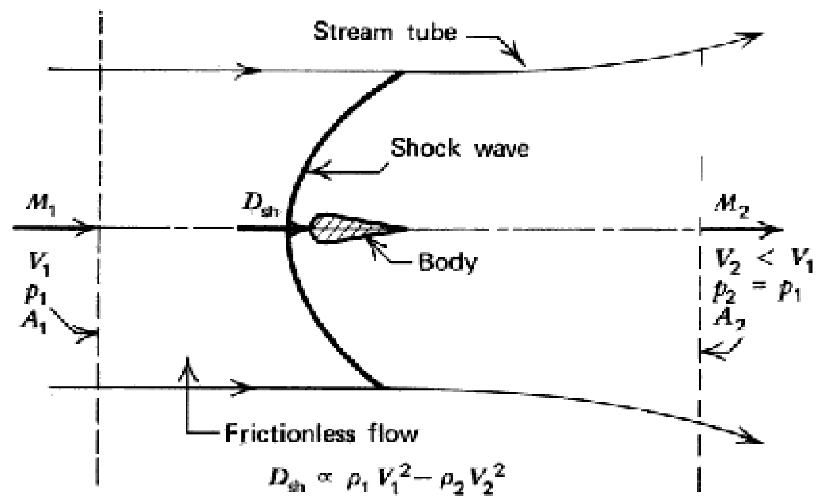
Figure: The diagram that reexplains the shock drag effect of a moving shock.

In section (5.2.4) it was shown that there is no shock drag in stationary shock. However, the shock or wave drag is very significant so much so that at one point it was considered the sound barrier . Consider the figure (5.7) where the stream lines are moving with the object speed. The other boundaries are stationary but the velocity at right boundary is not zero. The same arguments, as discussed before in the stationary case, are applied. What is different in the present case (as oppose to the stationary shock), one side has increase the momentum of the control volume. This increase momentum in the control volume causes the shock drag. In way, it can be view as continuous acceleration of the gas around the body from zero. Note this drag is only applicable to a moving shock (unsteady shock).

The moving shock is either results from a body that moves in gas or from a sudden imposed boundary like close or open valve[5.5] In the first case, the forces/energy flows from body to gas and there for there is a need for large force to accelerate the gas over extremely short distance (shock thickness). In the second case, the gas contains the energy

(as high pressure, for example in the open valve case) and the energy potential is lost in the shock process (like shock drag).

For some strange reasons, this topic has several misconceptions that even appear in many popular and good textbooks[5,6]. Consider the following example taken from such a book.



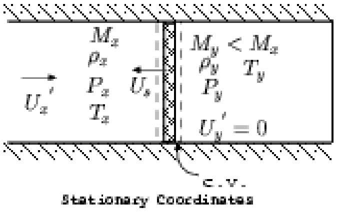
Development of shock (or wave) drag.

Figure: The diagram for the common explanation for shock or wave drag effect a shock. Please notice the strange notations (e.g. V and not U) and they result from a verbatim copy.

Shock Result from a Sudden and Complete Stop

The general discussion can be simplified in the extreme case when the shock is moving from a still medium. This situation arises in many cases in the industry, for example, in a sudden and complete closing of a valve. The sudden closing of the valve must result in a zero velocity of the gas. This shock is viewed by some as a reflective shock. The information propagates upstream in which the gas velocity is converted into temperature. In many such cases the steady state is established quite rapidly. In such a case, the shock

velocity ``downstream" is U_s. Equations (5.42) to (5.53) can be transformed into simpler equations when M_x is zero and U_s is a positive value.



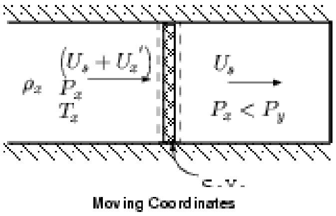


Figure: Comparison between a stationary shock and a moving shock in a stationary medium in ducts.

The ``upstream" Mach number reads

$$M_x = \frac{U_s + U_x}{c_x} = M_{sx} + M_x \tag{5.54}$$

The ``downstream" Mach number reads

$$M_y = \frac{|U_s|}{c_y} = M_{sy} \tag{5.55}$$

Again, the shock is moving to the left. In the moving coordinates, the observer (with the shock) sees the flow moving from the left to the right. The flow is moving to the right. The upstream is on the left of the shock. The stagnation temperature increases by

$$T_{0y} - T_{0x} = U_s \left(\frac{T_x}{c_x} \frac{k-1}{2} \left(M_{sx} + 2M_x \right) - \frac{T_y}{c_y} \frac{k-1}{2} \left(M_{sy} \right) \right) \tag{5.56}$$

The prominent question in this situation is what will be the shock wave velocity for a given fluid velocity, $\underline{U_x{}'}$, and for a given specific heat ratio. The ``upstream" or the ``downstream" Mach number is not known even if the pressure and the temperature downstream are given. The difficulty lies in the jump from the stationary coordinates to the moving coordinates. It turns out that it is very useful to use the dimensionless parameter M_{sx}, or M_{sy} instead of the velocity because it combines the temperature and the velocity into one parameter.

The relationship between the Mach number on the two sides of the shock are tied through equations (5.54) and (5.55) by

$$\left(M_y \right)^2 = \frac{\left(M_x{}' + M_{sx} \right)^2 + \frac{2}{k-1}}{\frac{2k}{k-1} \left(M_x{}' + M_{sx} \right)^2 - 1} \tag{5.57}$$

And substituting equation (5.57) into (5.48) results in

$$M_x = \overbrace{\sqrt{\frac{T_x}{T_y}}}^{f(M_{sx})} \sqrt{\frac{\left(M_x{}' + M_{sx} \right)^2 + \frac{2}{k-1}}{\frac{2k}{k-1} \left(M_x{}' + M_{sx} \right)^2 - 1}} \tag{5.58}$$

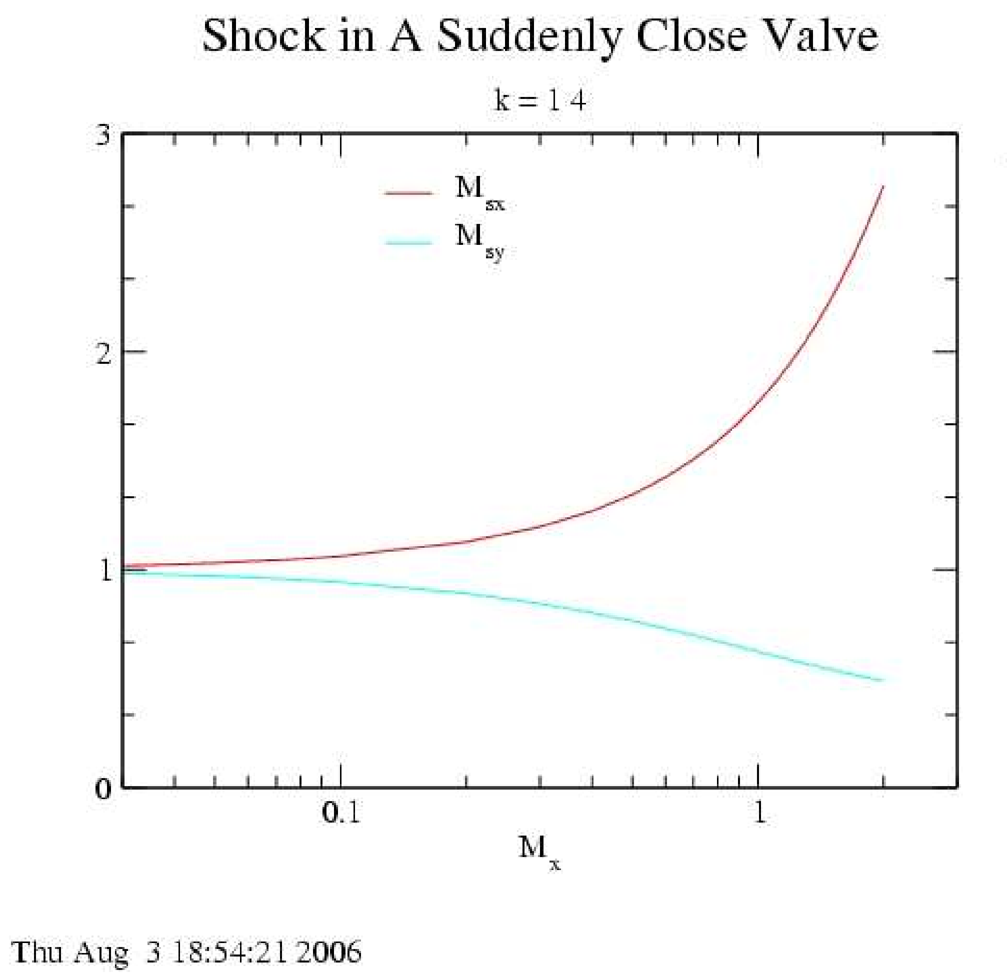


Figure: The moving shock Mach numbers as a result of a sudden and complete stop.

The temperature ratio in equation (5.58) and the rest of the right-hand side show clearly that M_{sx} has four possible solutions (fourth-order polynomial M_{sx} has four solutions). Only one real solution is possible. The solution to equation (5.58) can be obtained by

several numerical methods. Note, an analytical solution can be obtained for equation (5.58) but it seems utilizing numerical methods is much more simple. The typical method is the ``smart" guessing of M_{sx}. For very small values of the upstream Mach number, M_x' ~ ε equation (5.58) provides that M_{sx}~1 + 1/2ε and M_{sy}~1 - 1/2ε (the coefficient is only approximated as 0.5) as shown in Figure (5.11). From the same figure it can also be observed that a high velocity can result in a much larger velocity for the reflective shock. For example, a Mach number close to one (1), which can easily be obtained in a Fanno flow, the result is about double the sonic velocity of the reflective shock. Sometimes this phenomenon can have a tremendous significance in industrial applications.

Note that to achieve supersonic velocity (in stationary coordinates) a diverging-converging nozzle is required. Here no such device is needed! Luckily and hopefully, engineers who are dealing with a supersonic flow when installing the nozzle and pipe systems for gaseous mediums understand the importance of the reflective shock wave.

Two numerical methods and the algorithm employed to solve this problem for given, M_x', is provided herein:

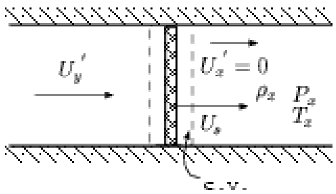
- Guess $M_x > 1$,
- Using shock table or use Potto--GDC to calculate temperature ratio and M_y,
- Calculate the $M_x = M_x'$ - sqrt(T_x/T_y)M_y
- Compare to the calculated M_x' to the given M_x'. and adjust the new guess $M_x > 1$ accordingly.

The second method is ``successive substitutions," which has better convergence to the solution initially in most ranges but less effective for higher accuracies.

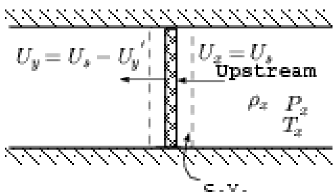
- Guess $M_x = 1 + M_x'$,
- \label{shock:list:mvshock} using the shock table or use Potto--GDC to calculate the temperature ratio and M_y ,
- calculate the $M_x = M_x'$ - sqrt(T_x/T_x) M_y
- Compare the new M_x approach the old M_x, if not satisfactory use the new M_x' to calculate M_x= 1 + M_x' then return to part \eqref{shock:list:mvshock}.

General Velocities Issues

When a valve or membrane is suddenly opened, a shock is created and propagates downstream. With the exception of close proximity to the valve, the shock moves in a constant velocity (5.12(a)). Using a coordinates system which moves with the shock results in a stationary shock and the flow is moving to the left see Figure (5.12(b)). The ``upstream'' will be on the right (see Figure (5.12(b))).



stationary coordinates



moving coordinates

Figure: A shock moves into a still medium as a result of a sudden and complete opening of a valve

Similar definitions of the right side and the left side of the shock Mach numbers can be utilized. It has to be noted that the ``upstream'' and ``downstream'' are the reverse from the previous case. The ``upstream'' Mach number is

$$M_x = \frac{U_s}{c_x} = M_{sx} \tag{5.59}$$

The ``downstream'' Mach number is

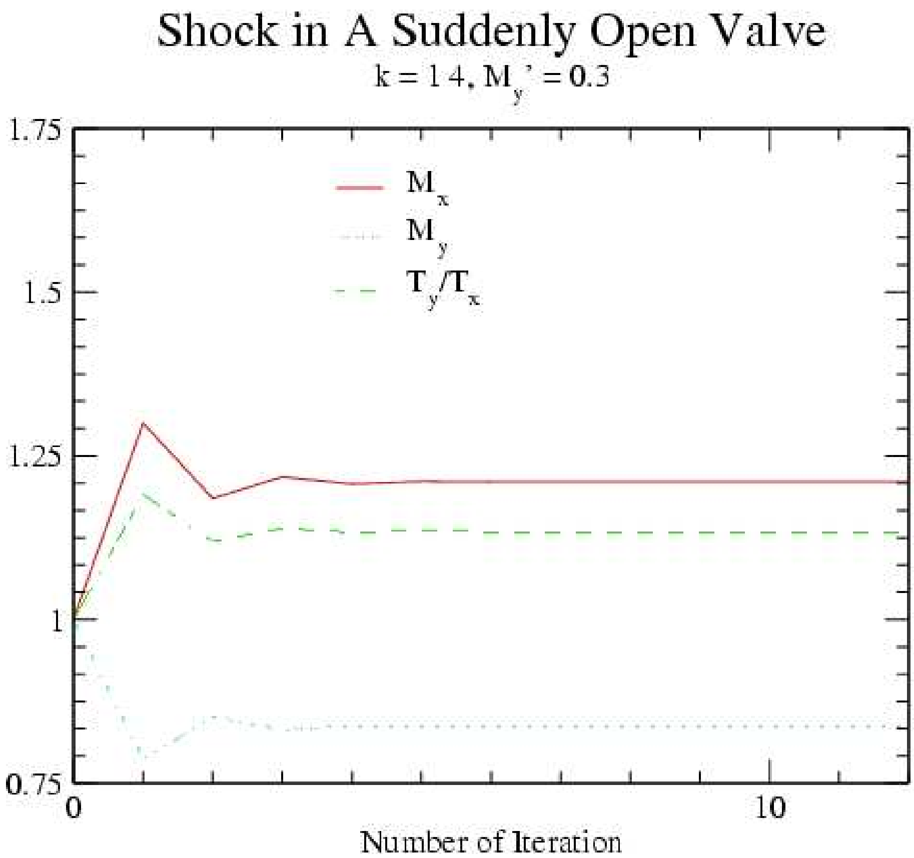
$$M_y = \frac{U_s - {U_y}'}{c_y} = M_{sy} - {M_y}' \tag{5.60}$$

Note that in this case the stagnation temperature in stationary coordinates changes (as in the previous case) whereas the thermal energy (due to pressure difference) is converted into velocity. The stagnation temperature (of moving coordinates) is

$$T_{0_y} - T_{0_x} = T_y\left(1 + \frac{k-1}{2}\left(M_{sy} - M_y\right)^2\right) - T_x\left(1 + \frac{k-1}{2}\left(M_x\right)^2\right) = 0 \tag{5.61}$$

A similar rearrangement to the previous case results in

$$T_{0_y}{}' - T_{0_x}{}' = T_y\left(1 + \frac{k-1}{2}\left(-2M_{sy}M_y + {M_y}^2\right)^2\right) \tag{5.62}$$



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$M_y{'} = 0.3$

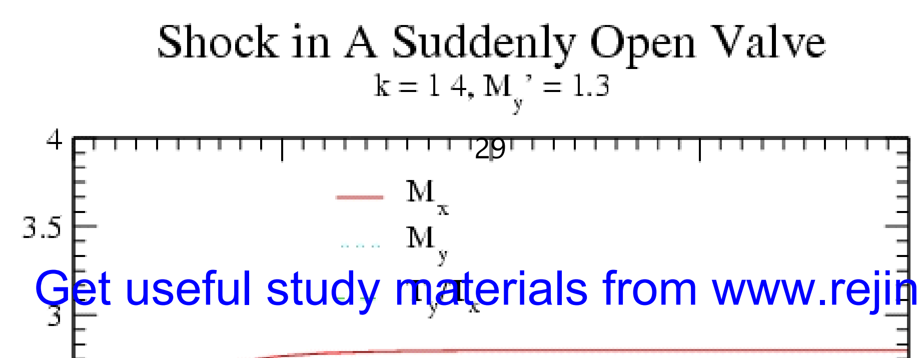


Figure 5.13: The number of iterations to achieve convergence.

The same question that was prominent in the previous case appears now, what will be the shock velocity for a given upstream Mach number? Again, the relationship between the two sides is

$$M_{sy} = {M_y}' + \sqrt{\frac{(M_{sx})^2 + \frac{2}{k-1}}{\frac{2k}{k-1}(M_{sx})^2 - 1}} \tag{5.63}$$

Since M_{sx} can be represented by M_{sy} theoretically equation (5.63) can be solved. It is common practice to solve this equation by numerical methods. One such methods is ``successive substitutions." This method is applied by the following algorithm:

- Assume that $M_x = 1.0$.
- \label{shock:item:openValve} Calculate the Mach number M_y by utilizing the tables or Potto--GDC.
- Utilizing

  $M_x = \text{sqrt}(T_y / T_x) (M_y + M_y')$

  calculate the new ``improved" M_x.
- Check the new and improved M_x against the old one. If it is satisfactory, stop or return to stage \eqref{shock:item:openValve}.

Piston Velocity

When a piston is moving, it creates a shock that moves at a speed greater than that of the piston itself. The unknown data are the piston velocity, the temperature, and, other conditions ahead of the shock. Therefore, no Mach number is given but pieces of information on both sides of the shock. In this case, the calculations for U_s can be obtained from equation (5.24) that relate the shock velocities and Shock Mach number as

$$\frac{U_x}{U_y} = \frac{M_{sx}}{M_{sx} - \frac{{U_y}'}{c_x}} = \frac{(k+1){M_{sx}}^2}{2 + (k-1){M_{sx}}^2} \tag{5.64}$$

Equation (5.64) is a quadratic equation for M_{sx}. There are three solutions of which the first one is M_{sx}=0 and this is immediately disregarded. The other two solutions are

$$M_{sx} = \frac{(k+1)U_y{}' \pm \sqrt{\left[U_y{}'(1+k)\right]^2 + 16{c_x}^2}}{4\, c_x} \tag{5.65}$$

The negative sign provides a negative value which is disregarded, and the only solution left is

$$M_{sx} = \frac{(k+1)U_y{}' + \sqrt{\left[U_y{}'(1+k)\right]^2 + 16{c_x}^2}}{4\, c_x} \tag{5.66}$$

or in a dimensionless form

$$M_{sx} = \frac{(k+1)M_{yx}{}' + \sqrt{\left[M_{yx}{}'(1+k)\right]^2 + 16}}{4} \tag{5.67}$$

Where the ``strange'' Mach number is $M_{sx}' = U_x' / C_x$. The limit of the equation when $c_x \Rightarrow \infty$ leads to

$$M_{sx} = \frac{(k+1)M_{yx}{}'}{4} \tag{5.68}$$

As one additional ``strange'' it can be seen that the shock is close to the piston when the gas ahead of the piston is very hot. This phenomenon occurs in many industrial

applications, such as the internal combustion engines and die casting. Some use equation (5.68) to explain the next Shock-Choke phenomenon.

Shock-Choke Phenomenon

Assuming that the gas velocity is supersonic (in stationary coordinates) before the shock moves, what is the maximum velocity that can be reached before this model fails? In other words, is there a point where the moving shock is fast enough to reduce the ``upstream'' relative Mach number below the speed of sound? This is the point where regardless of the pressure difference is, the shock Mach number cannot be increased.

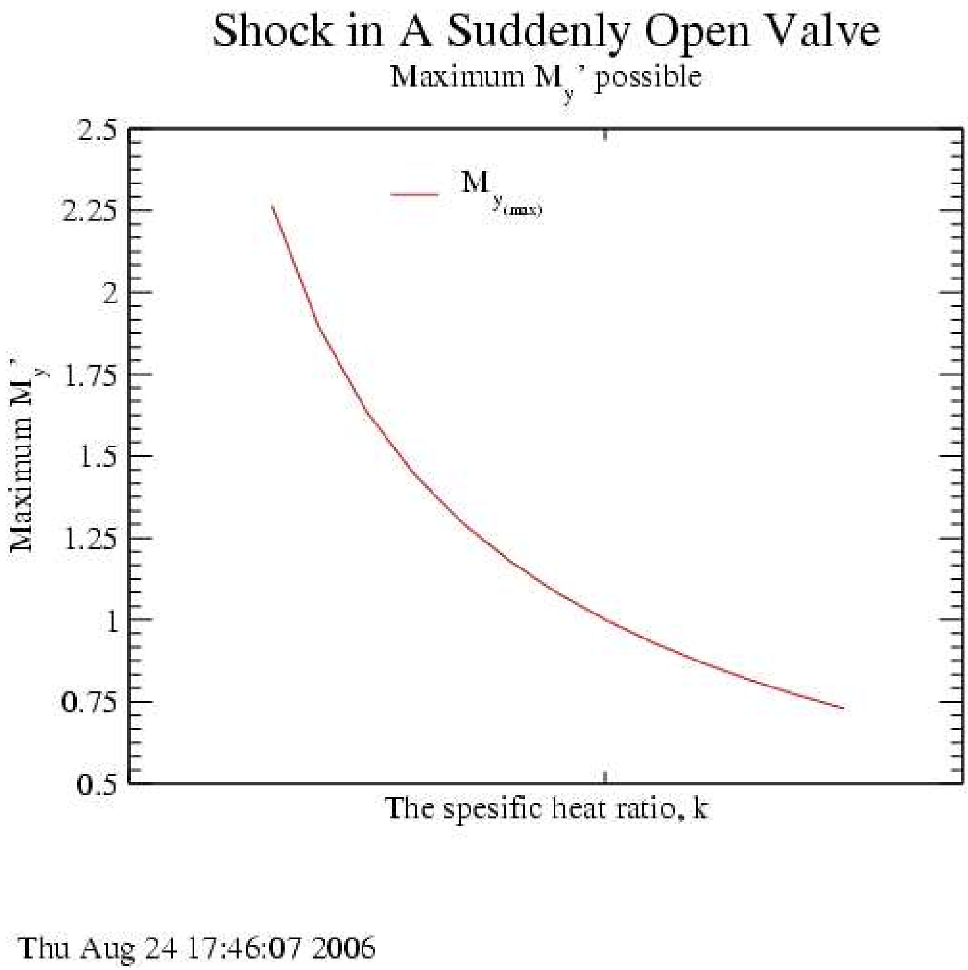


Figure: The maximum of ``downstream'' Mach number as a function of the specific heat, k.

This shock-choking phenomenon is somewhat similar to the choking phenomenon that was discussed earlier in a nozzle flow and in other pipe flow models (later chapters). The difference is that the actual velocity has no limit. It must be noted that in the previous

case of suddenly and completely closing of valve results in no limit (at least from the model point of view). To explain this phenomenon, look at the normal shock. Consider when the ``upstream" Mach approaches infinity, $M_x = M_{sx} \to \infty$, and the downstream Mach number, according to equation (5.38), is approaching to $(k-1)/2k$. One can view this as the source of the shock-choking phenomenon. These limits determine the maximum velocity after the shock since $U_{max} = c_y M_y$. From the upstream side, the Mach number is

$$M_x = M_{sx} = \infty \sqrt{\frac{T_y}{T_x}} \left(\frac{k-1}{2k}\right) \tag{5.69}$$

Thus, the Mach number is approaching infinity because of the temperature ratio but the velocity is finite.

To understand this limit, consider that the maximum Mach number is obtained when the pressure ratio is approaching infinity $\frac{P_y}{P_x} \to \infty$. By applying equation (5.23) to this situation the following is obtained:

$$M_{sx} = \sqrt{\frac{k+1}{2k}\left(\frac{P_x}{P_y} - 1\right) + 1} \tag{5.70}$$

and the mass conservation leads to

$$U_y \rho_y = U_s \rho_x$$

$$\left(U_s - U_y{'}\right) \rho_y = U_s \rho_x$$

$$M_y{'} = \sqrt{\frac{T_y}{T_x}} \left(1 - \frac{\rho_x}{\rho_y}\right) M_{sx} \tag{5.71}$$

Substituting equations (5.26) and (5.25) into equation (5.71) results in

$$M_y{'} = \frac{1}{k}\left(1 - \frac{P_y}{P_x}\right) \sqrt{\frac{\frac{2k}{k+1}}{\frac{P_y}{P_x} + \frac{k-1}{k+1}}} \times \sqrt{\frac{1 + \left(\frac{k+1}{k-1}\right)\left(\frac{P_y}{P_x}\right)}{\left(\frac{k+1}{k-1}\right) + \left(\frac{P_y}{P_x}\right)}} \tag{5.72}$$

When the pressure ratio is approaching infinity (extremely strong pressure ratio), the results is

$$M_y{'} = \sqrt{\frac{2}{k(k-1)}} \tag{5.73}$$

What happens when a gas with a Mach number larger than the maximum Mach number possible is flowing in the tube? Obviously, the semi steady state described by the moving shock cannot be sustained. A similar phenomenon to the choking in the nozzle and later in an internal pipe flow is obtained. The Mach number is reduced to the maximum value very rapidly. The reduction occurs by an increase of temperature after the shock or a stationary shock occurs as it will be shown in chapters on internal flow.

k	M_x	M_y	$M_y{'}$	$\frac{T_y}{T_x}$

1.30	1073.25	0.33968	2.2645	169842.29
1.40	985.85	0.37797	1.8898	188982.96
1.50	922.23	0.40825	1.6330	204124.86
1.60	873.09	0.43301	1.4434	216507.05
1.70	833.61	0.45374	1.2964	226871.99
1.80	801.02	0.47141	1.1785	235702.93
1.90	773.54	0.48667	1.0815	243332.79
2.00	750.00	0.50000	1.00000	250000.64
2.10	729.56	0.51177	0.93048	255883.78
2.20	711.62	0.52223	0.87039	261117.09
2.30	695.74	0.53161	0.81786	265805.36
2.40	681.56	0.54006	0.77151	270031.44
2.50	668.81	0.54772	0.73029	273861.85

<>

Table of maximum values of the shock-choking phenomenon.

The mass flow rate when the pressure ratio is approaching infinity, ∞, is

$$\frac{\dot{m}}{A} = U_y{}' \rho_y = M_y{}' c_y \rho_y = M_y{}' \overbrace{\sqrt{kRT_y}}^{c_y} \overbrace{\frac{P_y}{RT_y}}^{\rho_y}$$

$$= \frac{M_y{}' \sqrt{k} P_y}{\sqrt{RT_y}} \tag{5.74}$$

Equation (5.74) and equation (5.25) can be transferred for large pressure ratios into

$$\frac{\dot{m}}{A} \sim \sqrt{T_y}\frac{P_x}{T_x}\frac{k-1}{k+1} \tag{5.75}$$

Since the right hand side of equation (5.75) is constant, with the exception of $\sqrt{T_y}$ the mass flow rate is approaching infinity when the pressure ratio is approaching infinity. Thus, the shock-choke phenomenon means that the Mach number is only limited in stationary coordinates but the actual flow rate isn't.

Next Up Previous Index

Next: Partially Closed Valve **Up:** The Moving Shocks **Previous:** Shock-Choke Phenomenon **Index**

Partially Open Valve

The previous case is a special case of the moving shock. The general case is when one gas flows into another gas with a given velocity. The only limitation is that the ``downstream' gas velocity is higher than the ``upstream'' gas velocity as shown in Figure (5.17).

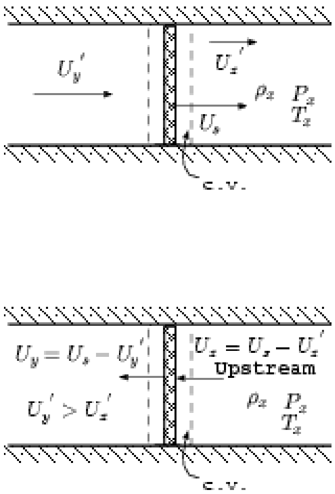


Figure: A shock moves into a moving medium as a result of a sudden and complete open valve.

The relationship between the different Mach numbers on the ``upstream'' side is

$$M_x = M_{sx} - M_x{}' \tag{5.76}$$

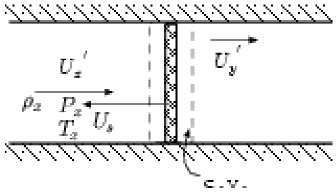
The relationship between the different Mach on the ``downstream'' side is

$$M_y = M_{sy} - M_y{}' \tag{5.77}$$

An additional parameter has be supplied to solve the problem. A common problem is to find the moving shock velocity when the velocity ``downstream'' or the pressure is suddenly increased. It has to be mentioned that the temperature ``downstream'' is unknown (the flow of the gas with the higher velocity). The procedure for the calculations can be done by the following algorithm:

(a)

Partially Closed Valve



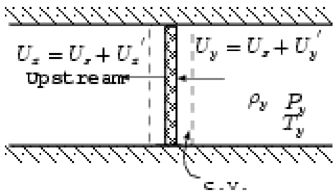


Figure: A shock as a result of a sudden and partially a valve closing or a narrowing the passage to the flow

The totally closed valve is a special case of a partially closed valve in which there is a sudden change and the resistance increases in the pipe. The information propagates upstream in the same way as before. Similar equations can be written:

$$U_x = U_s + U_x{}' \tag{5.78}$$

$$U_y = U_s + U_y{}' \tag{5.79}$$

$$M_x = M_s + M_x{}' \tag{5.80}$$

$$M_y = M_s + M_y{}' \tag{5.81}$$

For given static Mach numbers the procedure for the calculation is as follows:

(a)

Assume that $M_x = M_x{}' + 1$.

(b)

. Calculate the Mach number M_y by utilizing the tables or Potto-GDC

(c)

Calculate the ``downstream'' shock Mach number $M_{sy} = M_y - M_y{}'$

(d)

Utilizing

$$M_x = \sqrt{\frac{T_y}{T_x}}\,(M_{sy}) + M_x{}'$$

calculate the new ``improved'' M_x

(e)

Check the new and improved M_x against the old one. If it is satisfactory, stop or return to stage (b).

Shock Tube

The shock tube is a study tool with very little practical purposes. It is used in many cases to understand certain phenomena. Other situations can be examined and extended from these phenomena. A cylinder with two chambers connected by a diaphragm. On one side the pressure is high, while the pressure on the other side is low. When the diaphragm is ruptured the gas from the high pressure section flows into the low pressure section. When

the pressure is high enough, a shock is created that it travels to the low pressure chamber. This is the same case as in the suddenly opened valve case described previously. At the back of the shock, expansion waves occur with a reduction of pressure. The temperature is known to reach several thousands degrees in a very brief period of time. The high pressure chamber is referred to in the literature is the driver section and the low section is referred to as the expansion section.

Initially, the gas from the driver section is coalescing from small shock waves into a large shock wave. In this analysis, it is assumed that this time is essentially zero. Zone 1 is an undisturbed gas and zone 2 is an area where the shock already passed. The assumption is that the shock is very sharp with zero width. On the other side, the expansion waves are moving into the high pressure chamber i.e. the driver section. The shock is moving at a supersonic speed (it depends on the definition, i.e., what reference temperature is being used) and the medium behind the shock is also moving but at a velocity, U_2, which can be supersonic or subsonic in stationary coordinates. The velocities in the expansion chamber vary between three zones. In zone 3 is the original material that was in the high pressure chamber but is now the same pressure as zone 2. Zone 4 is where the gradual transition occurs between original high pressure to low pressure. The boundaries of zone 4 are defined by initial conditions. The expansion front is moving at the local speed of sound in the high pressure section. The expansion back front is moving at the local speed of sound velocity but the actual gas is moving in the opposite direction in U_2. In fact, material in the expansion chamber and the front are moving to the left while the actual flow of the gas is moving to the right (refer to Figure (5.20)). In zone 5, the velocity is zero and the pressure is in its original value.

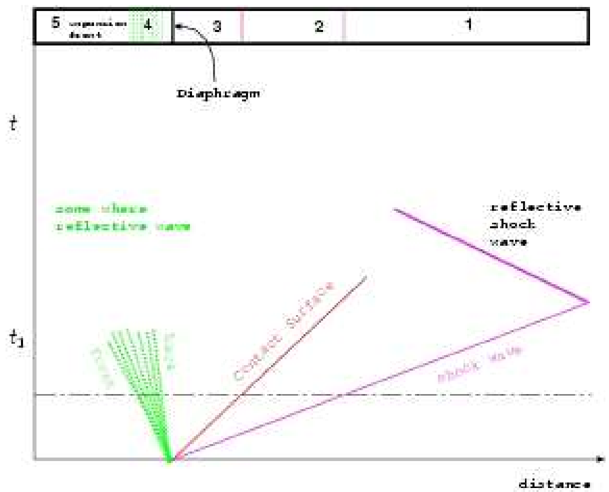


Figure 5.20: The shock tube schematic with a pressure "diagram."

The properties in the different zones have different relationships. The relationship between zone 1 and zone 2 is that of a moving shock into still medium (again, this is a case of sudden opened valve). The material in zone 2 and 3 is moving at the same velocity (speed) but the temperature and the entropy are different, while the pressure in the two zones are the same. The pressure, the temperature and their properties in zone 4 aren't constant and continuous between the conditions in zone 3 to the conditions in zone 5. The expansion front wave velocity is larger than the velocity at the back front expansion wave velocity. Zone 4 is expanding during the initial stage (until the expansion reaches the wall).

The shock tube is a relatively small length $1 - 2[m]$ and the typical velocity is in the range of the speed of sound, $c \sim \sqrt{340}$ thus the whole process takes only a few milliseconds or less. Thus, these kinds of experiments require fast recording devices (a relatively fast camera and fast data acquisition devices.). A typical design problem of a

shock tube is finding the pressure to achieve the desired temperature or Mach number. The relationship between the different properties was discussed earlier and because it is a common problem, a review of the material is provided thus far.

The following equations were developed earlier and are repeated here for clarification. The pressure ratio between the two sides of the shock is

$$\frac{P_2}{P_1} = \frac{k-1}{k+1}\left(\frac{2k}{k-1}{M_{s1}}^2 - 1\right) \tag{5.82}$$

Rearranging equation (5.82) becomes

$$M_{s1} = \sqrt{\frac{k-1}{2k} + \frac{k+1}{2k}\frac{P_2}{P_1}} \tag{5.83}$$

Or expressing the velocity as

$$U_s = M_{s1}c_1 = c_1\sqrt{\frac{k-1}{2k} + \frac{k+1}{2k}\frac{P_2}{P_1}} \tag{5.84}$$

And the velocity ratio between the two sides of the shock is

$$\frac{U_1}{U_2} = \frac{\rho_2}{\rho_2} = \frac{1 + \frac{k+1}{k-1}\frac{P_2}{P_1}}{\frac{k+1}{k-1}\frac{P_2}{P_1}} \tag{5.85}$$

The fluid velocity in zone 2 is the same

$$U_2{}' = U_s - U_2 = U_s\left(1 - \frac{U_2}{U_s}\right) \tag{5.86}$$

From the mass conservation, it follows that

$$\frac{U_2}{U_s} = \frac{\rho_1}{\rho_2} \tag{5.87}$$

$$U_2{}' = c_1 \sqrt{\frac{k-1}{2k} + \frac{k+1}{2k}\frac{P_2}{P_1}} \sqrt{1 - \frac{\frac{k+1}{k-1} + \frac{P_2}{P_1}}{1 + \frac{k+1}{k-1}\frac{P_2}{P_1}}} \tag{5.88}$$

After rearranging equation (5.88) the result is

$$U_2{}' = \frac{c_1}{k}\left(\frac{P_2}{P_1} - 1\right)\sqrt{\frac{\frac{2k}{k+1}}{\frac{P_2}{P_1}\frac{k-1}{1+k}}} \tag{5.89}$$

On the isentropic side, in zone 4, taking the derivative of the continuity equation, $d(\rho U) = 0$, and dividing by the continuity equation the following is obtained:

$$\frac{d\rho}{\rho} = -\frac{dU}{c} \tag{5.90}$$

Since the process in zone 4 is isentropic, applying the isentropic relationship ($T \propto \rho^{k-1}$) yields

$$\frac{c}{c_5} = \sqrt{\frac{T}{T_5}} = \left(\frac{\rho}{\rho_5}\right)^{\frac{k-1}{2}} \tag{5.91}$$

From equation (5.90) it follows that

$$dU = -c\frac{d\rho}{\rho} = c_5\left(\frac{\rho}{\rho_5}\right)^{\frac{k-1}{2}} d\rho \tag{5.92}$$

Equation (5.92) can be integrated as follows:

$$\int_{U_5=0}^{U_3} dU = \int_{\rho_5}^{\rho_3} c_5\left(\frac{\rho}{\rho_5}\right)^{\frac{k-1}{2}} d\rho \tag{5.93}$$

The results of the integration are

$$U_3 = \frac{2c_5}{k-1}\left(1 - \left(\frac{\rho_3}{\rho_5}\right)^{\frac{k-1}{2}}\right) \tag{5.94}$$

Or in terms of the pressure ratio as

$$U_3 = \frac{2c_5}{k-1}\left(1 - \left(\frac{P_3}{P_5}\right)^{\frac{k-1}{2k}}\right) \tag{5.95}$$

As it was mentioned earlier the velocity at points $2'$ and 3 are identical, hence equation (5.95) and equation (5.89) can be combined to yield

$$\frac{2c_5}{k-1}\left(1-\left(\frac{P_3}{P_5}\right)^{\frac{k-1}{2k}}\right)=\frac{c_1}{k}\left(\frac{P_2}{P_1}-1\right)\sqrt{\frac{\frac{2k}{k+1}}{\frac{P_2}{P_1}\frac{k-1}{1+k}}} \tag{5.96}$$

After some rearrangement, equation (5.96) is transformed into

$$\frac{P_5}{P_1}=\frac{P_2}{P_1}\left(1-\frac{(k-1)\frac{c_1}{c_5}\left(\frac{P_5}{P_3}-1\right)}{\sqrt{2k}\sqrt{2k+(k+1)\left(\frac{P_2}{P_1}-1\right)}}\right)^{-\frac{2k}{k-1}} \tag{5.97}$$

Or in terms of the Mach number, M_{s1}

$$\frac{P_5}{P_1}=\frac{k_1-1}{k+1+1}\left(\frac{2k}{k_1-1}{M_{s1}}^2-1\right)\left[1-\frac{\frac{k-1}{k+1}\frac{c_1}{c_5}\left({M_{s1}}^2-1\right)}{M_{s1}}\right]^{-\frac{2k}{k-1}} \tag{5.98}$$

Using the Rankine-Hugoniot relationship and the perfect gas model, the following is obtained:

$$\frac{T_2}{T_1}=\frac{1+\frac{k_1-1}{k_1+1}\frac{P_2}{P_1}}{1+\frac{k_1-1}{k_1+1}\frac{P_1}{P_2}} \tag{5.99}$$

By utilizing the isentropic relationship for zone 3 to 5 results in

$$\frac{T_3}{T_5} = \left(\frac{P_3}{P_5}\right)^{\frac{k_5-1}{k_5}} = \left(\frac{\frac{P_2}{P_1}}{\frac{P_5}{P_1}}\right)^{\frac{k_5-1}{k_5}} \tag{5.100}$$

1 What is the normal shock?

When the shock waves are right angles to the direction of flow and the rise in pressure is abrupt are called normal shock waves.

2 What is meant by normal shock as applied to compressible flow?

Compression wave front being normal to the direction of compressible fluid flow. It occurs when the flow is decelerating from supersonic flow. The fluid properties jump across the normal shock.

3 Shock waves cannot develop in subsonic flow? State the reason.

Shocks are introduced to increase the pressure and hence it is a deceleration process. Therefore, shocks are possible only when the fluid velocity is maximum. In a subsonic flow, the velocity of fluid is less then the critical velocity and hence deceleration is not possible.

Thus, shock waves cannot develop in subsonic flow.

4 Define oblique shock where it occurs.

The shock wave which is inclined at an angle to the two dimensional flow direction is called as oblique shock. When the flow is supersonic, the oblique shock occurs at the corner due to the turning of supersonic flow.

5 Give the difference between normal and oblique shock.

NORMAL SHOCK OBLIQUE SHOCK

(a) The shock waves are right angles to the direction of flow.

(b) May be treated as one dimensional analysis.

(a) The shock waves are inclined at an angle to the direction of flow.

(b) Oblique shock is two dimensional analyses.

6 What is Prandtl-Meyer relation? What its significance?

The fundamental relation between gas velocities before and after the normal shock and the critical velocity of sound is known as Prandtl-Meyer relation.

i.e., (i) cx x cy = a*2

and (ii) M*x x M* y = 1

it signifies the velocities (before and after the shock) with the critical velocity of sound and the product of mach numbers before and after the shock is unity.

7 How can you define strong shocks mathematically?

Shock strength is proportional to (Mx2 – 1), strong shocks are a result of very high values of the upstream Mach number.

8 How can you determine the mach number of supersonic flows?

By having a pitot tube along with a wall lapping can be used to determine the Mach number of a supersonic stream. The introduction of the pitot tube produces a curved shock a little distance upstream of its mouth.

9 Define supersonic wind tunnels.

A supersonic wind tunnel consists of a nozzle, test section and the diffuser. Normal shocks have applications in supersonic wind tunnels where the diffusion of the supersonic flow after the test section takes place through a shock wave.

10 What do you understand by Oblique shock wave?

When the direction of flow is inclined at an oblique angle to the shock wave it is known as "oblique Shock Wave".

Flow with normal shock

1) The state of a gas (γ=1.3,R =0.469 KJ/Kg K) upstream of a normal shock is given by the following data: Mx =2.5, px= 2bar ,Tx =275K calculate the Mach number ,pressure,temperature and velocity of the gas downstream of the shock;

check the calculated values with those give in the gas tables. (16)

2) The ratio of th exit to entry area in a subsonic diffuser is 4.0 .The Mach number of a jet of air

approaching the diffuser at p0=1.013 bar, T =290 K is 2.2 .There is a standing normal shock wave just outside the diffuser entry. The flow in the diffuser is isentropic . Determine at the exit of the diffuser.

a) Mach number , (4)

b) Temperature, and (4)

c) Pressure (4)

d) What is the stagnation pressure loss between the initial and final states of the flow ? (4)

3) The velocity of a normal shock wave moving into stagnant air (p=1.0 bar, t=17° C) is 500 m/s . If the area of cross- section of the duct is constant determine (a) pressure (b) temperature (c) velocity of air (d) stagnation temperature and (e) the mach number imparted upstream of the wave front. (16)

4) The following data refers to a supersonic wind tunnel:

Nozzle throat area =200cm²

Test section cross- section =337.5cm²

Working fluid ;air (γ =1.4, Cp =0.287 KJ/Kg K)

Determine the test section Mach number and the diffuser throat area if a normal shock is located in the test section. (16)

5) A supersonic diffuser for air (γ =1.4) has an area ratio of 0.416 with an inlet Mach number of 2.4 (design value). Determine the exit Mach number and the design value of the pressure ratio across the diffuser for isentropic flow. At an off- design value of the inlet Mach number (2.7) a normal shock occurs inside the diffuser .Determine the upstream Mach number and area ratio at the section where the shock occurs, diffuser efficiency and the pressure ratio across the diffuser. Depict graphically the static pressure distribution at off design. (16)

6) Starting from the energy equation for flow through a normal shock obtain the following relations (or) prandtl – meyer relation $C_x C_y = a^{*2}$ $M^*_x M^*_y = 1$ (16)

Flow with oblique shock waves:

7) Air approaches a symmetrical wedge (δ =15°) at a Mach number of 2.0.Determine for the strong and weak waves (a) wave angle (b) pressure ratio (c) density ratio,(d) temperature ratio and (e)downstream Mach number Verify these values using Gas tables for normal shocks. (16)

8) A gas (γ =1.3) at p1 =345 mbar, T1= 350 K and M1=1.5 is to be isentropically expanded to 138 mbar. Determine (a) the deflection angle ,(b) final Mach number and (c) the temperature of the gas. (16)

9) A jet of air at Mach number of 2.5 is deflected inwards at the corner of a curved wall..The wave angle at the corner is 60°.Determine the deflection angle of the wall, pressure and temperature ratios and final Mach number. (16)

10) Derive the Rankine –Hugoniot relation for an oblique shock

Þ2 /þ 1 = γ +1 p2 γ +1 p2

-------- --- +1 -------- + ------

γ -1 p1 γ -1 p1

Compare graphically the variation of density ratio with the intial Mach number in isentropic flow

and flow with oblique shock. (16)

11) The Mach number at the exit of a combustion chamber is 0.9. The ratio of stagnation temperature at exit and entry is 3.74.If the pressure and temperature of a gas at exit are2.5 bar and 1000°C respectively determine (a) Mach number ,pressure and temperature of the gas at entry,(b) the heat supplied per Kg of the gas and (c) the maximum heat that can be supplied. Take γ =1.3 and Cp = 1.218 KJ/Kg K (16)

12) The conditions of a gas in a combuster at entry are: P1=0.343 bar,T1= 310K ,C1= 60m/s Determine the Mach number ,pressure,temperature and velocity at the exit if the increase in stagnation enthalpy of the gas between entry and exit is 1172.5KJ/Kg.Take Cp=1.005KJ/kg, γ =1.4. (16)

13) A combustion chamber in a gas turbine plant receives air at 350 K , 0.55 bar and 75m/s .The air –fuel ratio is 29 and the calorific value of the fuel is 41.87

MJ/Kg.Taking γ =1.4 and R =0.287 KJ/Kg K for the gas determine :

a) The initial and final Mach number, (4)

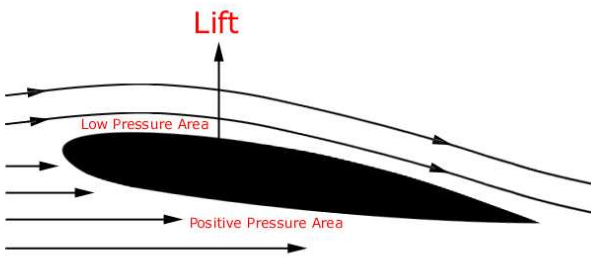
b) Final pressure,temperature and velocity of the gas, (4)

c) Percent stagnation pressure loss in the combustion chamber and (4),

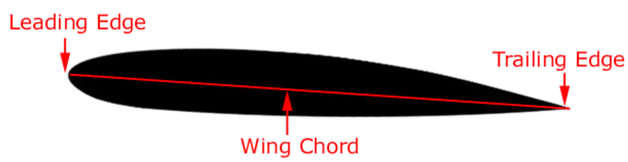
d) The maximum stagnation temperature attainable. (4)

14) Obtain an equation representing the rayleigh line. Draw Rayleigh lines on the h-s and p-v planes for two different values of the mass flux. Show that the slope of the Rayleigh line on the p-v plane is {dP/dV} r = $þ^2$ c^2 (16)

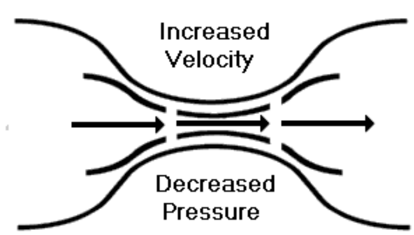
UNIT IV JET PROPULSION



An airfoil is a device which gets a useful reaction from air moving over its surface. When an airfoil is moved through the air, it is capable of producing lift. Wings, horizontal tail surfaces, vertical tails surfaces, and propellers are all examples of airfoils.



Generally the wing of small aircraft will look like the cross-section of the figure above. The forward part of an airfoil is rounded and is called the leading edge. The aft part is narrow and tapered and is called the trailing edge. A reference line often used in discussing airfoils is the *chord*, an imaginary straight line joining the extremities of the leading and trailing edges.

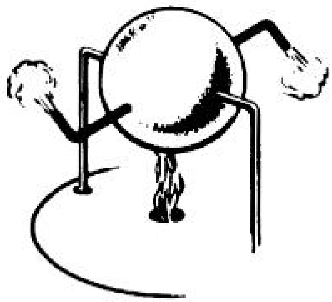


Bernoulli's Principle: To understand how lift is produced, we must examine a phenomenon discovered many years ago by the scientist Bernoulli and later called Bernoulli's Principle: The pressure of a fluid (liquid or gas) decreases at points where the speed of the fluid increases. In other words, Bernoulli found that within the same fluid, in this case air, high speed flow is associated with low pressure, and low speed flow with high pressure. This principle was first used to explain changes in the pressure of fluid flowing within a pipe whose cross-sectional area varied. In the wide section of the gradually narrowing pipe, the fluid moves at low speed, producing high pressure. As the pipe narrows it must contain the same amount of fluid. In this narrow section, the fluid moves at high speed, producing low pressure.

An important application of this phenomenon is made in giving lift to the wing of an airplane, an airfoil. The airfoil is designed to increase the velocity of the airflow above its surface, thereby decreasing pressure above the airfoil. Simultaneously, the impact of the air on the lower surface of the airfoil increases the pressure below. This combination of pressure decrease above and increase below produces lift.

Probably you have held your flattened hand out of the window of a moving automobile. As you inclined your hand to the wind, the force of air pushed against it forcing your hand to rise. The airfoil (in this case, your hand) was deflecting the wind which, in turn, created an equal and opposite dynamic pressure on the lower surface of the airfoil, forcing it up and back. The upward component of this force is lift; the backward component is drag.

Jet Engine Theory



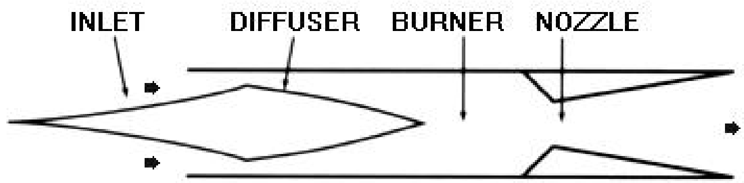
Over the course of the past last half century, jet-powered flight has vastly changed the way we all live. However, the basic principle of jet propulsion is neither new nor complicated.

Centuries ago in 100 A.D., Hero, a Greek philosopher and mathematician, demonstrated jet power in a machine called an "aeolipile." A heated, water filled steel ball with nozzles spun as steam escaped. Why? The principle behind this phenomenon was not fully understood until 1690 A.D. when Sir Isaac Newton in England formulated the principle of Hero's jet propulsion "aeolipile" in scientific terms. His Third Law of Motion stated: "Every action produces a reaction ... equal in force and opposite in direction."

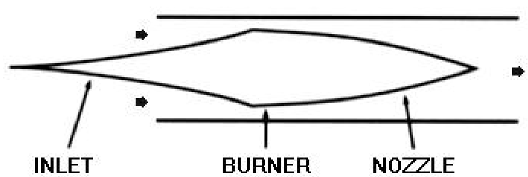
The jet engine of today operates according to this same basic principle. Jet engines contain three common components: the **compressor**, the **combustor**, and the **turbine**. To this basic engine, other components may be added, including:

- A **nozzle** to recover and direct the gas energy and possibly divert the thrust for vertical takeoff and landing as well as changing direction of aircraft flight.
- An **afterburner**or **augmentor**, a long "tailpipe" behind the turbine into which additional fuel is sprayed and burned to provide additional thrust.
- A **thrust reverser**, which blocks the gas rushing toward the rear of the engine, thus forcing the gases forward to provide additional braking of aircraft.
- A **fan** in front of the compressor to increase thrust and reduce fuel consumption.

- An additional **turbine** that can be utilized to drive a **propeller** or **helicopter rotor**.

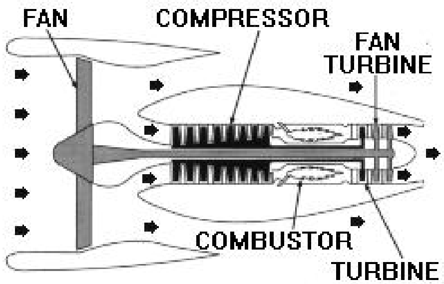


A ramjet has no moving parts and achieves compression of intake air by the forward speed of the air vehicle. Air entering the intake of a supersonic aircraft is slowed by aerodynamic diffusion created by the inlet and diffuser to velocities comparable to those in a turbojet augmentor. The expansion of hot gases, after fuel injection and combustion, accelerates the exhaust air to a velocity higher than that at the inlet and creates positive push.

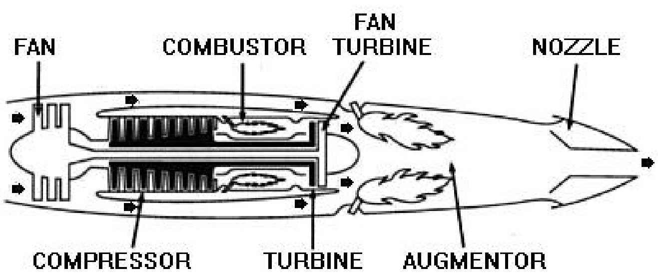


Scramjet is an acronym for Supersonic Combustion Ramjet. The scramjet differs from the ramjet in that combustion takes place at supersonic air velocities through the engine. It is mechanically simple, but vastly more aerodynamically complex than a jet engine. Hydrogen is normally the fuel used.

The Turbofan Engine

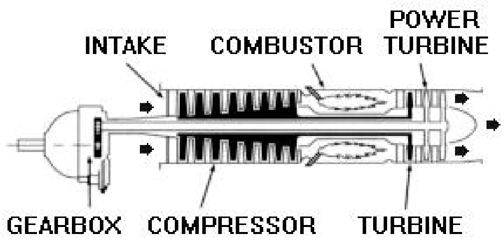


A turbofan engine is basically a turbojet to which a fan has been added. Large fans can be placed at either the front or rear of the engine to create high bypass ratios for subsonic flight. In the case of a front fan, the fan is driven by a second turbine, located behind the primary turbine that drives the main compressor. The fan causes more air to flow around (bypass) the engine. This produces greater thrust and reduces specific fuel consumption.



For supersonic flight, a low bypass fan is utilized, and an augmentor (afterburner) is added for additional thrust.

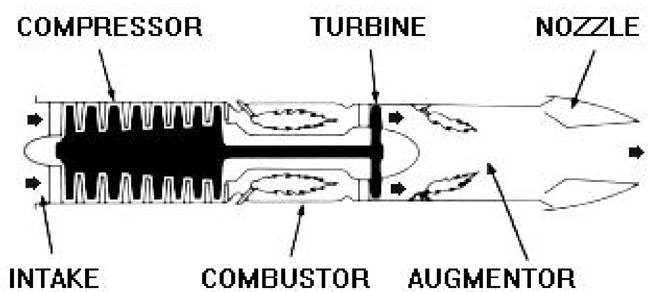
The Turboprop/Turbo shaft Engine



A turboprop engine uses thrust to turn a propeller. As in a turbojet, hot gases flowing through the engine rotate a turbine wheel that drives the compressor. The gases then pass through another turbine, called a power turbine. This power turbine is coupled to the shaft, which drives the propeller through gear connections.

A turboshaft is similar to a turboprop engine, differing primarily in the function of the turbine shaft. Instead of driving a propeller, the turbine shaft is connected to a transmission system that drives helicopter rotor blades; electrical generators, compressors and pumps; and marine propulsion drives for naval vessels, cargo ships, high speed passenger ships, hydrofoils and other vessels.

The Turbojet Engine



The turbojet is the basic engine of the jet age. Air is drawn into the engine through the front intake. The compressor squeezes the air to many times normal atmospheric pressure and forces it into the combustor. Here, fuel is sprayed into the compressed air, is ignited and burned continuously like a blowtorch. The burning gases expand rapidly rearward

and pass through the turbine. The turbine extracts energy from the expanding gases to drive the compressor, which intakes more air. After leaving the turbine, the hot gases exit at the rear of the engine, giving the aircraft its forward push ... action, reaction !

For additional thrust or power, an afterburner or augmentor can be added. Additional fuel is introduced into the hot exhaust and burned with a resultant increase of up to 50 percent in engine thrust by way of even higher velocity and more push.

1 What is meant by a jet propulsion system?

It is the propulsion of a jet aircraft (or)other missiles by the reaction of jet coming out with high velocity. The jet propulsion in used when the oxygen is obtained from the surrounding atmosphere.

2 How will you classify propulsive engines?

The jet propulsion engines are classified into

i. Air breathing engines and

ii. Rocket engines which do not use atmospheric air.

3 What is the difference between shaft propulsion and jet propulsion?

a) The power to the propeller is transmitted through a reduction gear

b) At higher altitude, the performance is poor. Hence it is suitable for lower altitudes.

c) With increasing speeds and size of the aircrafts, the shaft propulsion engine becomes too complicated.

d) Propulsive efficiency is less. There is no reduction gear.

b) Suitable for higher altitudes.

c) Construction is simpler.

d) More.

4 List the different types of jet engines.

i. Turbo-jet

ii. Turpo-prop engine,

iii. Ram jet engine,

iv. Pulse jet engines.

5 Define the principle of Ram jet engine.

The principle of jet engine is obtained from the application of Newton's law of motion. We know that when a fluid is accelerated, a force is required to produce this acceleration is the fluid and at the same time, there is an equal and opposite reaction force of the fluid on the engine is known as the thrust, and therefore the principle of jet propulsion is based on the reaction principle.

6 Give the components of a turbo jet.

i. Diffuser

ii. Mechanical compressor,

iii. Combustion chamber,

iv. Turbine and

v. Exhaust nozzle.

7 Give the difference between pulse jet and ram jet engine.

PULSE JET

RAM JET

a)Mechanical valve arrangements are used during combustion.

b) The stagnation temperature at the

diffuser exit is comparatively less. a) Works without the aid of any mechanical device and needs no moving parts.

b) Since the mach number in Ram jet engine is supersonic, the stagnation temperature is very high

8 Give the difference between turbojet and ram jet engine.

a)Compressor and turbine are used.

b) Lower thrust and propulsive efficiency at lower speeds.

c) Construction cost is more a) Compressor and turbine are not used but diffuser and nozzle are used.

b) It provides high thrust per unit weight.

c) In the absence of rotating machines, the construction is simple and cheap.

9 What is the difference between turbo prop engine and turbo jet engine.

a) The specific fuel consumption based on thrust is low.

b) Propulsive efficiency within the range of operation is higher.

c) On account of higher thrust at low speeds the take-off role is short and requiring shorter runway.

d) Use of centrifugal compressor stages increases the frontal area.

e) Higher weight per unit thrust.

a) TSFC is comparatively higher at lower speeds and altitudes.

b) Propulsive efficiency is low.

c) Take – off role is longer and requiring longer run way.

d) Lower Frontal area

e) Lower weight per unit thrust.

10 What is ram effect?

When an aircraft flies with high velocity, the incoming air is compressed to high pressure without external work at the expense of velocity energy is known as "ram effect".

PART B

1) Aircraft speed of 525 Kmph. The data for the engine is given below

Inlet diffuser efficience =0.875

Compressor efficieny =0.790

Velocity of air at compressor entry =90m/s

Properties of air :γ =1.4, Cp =1.005 KJ/kg K (16)

2) The diameter of the propeller of an aircraft is 2.5m; It flies at a speed of 500Kmph at an altitude of 8000m. For a flight to jet speed ratio of 0.75 determine (a) the flow rate of air

through the propeller, (b) thrust produced (c) specific thrust, (d) specific impulse and (e) the thrust power. (16)

3) An aircraft flies at 960Kmph. One of its turbojet engines takes in 40 kg/s of air and expands the gases to the ambient pressure .The air –fuel ratio is 50 and the lower calorific value of the fuel is 43 MJ/Kg .For maximum thrust power determine (a)jet velocity (b) thrust (c) specific thrust (d) thrust power (e) propulsive, thermal and overall efficiencies and (f) TSFC(16)

4) A turbo jet engine propels an aircraft at a Mach number of 0.8 in level flight at an altitude of

10 km .The data for the engine is given below:

Stagnation temperature at the turbine inlet =1200K

Stagnation temperature rise through the compressor =175 K

Calorific value of the fuel =43 MJ/Kg

Compressor efficiency =0.75

Combustion chamber efficiency =0.975

Turbine efficiency =0.81

Mechanical efficiency of the power transmission between turbine and compressor =0.98

Exhaust nozzle efficiency=0.97

Specific impulse =25 seconds

Assuming the same properties for air and combustion gases calculate

- Fuel –air ratio, (2)
- Compressor pressure ratio, (4)
- Turbine pressure ratio, (4)
- Exhaust nozzles pressure ratio ,and (4)
- Mach number of exhaust jet (2)

5) A ramjet engine operates at M=1.5 at an altitude of 6500m.The diameter of the inlet diffuser

at entry is 50cm and the stagnation temperature at the nozzle entry is 1600K.The calorific value

of the fuel used is 40MJ/Kg .The properties of the combustion gases are same as those of air (γ

=1.4, R=287J/Kg K). The velocity of air at the diffuser exit is negligible Calculate (a) the efficiency of the ideal cycle, (b) flight speed (c) air flow rate (d) diffuser pressure

ratio (e) fuel –ratio (f)nozzle pressure ratio (g) nozzle jet Mach number (h) propulsive efficiency (i) and thrust. Assume the following values:□D =0.90,□B =0.98, □ j= 0.96.Stagnation pressure loss in the combustion chamber =0.002Po2. (16)

6) A rocket flies at 10,080 Kmph with an effective exhaust jet velocity of 1400m/s and propellant flow rate of 5.0Kg/s .If the heat of reaction of the propellants is 6500KJ/Kg of the propellat mixture determine;

a) Propulsion efficiency and propulsion power, (6)

b) Engine output and thermal efficiency ,and (6)

c) Overall efficiency. (4)

7) Determine the maximum velocity of a rocket and the altitude attained from the following data:

Mass ratio =0.15

Burn out time =75s

Effective jet velocity =2500m/s

What are the values of the velocity and altitude losses due to gravity? Ignore drag and assume vertical trajectory . (16)

8) A missile has a maximum flight speed to jet speed ratio of 0.2105 and specific impulse equal to 203.88 seconds .Determine for a burn out time of 8 seconds

a) Effective jet velocity (4)

b) Mass ratio and propellant mass functions (4)

c) Maximum flight speed, and (4)

d) Altitude gain during powered and coasting flights (4)

UNIT V SPACE PROPULSION

Rocket Propulsion

In the section about the rocket equation we explored some of the issues surrounding the performance of a whole rocket. What we didn't explore was the heart of the rocket, the motor. In this section we'll look at the design of motors, the factors which affect the performance of motors, and some of the practical limitations of motor design. The first part of this section is necessarily descriptive as the chemistry, thermodynamics and maths associated with motor design are beyond the target audience of this website.

General Principles of a Rocket Motor

In a rocket motor a chemical reaction is used to generate hot gas in a confined space called the combustion chamber. The chamber has a single exit through a constriction called the throat. The pressure of the hot gas is higher than the surrounding atmosphere, thus the gas flows out through the constriction and is accelerated.

Propellants

The chemical reaction in model rocket motors is referred to as an "exothermal redox" reaction. The term "exothermal" means that the reaction gives off heat, and in the case of rocket motors this heat is mainly absorbed by the propellants raising their temperature. The term "redox" means that it is an oxidation/reduction reaction, in other words one of the chemicals transfers oxygen atoms to another during the reaction. The two chemicals are called the oxidising agent and the reducing agent.

The most popular rocket motors are black powder motors, where the oxidising agent is saltpetre and the reducing agents are sulphur and carbon. Other motors include Potassium or ammonium perchlorate as the oxidising agent and mixtures of hydrocarbons and fine powdered metals as the reducing agents. Other chemicals are often added such as retardants to slow down the rate of burn, binding agents to hold the fuel together (often these are the hydrocarbons used in the reaction), or chemicals to colour the flame or

smoke for effects. In hybrid motors a gaseous oxidiser, nitrous oxide, reacts with a hydrocarbon, such as a plastic, to produce the hot gas.

Energy Conversion

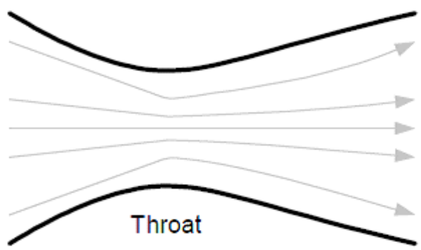
This reaction releases energy in the form of heat, and by confining the gas within the combustion chamber we give it energy due to its pressure. We refer to the energy of this hot pressurised gas as its "enthalpy". By releasing the gas through the throat the rocket motor turns the enthalpy of the gas into a flow of the gas with kinetic energy. It is this release of energy which powers the rocket. So the energy undergoes two conversions:

• Chemical energy to enthalpy

• Enthalpy to kinetic energy

The conversion from chemical energy to enthalpy takes place in the combustion chamber. To obtain the maximum enthalpy it is clearly important to have a reaction which releases lots of heat and generates lots of high energy molecules of gas to maximise pressure. here is clearly a limit to the temperature & pressure, as the combustion chamber may melt or split if these are too high. The designer has a limitation placed on his choice of reagents in that the reaction must not heat the combustion chamber to a point where it is damaged, nor must the pressure exceed that which the chamber can survive.

Changing enthalpy to kinetic energy takes place in the throat and the nozzle. Our mass of hot gas flows into the throat, accelerating as the throat converges. If we reduce the chameter of the throat enough, the flow will accelerate to the speed of sound, at which pint something unexpected occurs. As the flow diverges into the nozzle it continues to accelerate beyond the speed of sound, the increase in velocity depending on the increase in area. This type of nozzle is called a De Laval nozzle.



You will recall that the kinetic energy of a body can be calculated from:

If we consider a small volume of gas, it will have a very low mass. As we accelerate this gas it gains kinetic energy proportional to the square of the velocity, so if we double the velocity we get four times the kinetic energy. The velocity of the supersonic flow increases proportional to the increase in area of the nozzle, thus the kinetic energy increases by the fourth power of the increase in nozzle diameter. Thus doubling the nozzle diameter increases the kinetic energy by 16 times! The De Laval nozzle make rocket motors possible, as only such high velocity flows can generate the energy required to accelerate a rocket.

In model rockets the reaction is chemical generally short lived, a few seconds at most, sothe amount of heat transferred to the structural parts of the motor is limited. Also, the liner of the motor casing acts to insulate the casing from the rapid rise in temperature which would result from a reaction in direct contact with the metal casing. Model rocket motors also run at quite low pressure, well below the limits if the motor casing, further protecting the casing. It can be seen that the enthalpy of a model rocket motor is thus quite low. In large launch vehicles such as Ariane, the pressure and temperature are high, the burn may last several minutes, and the mass budget for the designer is very tight. Designing motors for these purposes is highly complex.

Thrust

The basic principles of a rocket motor are relatively straightforward to understand. In

rocketry the motor exists to accelerate the rocket, and thus it has to develop a force called "thrust". One of several definitions of force is that:

Force = rate of change of momentum

If we ignore (for a few paragraphs) any external effects we can say that the thrust is entirely due to the momentum of the propellant, a force called the "momentum thrust". If we denote the thrust as F and the momentum as P, then mathematically:

$$F = \frac{dP}{dt}$$

Sometimes for mathematical clarity we us the notation of P with a dot on top to denote the first derivative of P, and with 2 dots for the second derivative. Thus, in this new notation:

$$F = \frac{dP}{dt} = \dot{P}$$

You may also recall from the section on the rocket equation that momentum is the product of the mass and velocity. Thus we can say that the momentum of the flow from the nozzle of the rocket has a momentum:

$$P = mv_e$$

Thus:

$$F = \frac{dP}{dt} = \dot{P} = \frac{d(mv_e)}{dt}$$

If the exhaust velocity remains constant, which is a reasonable assumption, we arrive at the equation:

$$F = v_e \frac{d(m)}{dt} = \dot{m}v_e$$

The term "m-dot" is known as the mass flow rate, in other words the rate at which mass is ejected through the nozzle in kg/sec. In other words this is the rate at which the rocket burns fuel. This is an interesting relationship, which can be expressed in words as:

Momentum Thrust = mass flow rate x exhaust velocity

Flow expansion

The propellant is accelerated into the atmosphere. As it leaves the nozzle the propellant has an exit pressure Pexit and enters an atmosphere which has a pressure Patm. The transition from one pressure to the other cannot happen instantaneously as any pressure difference will cause a flow of high pressure fluid into the low pressure region. So what does this do to the thrust?

So the force (a component of thrust called "pressure thrust") depends on the pressure difference and the area of the

$$F = \dot{m}v_e + A(P_{exit} - P_{atm})$$

This suggests that we should aim for a maximum pressure in the nozzle so that the pressure thrust combines with the momentum thrust to produce a greater overall thrust. In fact, this intuitively correct result is wrong! If Pexit > Patm the exhaust gases will expand in all directions when they leave the nozzle, not only the direction of thrust. The total thrust of such a motor is less than could be delivered by just momentum thrust. We call this an "under expanded" flow as the propellant needs to expand more within the nozzle.

So what if Pexit < Patm? In this circumstance the atmosphere will try to flow back into the nozzle. This causes sudden transitions from supersonic to subsonic flow to occur in the nozzle setting up shock waves. These shock weaves turn some of the kinetic energy of

the flow back into enthalpy, reducing the overall thrust. We call the flow "over expanded" as the flow expands too much in the nozzle reducing the overall pressure.

The ideal situation is when Pexit = Patm which only occurs over a narrow range of altitudes. This is not a major problem for modellers, as the burns tend to occur at low altitudes and over a relatively narrow range of atmospheric pressures. It is easy to design motors which are efficient over this range. It is a real problem for manufacturers of launch vehicles as the motor may burn from sea level to several tens of miles above sea level. It is normal practice on major launchers to tune the motor for an altitude around the middle of the range of pressures and accept some loss of efficiency at the start and end of the burn.

This effect is very pronounced on the Saturn V rocket. Next time you see any video of a launch, watch the plume. At launch it is long and thin as the flow is over expanded. At high altitudes the plume is very wide, exhibiting under expansion.

Specific Impulse

How do we measure the effectiveness of a rocket motor? In cars we compare the effectiveness of motors through measures like miles per gallon of fuel, and time for 0-60 mph. The equivalent in rocket motors is called the specific impulse (Isp). Isp is defined as:

$$I_{sp} = \frac{Ft}{W}$$

Where F is the thrust in Newtons, t is the duration of the burn in seconds, and W is the weight of fuel in Newtons. Overall this gives a measure of the impulse Ft provided by a weight of fuel W. If we think about this, both F and W are forces, thus SI has the units of seconds. If we imagine rocket motor with an Isp of 300 seconds, then Newton of fuel (i.e. 1 kg under the acceleration due to earth's gravity at sea level) will give 1 Newton of thrust for 300 seconds. The same amount of fuel could also give 150 Newtons of thrust for 2 seconds. It can be seen that the notion of Isp gives a measure of the effectiveness of

a motor and fuel combination which is independent of the rate at which the fuel burns. Some typical values of Isp are:

Black powder: 20-40 seconds

Ammonium perchlorate and aluminium: 100-150 seconds

Liquid oxygen and liquid hydrogen: 400 seconds.

By considering the mass flow rate of the motor as instantaneously constant, we can modify equation 1 to read:

$$F = v_e \frac{m}{t}$$

We also know that the weight of fuel W is the mass of fuel multiplied by the acceleration due to gravity, so that

Types of Rocket Engines

rocket or **rocket vehicle** is a missile, spacecraft, aircraft or other vehicle which obtains thrust from a rocket engine. In all rockets, the exhaust is formed entirely from propellants carried within the rocket before use. Rocket engines work by action and reaction. Rocket engines push rockets forwards simply by throwing their exhaust backwards extremely fast.

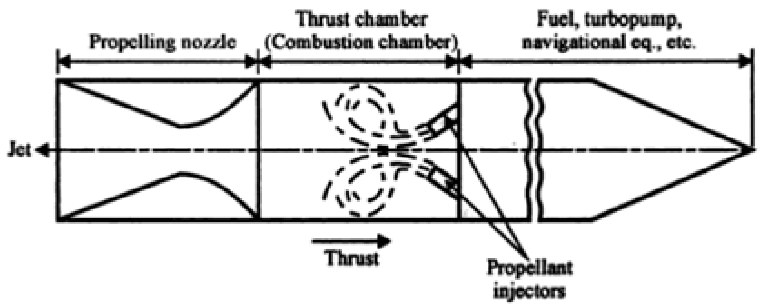


FIGURE 13.1 : A liquid propellant rocket

Rockets for military and recreational uses date back to the 13th century.[2] Significant scientific, interplanetary and industrial use did not occur until the 20th century, when rocketry was the enabling technology of the Space Age, including setting foot on the moon.

Rockets are used for fireworks, weaponry, ejection seats, launch vehicles for artificial satellites, human spaceflight and exploration of other planets. While comparatively inefficient for low speed use, they are very lightweight and powerful, capable of generating large accelerations and of attaining extremely high speeds with reasonable efficiency.

Chemical rockets are the most common type of rocket and they typically create their exhaust by the combustion of rocket propellant. Chemical rockets store a large amount of energy in an easily released form, and can be very dangerous. However, careful design, testing, construction and use minimize risks.

Rocket vehicles are often constructed in the archetypal tall thin "rocket" shape that takes off vertically, but there are actually many different types of rockets including, tiny models such as balloon rockets, water rockets, skyrockets or small solid rockets that can be purchased at a hobby store

- missiles
- space rockets such as the enormous Saturn V used for the Apollo program
- rocket cars
- rocket bike
- rocket powered aircraft (including rocket assisted takeoff of conventional aircraft- JATO)
- rocket sleds
- rocket trains
- rocket torpedos
- rocket powered jet packs
- rapid escape systems such as ejection seats and launch escape systems
- space probes

Propellants: A **propellant** is a material that is used to move ("propel") an object. The material is usually expelled by gas pressure through a nozzle. The pressure may be from a compressed gas, or a gas produced by a chemical reaction. The exhaust material may be a gas, liquid, plasma, or, before the chemical reaction, a solid, liquid or gelled. Common chemical propellants consist of a fuel; like gasoline, jet fuel, rocket fuel, and an oxidizer.

Propellant used for propulsion

Technically, the word **propellant** is the general name for chemicals used to create thrust. For vehicles, the term propellant refers only to chemicals that are stored within the vehicle prior to use, and excludes atmospheric gas or other material that may be collected in operation.

Amongst the English-speaking laymen, used to having fuels propel vehicles on Earth, the word **fuel** is inappropriately used. In Germany, the word *Treibstoff*—literally "drive-stuff"—is used; in France, the word *ergols* is used; it has the same Greek roots as hypergolic, a term used in English for propellants which combine spontaneously and do not have to be set ablaze by auxiliary ignition system.

In rockets, the most common combinations are *bipropellants*, which use two chemicals, a fuel and an oxidiser. There is the possibility of a tripropellant combination, which takes advantage of the ability of substances with smaller atoms to attain a greater exhaust velocity, and hence propulsive efficiency, at a given temperature.

Although not used in practice, the most developed tripropellant systems involves adding a third propellant tank containing liquid hydrogen to do this.

Solid propellant

In ballistics and pyrotechnics, a **propellant** is a generic name for chemicals used for propelling projectiles from guns and other firearms. Propellants are usually made from low explosive materials, but may include high explosive chemical ingredients that are diluted and burned in a controlled way (deflagration) rather than detonation. The controlled burning of the propellant composition usually produces thrust by gas pressure and can accelerate a projectile, rocket, or other vehicle. In this sense, common or well known **propellants** include, for firearms, artillery and solid propellant rockets:

Gun propellants, such as:

- Gunpowder (black powder)
- Nitrocellulose-based powders
- Cordite
- Ballistite
- Smokeless powders

Composite propellants made from a solid oxidizer such as ammonium perchlorate or ammonium nitrate, a rubber such as HTPB, or PBAN (may be replaced by energetic polymers such as polyglycidyl nitrate or polyvinyl nitrate for extra energy) , optional high explosive fuels (again, for extra energy) such as RDX or nitroglycerin, and usually a powdered metal fuel such as aluminum.

Some amateur propellants use potassium nitrate, combined with sugar, epoxy, or other

fuels / binder compounds.

Potassium perchlorate has been used as an oxidizer, paired with asphalt, epoxy, and other binders.

Grain

Propellants are used in forms called grains. A grain is any individual particle of propellant regardless of the size or shape. The shape and size of a propellant grain determines the burn time, amount of gas and rate produced from the burning propellant and consequently thrust vs time profile.

There are three types of burns that can be achieved with different grains.

Progressive Burn

Usually a grain with multiple perforations or a star cut in the center providing a lot of surface area.

Digressive Burn

Usually a solid grain in the shape of a cylinder or sphere.

Neutral Burn

Usually a single perforation; as outside surface decreases the inside surface increases at the same rate.

Composition

There are four different types of solid propellant compositions:

Single Based Propellant: A single based propellant has nitrocellulose as its chief explosives ingredient. Stabilizers and other additives are used to control the chemical stability and enhance the propellant's properties.

Double Based Propellant: Double based propellants consist of nitrocellulose with nitroglycerin or other liquid organic nitrate explosives added. Stabilizers and other additives are used also. Nitroglycerin reduces smoke and increases the energy output. Double based propellants are used in small arms, cannons, mortars and rockets.

Triple Based Propellant

Triple based propellants consist of nitrocellulose, nitroquanidine, nitroglycerin or other liquid organic nitrate explosives. Triple based propellants are used in cannons.

Composite

Composites contain no nitrocellulose, nitroglycerin, nitroquanidine or any other organic nitrate. Composites usually consist of a fuel such as metallic aluminum, a binder such as synthetic rubber, and an oxidizer such as ammonium perchlorate. Composite propellants are used in large rocket motors.

Liquid propellant

Common propellant combinations used for liquid propellant rockets include:

- Red fuming nitric acid (RFNA) and kerosene or RP-1
- RFNA and Unsymmetrical dimethyl hydrazine (UDMH)
- Dinitrogen tetroxide and UDMH, MMH and/or hydrazine
- Liquid oxygen and kerosene or RP-1
- Liquid oxygen and liquid hydrogen
- Liquid oxygen and ethanol
- Hydrogen peroxide and alcohol or RP-1
- Chlorine pentafluoride and hydrazine

Common monopropellant used for liquid rocket engines include:

- Hydrogen peroxide
- Hydrazine
- Red fuming nitric acid (RFNA)

Introducing propellant into a combustion chamber

Rocket propellant is mass that is stored, usually in some form of propellant tank, prior to being ejected from a rocket engine in the form of a fluid jet to produce thrust.

Chemical rocket propellants are most commonly used, which undergo exothermic chemical reactions which produce hot gas which is used by a rocket for propulsive purposes. Alternatively, a chemically inert reaction mass can be heated using a high-energy power source via a heat exchanger, and then no combustion chamber is used.

A solid rocket motor:

Solid rocket propellants are prepared as a mixture of fuel and oxidizing components called 'grain' and the propellant storage casing effectively becomes the combustion chamber. Liquid-fueled rockets typically pump separate fuel and oxidiser components into the combustion chamber, where they mix and burn. Hybrid rocket engines use a combination of solid and liquid or gaseous propellants. Both liquid and hybrid rockets use *injectors* to introduce the propellant into the chamber. These are often an array of simple jets- holes through which the propellant escapes under pressure; but sometimes may be more complex spray nozzles. When two or more propellants are injected the jets usually deliberately collide the propellants as this breaks up the flow into smaller droplets that burn more easily.

Rocket Ignition :

Rocket fuels, hypergolic or otherwise, must be mixed in the right quantities to have a controlled rate of production of hot gas. A hard start indicates that the quantity of combustible propellant that entered the combustion chamber prior to ignition was too large. The result is an excessive spike of pressure, possibly leading to structural failure or even an explosion (sometimes facetiously referred to as "spontaneous disassembly").

Avoiding hard starts involves careful timing of the ignition relative to valve timing or varying the mixture ratio so as to limit the maximum pressure that can occur or simply ensuring an adequate ignition source is present well prior to propellant entering the chamber.

Explosions from hard starts often cannot happen with purely gaseous propellants, since the amount of the gas present in the chamber is limited by the injector area relative to the throat area, and for practical designs propellant mass escapes too quickly to be an issue.

A famous example of a hard start was the explosion of Wernher von Braun's "1W" engine during a demonstration to General Dornberger on December 21, 1932. Delayed ignition allowed the chamber to fill with alcohol and liquid oxygen, which exploded violently. Shrapnel was embedded in the walls, but nobody was hit.

Rocket Combution:

Combustion chamber

For chemical rockets the combustion chamber is typically just a cylinder, and flame holders are rarely used. The dimensions of the cylinder are such that the propellant is able to combust thoroughly; different propellants require different combustion chamber sizes for this to occur. This leads to a number called *L*

L = Vc/At

where:

Vc is the volume of the chamber

At is the area of the throat

L* is typically in the range of 25–60 inches (0.63–1.5 m).

The combination of temperatures and pressures typically reached in a combustion chamber is usually extreme by any standards. Unlike in air-breathing jet engines, no atmospheric nitrogen is present to dilute and cool the combustion, and the temperature can reach true stoichiometric. This, in combination with the high pressures, means that the rate of heat conduction through the walls is very high.

Rocket nozzles

Typical temperatures (T) and pressures (p) and speeds (v) in a De Laval Nozzle

The large bell or cone shaped expansion nozzle gives a rocket engine its characteristic shape.

In rockets the hot gas produced in the combustion chamber is permitted to escape from the combustion chamber through an opening (the "throat"), within a high expansion-ratio 'de Laval' nozzle.

Provided sufficient pressure is provided to the nozzle (about 2.5-3x above ambient pressure) the nozzle *chokes* and a supersonic jet is formed, dramatically accelerating the gas, converting most of the thermal energy into kinetic energy.

The exhaust speeds vary, depending on the expansion ratio the nozzle is designed to give, but exhaust speeds as high as ten times the speed of sound of sea level air are not

uncommon.

Rocket thrust is caused by pressures acting in the combustion chamber and nozzle. From Newton's third law, equal and opposite pressures act on the exhaust, and this accelerates it to high speeds.

About half of the rocket engine's thrust comes from the unbalanced pressures inside the combustion chamber and the rest comes from the pressures acting against the inside of the nozzle (see diagram). As the gas expands (adiabatically) the pressure against the nozzle's walls forces the rocket engine in one direction while accelerating the gas in the other.

Propellant efficiency

For a rocket engine to be propellant efficient, it is important that the maximum pressures possible be created on the walls of the chamber and nozzle by a specific amount of propellant; as this is the source of the thrust. This can be achieved by all of:

- Heating the propellant to as high a temperature as possible (using a high energy fuel, containing hydrogen and carbon and sometimes metals such as aluminium, or even using nuclear energy)
- Using a low specific density gas (as hydrogen rich as possible)
- Using propellants which are, or decompose to, simple molecules with few degrees of freedom to maximize translational velocity.

Since all of these things minimise the mass of the propellant used, and since pressure is proportional to the mass of propellant present to be accelerated as it pushes on the engine, and since from Newton's third law the pressure that acts on the engine also reciprocally acts on the propellant, it turns out that for any given engine the speed that the propellant leaves the chamber is unaffected by the chamber pressure (although the thrust is proportional). However, speed is significantly affected by all three of the above factors and the exhaust speed is an excellent measure of the engine propellant efficiency. This is termed *exhaust velocity*, and after allowance is made for factors that can reduce it, the **effective exhaust velocity** is one of the most important parameters of a rocket engine

(although weight, cost, ease of manufacture etc. are usually also very important).
For aerodynamic reasons the flow goes sonic ("chokes") at the narrowest part of the nozzle, the 'throat'. Since the speed of sound in gases increases with the square root of temperature, the use of hot exhaust gas greatly improves performance. By comparison, at room temperature the speed of sound in air is about 340 m/s while the speed of sound in the hot gas of a rocket engine can be over 1700 m/s; much of this performance is due to the higher temperature, but additionally rocket propellants are chosen to be of low molecular mass, and this also gives a higher velocity compared to air.

Expansion in the rocket nozzle then further multiplies the speed, typically between 1.5 and 2 times, giving a highly collimated hypersonic exhaust jet. The speed increase of a rocket nozzle is mostly determined by its area expansion ratio—the ratio of the area of the throat to the area at the exit, but detailed properties of the gas are also important. Larger ratio nozzles are more massive but are able to extract more heat from the combustion gases, increasing the exhaust velocity.

Nozzle efficiency is affected by operation in the atmosphere because atmospheric pressure changes with altitude; but due to the supersonic speeds of the gas exiting from a rocket engine, the pressure of the jet may be either below or above ambient, and equilibrium between the two is not reached at all altitudes (See Diagram).

Back pressure and optimal expansion

For optimal performance the pressure of the gas at the end of the nozzle should just equal the ambient pressure: if the exhaust's pressure is lower than the ambient pressure, then the vehicle will be slowed by the difference in pressure between the top of the engine and the exit; on the other hand, if the exhaust's pressure is higher, then exhaust pressure that could have been converted into thrust is not converted, and energy is wasted.

To maintain this ideal of equality between the exhaust's exit pressure and the ambient pressure, the diameter of the nozzle would need to increase with altitude, giving the

pressure a longer nozzle to act on (and reducing the exit pressure and temperature). This increase is difficult to arrange in a lightweight fashion, although is routinely done with other forms of jet engines. In rocketry a lightweight compromise nozzle is generally used and some reduction in atmospheric performance occurs when used at other than the 'design altitude' or when throttled. To improve on this, various exotic nozzle designs such as the plug nozzle, stepped nozzles, the expanding nozzle and the aerospike have been proposed, each providing some way to adapt to changing ambient air pressure and each allowing the gas to expand further against the nozzle, giving extra thrust at higher altitudes.

When exhausting into a sufficiently low ambient pressure (vacuum) several issues arise. One is the sheer weight of the nozzle- beyond a certain point, for a particular vehicle, the extra weight of the nozzle outweighs any performance gained. Secondly, as the exhaust gases adiabatically expand within the nozzle they cool, and eventually some of the chemicals can freeze, producing 'snow' within the jet. This causes instabilities in the jet and must be avoided.

On a De Laval nozzle, exhaust gas flow detachment will occur in a grossly over-expanded nozzle. As the detachment point will not be uniform around the axis of the engine, a side force may be imparted to the engine. This side force may change over time and result in control problems with the launch vehicle.

Thrust vectoring

Many engines require the overall thrust to change direction over the length of the burn. A number of different ways to achieve this have been flown:

- The entire engine is mounted on a hinge or gimbal and any propellant feeds reach the engine via low pressure flexible pipes or rotary couplings.
- Just the combustion chamber and nozzle is gimbled, the pumps are fixed, and

high pressure feeds attach to the engine

- multiple engines (often canted at slight angles) are deployed but throttled to give the overall vector that is required, giving only a very small penalty
- fixed engines with vernier thrusters
- high temperature vanes held in the exhaust that can be tilted to deflect the jet

Overall rocket engine performance

- Rocket technology can combine very high thrust (meganewtons), very high exhaust speeds (around 10 times the speed of sound in air at sea level) and very high thrust/weight ratios (>100) *simultaneously* as well as being able to operate outside the atmosphere, and while permitting the use of low pressure and hence lightweight tanks and structure.
- Rockets can be further optimised to even more extreme performance along one or more of these axes at the expense of the others.

Specific impulse

The most important metric for the efficiency of a rocket engine is impulse per unit of propellant, this is called specific impulse (usually written *Isp*). This is either measured as a speed (the *effective exhaust velocity Ve* in metres/second or ft/s) or as a time (seconds). An engine that gives a large specific impulse is normally highly desirable.

The specific impulse that can be achieved is primarily a function of the propellant mix (and ultimately would limit the specific impulse), but practical limits on chamber pressures and the nozzle expansion ratios reduce the performance that can be achieved.

Space flight:

Spaceflight is the act of travelling into or through outer space. Spaceflight can occur with spacecraft which may, or may not, have humans on board. Examples of human spaceflight include the Russian Soyuz program, the U.S. Space shuttle program, as well as the ongoing International Space Station. Examples of unmanned spaceflight include space probes which leave Earth's orbit, as well as satellites in orbit around Earth, such as

communication satellites.

Spaceflight is used in space exploration, and also in commercial activities like space tourism and satellite telecommunications. Additional non-commercial uses of spaceflight include space observatories, reconnaissance satellites and other earth observation satellites.

A spaceflight typically begins with a rocket launch, which provides the initial thrust to overcome the force of gravity and propels the spacecraft from the surface of the Earth. Once in space, the motion of a spacecraft—both when unpropelled and when under propulsion—is covered by the area of study called astrodynamics. Some spacecraft remain in space indefinitely, some disintegrate during atmospheric reentry, and others reach a planetary or lunar surface for landing or impact.

Types of spaceflight

Human spaceflight

The first human spaceflight was Vostok 1 on April 12, 1961, on which cosmonaut Yuri Gagarin of the USSR made one orbit around the Earth. In official Soviet documents, there is no mention of the fact that Gagarin parachuted the final seven miles.[3] The international rules for aviation records stated that "The pilot remains in his craft from launch to landing". This rule, if applied, would have "disqualified" Gagarins space-flight. Currently the only spacecraft regularly used for human spaceflight are Russian Soyuz spacecraft and the U.S. Space Shuttle fleet. Each of those space programs have used other spacecraft in the past. Recently, the Chinese Shenzhou spacecraft has been used three times for human spaceflight, and SpaceshipOne twice.

Sub-orbital spaceflight

On a sub-orbital spaceflight the spacecraft reaches space and then returns to the atmosphere after following a (primarily) ballistic trajectory. This is usually because of insufficient specific orbital energy, in which case a suborbital flight will last only a few minutes, but it is also possible for an object with enough energy for an orbit to have a trajectory that intersects the Earth's atmosphere, sometimes after many hours. Pioneer 1

was NASA's first space probe intended to reach the Moon. A partial failure caused it to instead follow a suborbital trajectory to an altitude of 113,854 kilometers (70,746 mi) before reentering the Earth's atmosphere 43 hours after launch.

The most generally recognized boundary of space is the Kármán line (actually a sphere) 100 km above sea level. (NASA alternatively defines an astronaut as someone who has flown more than 50 miles or 80 km above sea level.) It is not generally recognized by the public that the increase in potential energy required to pass the Kármán line is only about 3% of the orbital energy (potential plus kinetic energy) required by the lowest possible earth orbit (a circular orbit just above the Kármán line.) In other words, it is far easier to reach space than to stay there.

On May 17, 2004, Civilian Space eXploration Team launched the GoFast Rocket on a suborbital flight, the first amateur spaceflight. On June 21, 2004, SpaceShipOne was used for the first privately-funded human spaceflight.

Orbital spaceflight

A minimal orbital spaceflight requires much higher velocities than a minimal sub-orbital flight, and so it is technologically much more challenging to achieve. To achieve orbital spaceflight, the tangential velocity around the Earth is as important as altitude. In order to perform a stable and lasting flight in space, the spacecraft must reach the minimal orbital speed required for a closed orbit.

Interplanetary spaceflight

An artist's imaginative impression of a vehicle entering a wormhole for interstellar travel

Interplanetary travel is travel between planets within a single planetary system. In practice, the use of the term is confined to travel between the planets of the Solar System.

Interstellar spaceflight

Five spacecraft are currently leaving the Solar System on escape trajectories. The one farthest from the Sun is Voyager 1, which is more than 100 AU distant and is moving at 3.6 AU per year.[4] In comparison Proxima Centauri, the closest star other than the Sun, is 267,000 AU distant. It will take Voyager 1 over 74,000 years to reach this distance. Vehicle designs using other techniques, such as nuclear pulse propulsion are likely to be

able to reach the nearest star significantly faster.

Another possibility that could allow for human interstellar spaceflight is to make use of time dilation, as this would make it possible for passengers in a fast-moving vehicle to travel further into the future while aging very little, in that their great speed slows down the rate of passage of on-board time. However, attaining such high speeds would still require the use of some new, advanced method of propulsion.

Intergalactic spaceflight

Intergalactic travel involves spaceflight between galaxies, and is considered much more technologically demanding than even interstellar travel and, by current engineering terms, is considered science fiction.

1 Give the difference between Jet propulsion and Rocket propulsion.

a) Oxygen is obtained from the surrounding atmosphere for combustion purposes.

b) The jet consists of air plus combustion products.

c) Mechanical devices are also used a)The propulsion unit consists of its own oxygen supply for combustion purposes.

b) Jet consists of the exhaust gases
only.

c) Mechanical devices are not used.

2 Define Rocket propulsion.

If the propulsion unit contains its own oxygen supply for combustion purposes, the system is known as "Rocket propulsion".

3 Define thrust for a rocket engine and how it is produced.

The force that propels the rocket at a given velocity is known as thrust. This is produced due to the change in momentum flux of the outgoing gases as well as the difference between the nozzle exit pressure and the ambient pressure.

4 What are the types of rocket engines?

Rocket engines are classified in the following manner.

a) On the basis of source of energy employed

i. Chemical rockets,

ii. Solar rockets

iii. Nuclear rockets and

iv. Electrical rockets

b) On the basis of propellants used

i. Liquid propellant

ii. Solid propellant

iii. Hybrid propellant rockets.

5 Compare solid and liquid propellant rockets.

a) Solid fuels and oxidizers are used in rocket engines

b) Generally stored in combustion chamber (both oxidizer and fuel).

c) Burning in the combustion chamber is uncontrolled rate a) Liquid fuels and oxidizers are

used.

b) Separate oxidizer and fuel tanks are used for storing purposes.

c) Controlled rate

6 What are the types of liquid propellants used in rocket engines?

i. Mono propellants

ii. Bi – propellants

7 Give two liquid propellants.

Liquid fuels : Liquid hydrogen, UDMH, hydrazine

Solid fuels : Polymers, plastics and resin material

8 What is mono-propellants? Give example.

A liquid propellant which contains both the fuel and oxidizer in a single chemical is known as "mono propellant". e.g.,

i. Hydrogen peroxide

ii. Hydrazine

iii Nitroglycerine and

iv Nitromethane, etc.

9 Name some oxidizers used in rockets.

A liquid propellant which contains the fuel and oxidizer in separate units is known as bi-propellant. The commonly used bi-propellant combinations are:

OXIDIZER FUEL

a) Liquid oxygen

b) Hydrogen peroxide

c) Nitrogen tetroxide

d) Nitric acid

a) Gasoline

b) Liquid bydrogen

c) UDMH

d) Alcohol, ethanol

10 Name few advantages of liquid propellant rockets over solid propellant rockets.

i. Liquid propellant can be reused or recharged. Hence it is economical.

ii. Increase or decrease of speed is possible when it is in operation.

iii. Storing and transportation is easy as the fuel and oxidizer are kept separately.

iv. Specific impulse is very high.

PART B

1) Calculate the orbital and escape velocities of a rocket at mean sea level and an altitude of 300km from the following data:

Radius of earth at mean sea level =6341.6Km

Acceleration due to gravity at mean sea level =9.809 m/s² (16)

2) With a neat sketches the principle of operation of:

1. turbo fan engine and (8)

2. ram jet engine (8)

3) Explain the construction and operation of a ramjet engine and derive an expression for the ideal efficiency. (16)

4) Explain the construction and operation of a solid propellant rocket engine. Also name any four solid propellants.and state its advantages and disadvantages. (16)

5) What are the advantages and disadvantages of liquid propellants compared to solid propellants. (16)

6) Dicuss in detail the various propellants used in solid fuel rockets and liquid fuel system .Also
sketch the propellant feed-system for a liquid propellant rocket motor. (16)

7) Briefly explain the construction and working of :

A. Rocket engine (6)

B. Ramjet engine (6)

C. Pulsejet engine (4)

8) With the help of a neat sketch describe the working of a ramjet engine. Depict the various thermodynamic process occurring in it on h-s diagram. What is the effect of flight Mach number
on its efficiency? (16)

9) Explain with a neat sketch the working of a turbo-pump feed system used in a liquid propellant rocket? (16)