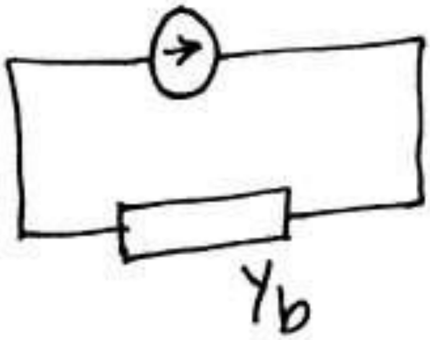
Define primitive network. EE - N/D - 07

U-I

Primitive network is defined as a network element may contain active and passive components.

<u>Impedance form</u>

<u>Admittance form</u>



$Y_b = \frac{1}{Z_b}$

A set of unconnected elements is defined as a primitive network.

If there is no mutual coupling between the diagonals are impedances (or) admittances.

Name the diagonal and off - diagonal elements of bus impedance matrix. EE - N/D - 13

U-I

The diagonal elements of bus impedance matrix are called as self impedance of the bus in the n/w.
The off-diagonal elements of bus impedance matrix are called as mutual impedance between the buses in the network

What is single line diagram? EE-N/D-11

U-I

A single line diagram is diagramatic representation of Power system and components in the network are represented by their symbol and connection between them are represented by a single line.

State the advantage of per unit analysis. EE - N/D - 16

U-I

The advantages of per unit are

1) Manufacturers usually specify the impedance values of equipments in p.u. of the equipment's rating. If any data is not available, it is easier to assume its per unit value than its numerical value.

2) Per unit data representation yields important information about relative magnitudes.

3) The transformer connections in 3 phase circuits do not affect the per unit value of impedance although the base voltages on the two sides do depend on the connections.

How are the loads represented in the reactance and Impedance diagram? EE-N/D-16

The loads in impedance diagram is shown in fig-1.

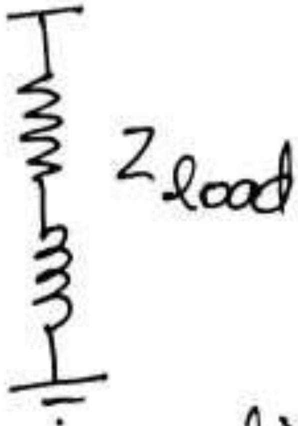
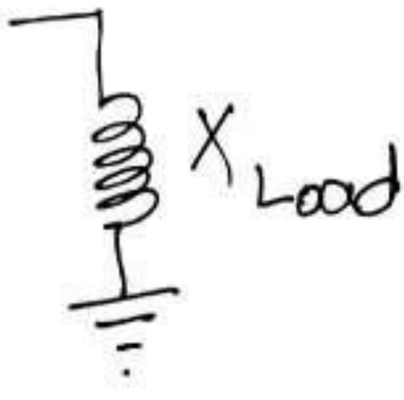


fig-1

The load in the reactance diagram is shown in fig-2.



What is the need for system analysis in planning and operation of power system? EE – N/D – 09

The operational planning covers the whole period ranging from the incremental stage of system development. The system operation engineers at various points like area, regional and national load despatch centre deals with the despatch of power

The power balance equations is

$$P_D = \sum_{i=1}^{N} P_{Gi} \quad i = 1, 2, \ldots, N$$

The operation of a power system must be reliable and uninterrupted. The reliability of power supply implies more than availability of power.

The loads must be fed at constant voltage and frequency.

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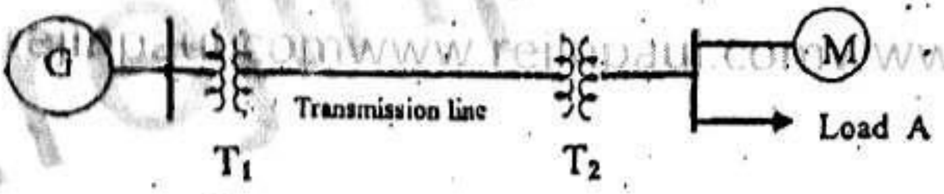
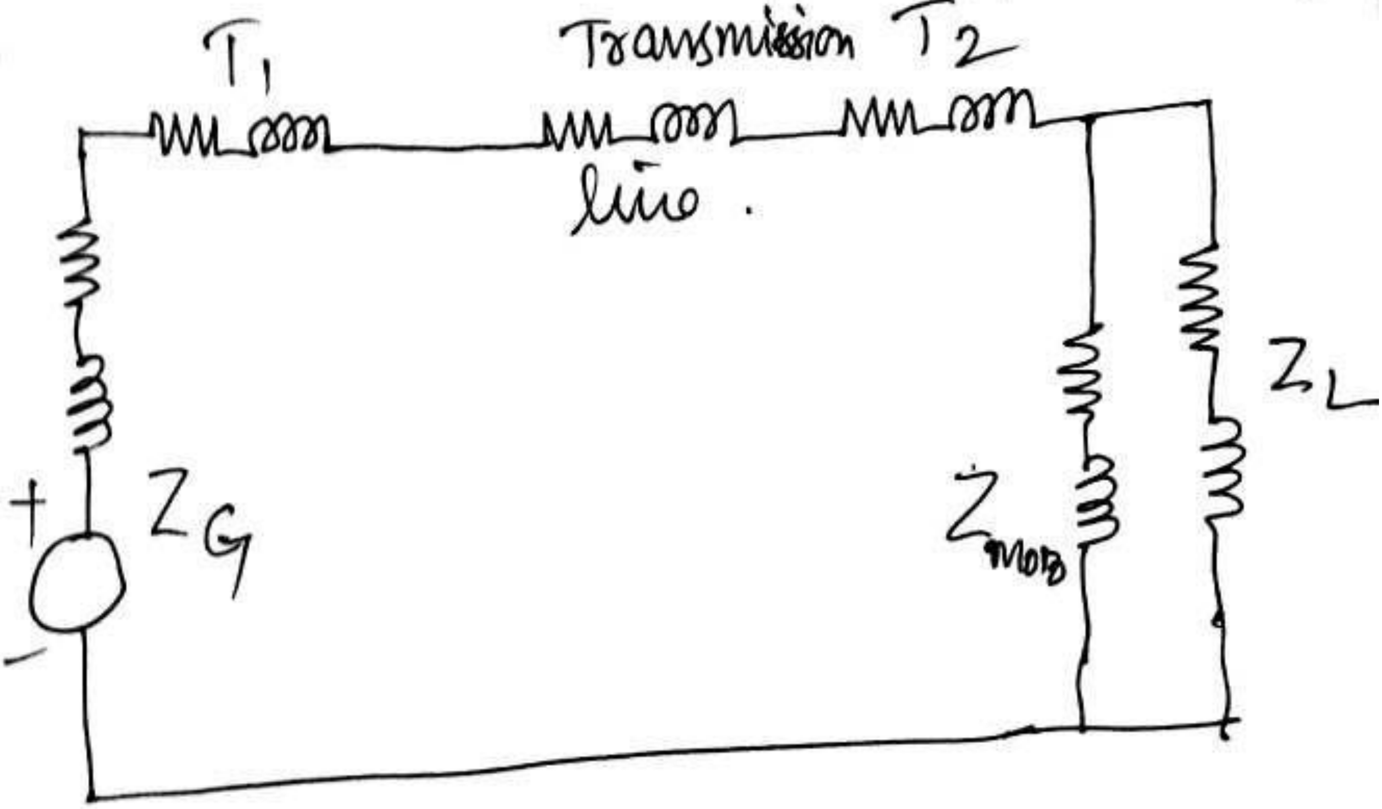
1. Draw the impedance diagram for the given single line representation of the power system. EE - M/J -14





If the reactance in ohms is 15 ohms, find the p.u value for a base of 15 KVA and 10 KV.

EE - N/J -12

U-I

<u>Data</u>

Reactance $= 15\,\Omega$

$kVA_b = 15$

$kV_b = 10$

$$\text{Base reactance} = \frac{V_b^2}{kVA_b}$$

$$= \frac{10^2}{15} = \frac{100}{15}$$

$$= 6.67\,\Omega$$

$$\text{P.u. reactance} = \frac{\text{Actual reactance}}{\text{Base reactance}}$$

$$= \frac{15}{6.67}$$

$$= 2.249 \text{ P.u.}$$

What is the advantage of building algorithm over other methods of forming Z_{Bus}? EE -

V-I

M/J - 09

The advantages of bus building algorithm are

1) Any modification of the network does not require complete rebuilding of Z_{bus}

2) Easy to compute the matrix Values.

What are the main divisions of power system? EE - N/D-14

U-I

The main division of power system are

1. Power system stability
2. Power system operation & control
3. Power system security
4. Power system Dynamics
5. Power system Economics.

. Define per unit value of an electrical quantity. Write equation for base impedance with to 3-phase system. EE - MIT - 09

U-I

Per unit is defined as the ratio of ~~between~~ actual quantity to base quantity.

$$\text{Per unit} = \frac{\text{Actual value}}{\text{Base value}}$$

What are the components of power system? EE- MCJ +2

U-I

The components of power system are

1) Synchronous generator
2) Transformer
3) Transmission line
4) Loads
5) Compensating equipments.

What are the functions of modern power system? EE - N/D - 13

U-I

~~The~~ The functions of modern Power system are

1. To calculate the voltage at every bus instentaneously.

2. If there is any fault on the system, faulty system will be isolated from the healthy ~~power~~ part of the system.

Write the equation for converting the p.u. impedance expressed in one base to another base.

U-I

EE - A/M - 10

The equation for converting P.U. impedance expressed in one base to another base is

$$Z_{p.u.\,new} = Z_{p.u.\,old} \times \left(\frac{kV_{b\,old}}{kV_{b\,new}}\right)^2 \times \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$$

How are the base values chose in per unit representation of a power system? EE - N/D -0

-08

The highest capacity of the rating will be chosen as the base value to Calculate the p.u. value of the Power system components.

The reactance of a generator designated X" is given as 0.2 per unit based on the generator's nameplate rating of 20 KV, 500 MVA. The base for calculations is 22 KV, 100 MVA. Find X" on the new base. EE - N/D - 07

U-I

$$X_{new} = 0.2 \times \left(\frac{20}{22}\right)^2 \times \left(\frac{100}{500}\right)$$

$$= 0.0331 \text{ p.u.}$$

1| Explain the modelling of generator, load, transmission line for ~~Generator model~~ short circuit, Power flow & Stability studies. (N/D – 08)

Generator model

The thevenin's equivalent circuit of the generator is the voltage source in series with the Thevenin's equivalent impedance as shown in fig-1

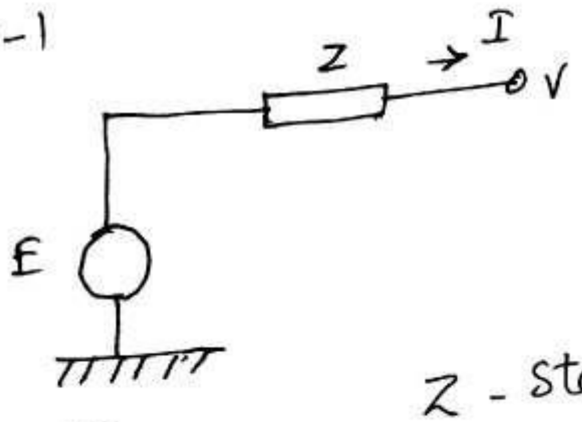
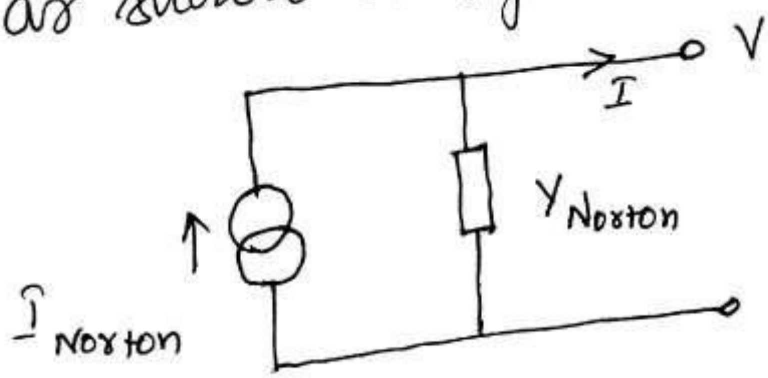


fig-1

$$E = V + IZ \quad \text{--- (1)}$$

Z - steady state impedance.

The Norton form of the equivalent circuit is the current source in parallel with the admittance as shown in fig-2.



$$I_{Norton} = \frac{E}{Z},$$

$$Y_{Norton} = \frac{1}{Z}$$

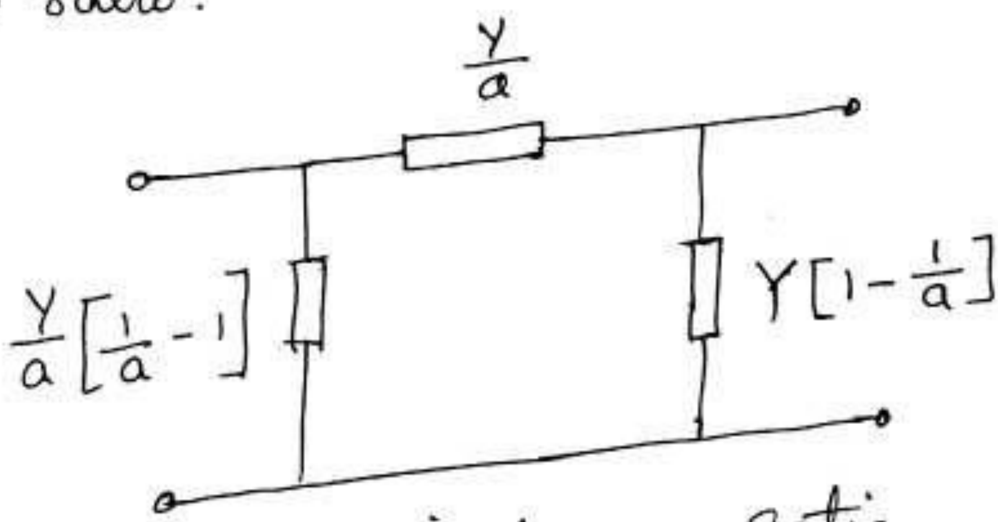
$$I_{Norton} = I + Y_{Norton} \cdot V$$

Transformer model

For unity turns ratio: Transformers are modelled by a series reactance, which is its leakage reactance.

For non-unity turns ratio (off nominal turns ratio)

Transformers are modelled by π circuits and the elements of the π circuit are functions of the tap ratio.



where a - off nominal turns ratio

Y - Admittance of the transformer $\left(\frac{1}{2}\right)$

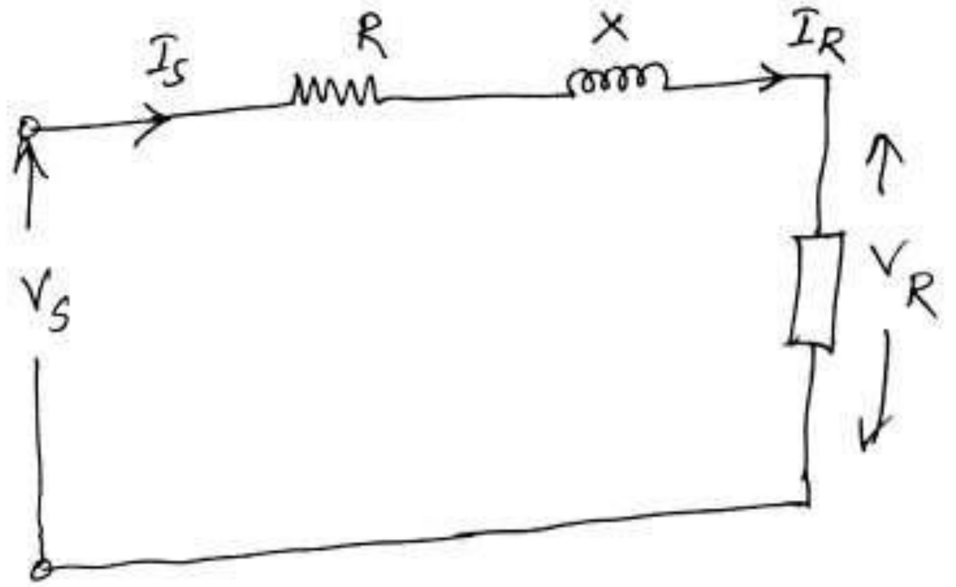
Transmission line

Transmission lines are modelled as short line, medium line and long line.

Short line

In short lines, resistance and inductance are assumed to be lumped.

→ The equivalent circuit of a short transmission line is as shown in fig.

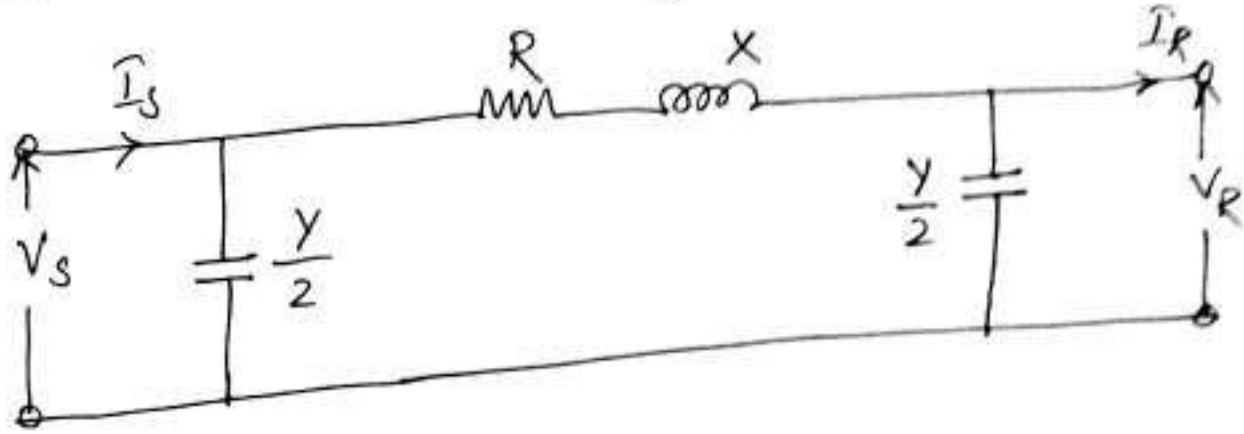


Medium lines (80 - 250 km)

(long lines) $\frac{Y'}{2}$ & Z'

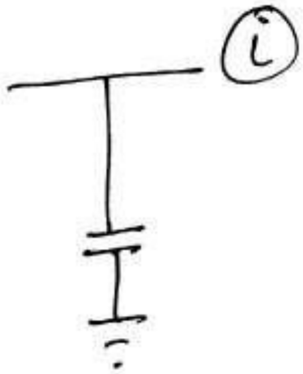
In medium lines, resistance and inductance are assumed to be lumped and the total shunt admittance is divided into two equal parts and placed at the receiving and sending ends.

π model is shown in figure.



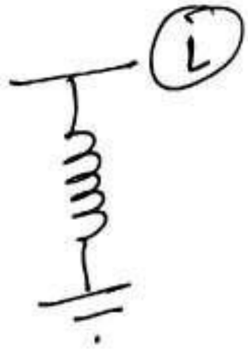
Shunt elements.

The shunt capacitor is connected to bus i as shown in fig.



If S is MVAR rating of shunt capacitor,
S_0 is base MVA.

The shunt reactor is connected to bus i as shown in fig.



If S is MVAR rating of shunt reactor,
S_0 is base MVA.

—x—

Describe the Z_{bus} building algorithm in detail.
M/J-14

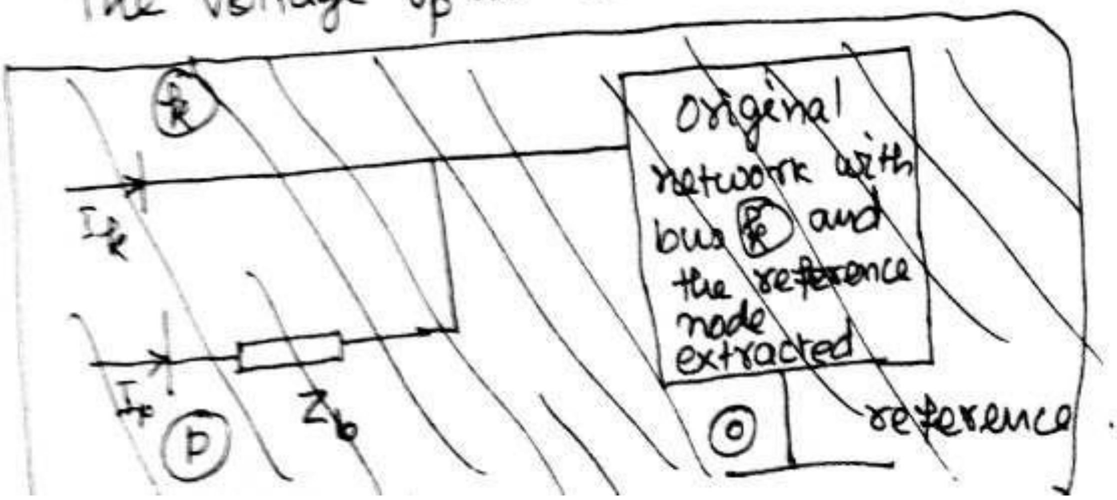
→ Z_{bus} is an important tool in power system analysis we now examine how an existing Z_{bus} may be modified to add new buses or to connect new lines to established buses.

→ Of course, we could create a new Y_{bus} and invert it, direct methods of modifying Z_{bus} are available and very much simpler than a matrix inversion even for a small number of buses.

Case 1 Adding Z_b from a new bus (P) to reference node

→ The addition of the new bus (P) connected to the reference node through Z_b without a connection to any of the buses of the original network cannot alter the original bus voltages when a current I_p is injected at the new bus.

The voltage V_p at the new bus is equal to $I_p \cdot Z_b$.

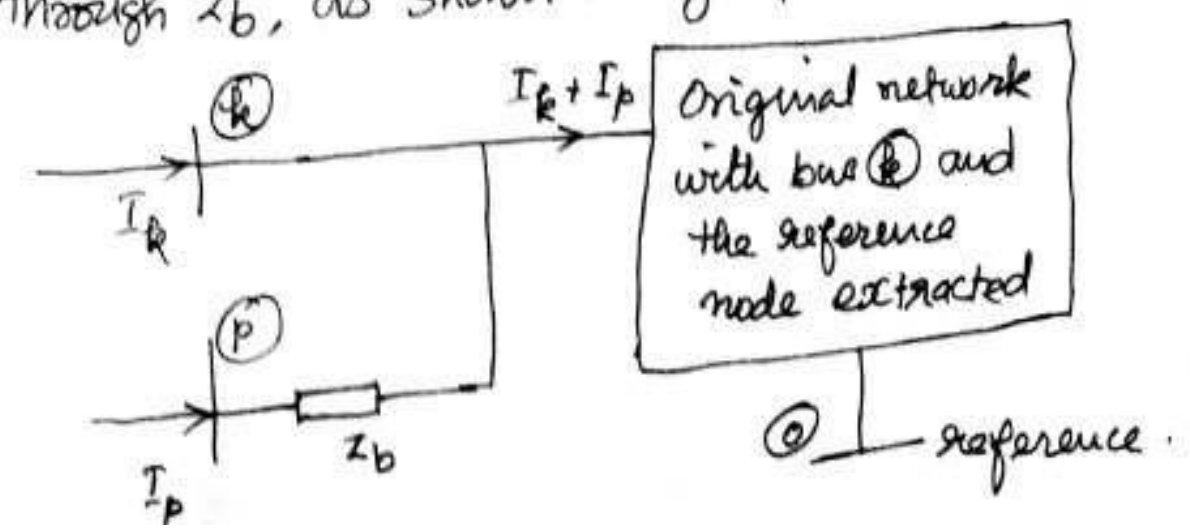


$$\begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_N^0 \\ V_p \end{bmatrix} = \underbrace{\begin{array}{c} \\ \\ \\ \\ \\ (p) \end{array}\begin{array}{c} \begin{array}{ccccc} & & & & (p) \end{array} \\ \left[\begin{array}{cccc|c} & & & & 0 \\ & Z_{orig} & & & 0 \\ & & & & \vdots \\ & & & & 0 \\ \hline 0 & 0 & \cdots & 0 & Z_b \end{array}\right] \end{array}}_{Z_{bus}(new)} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ I_p \end{bmatrix}$$

we note that the column vector of currents multiplied by the new Z_{bus} will not alter the Voltages of the original network and will result in the correct Voltage at new bus (p)

Case-2 Adding z_b from a new bus (p) to an existing bus (k)

The addition of a new bus (p) connected through z_b to an existing bus (k) with I_p injected at bus (p) will cause the current entering the original network at bus (k) to become the sum of I_k injected at bus (k) plus the current I_p coming through z_b, as shown in fig-1.



$$\begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_N^0 \\ \hline V_p \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} Z_{orig} & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \\ \hline \begin{matrix} 0 & 0 & \cdots & 0 \end{matrix} & Z_b \end{array}\right]}_{Z_{bus(new)}} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \\ \hline I_p \end{bmatrix}$$

The current I_p flowing into the network at bus k will increase the original voltage V_k^0 by the voltage $I_p Z_{kk}$ just like in equ.

$$V_k = V_k^0 + Z_{kk} \Delta I_k \quad \text{that is}$$

$$V_k = V_k^0 + I_p \cdot Z_{kk} \quad \text{--- (1)}$$

V_p will be larger than V_k by the voltage $I_p \cdot Z_b$. so,

$$V_p = V_k^0 + I_p Z_{kk} + I_p Z_b \quad \text{--- (2)}$$

and substituting for V_k^0 we obtain

$$V_p = I_1 Z_{k1} + I_2 Z_{k2} + \cdots + I_N Z_{kN} + I_p (Z_{kk} + Z_b) \quad \text{--- (3)}$$

we now see that the new row which must be added to Z_{orig} in order to find V_p is

$$Z_{k1} \; Z_{k2} \cdots Z_{kN} \; (Z_{kk} + Z_b)$$

✓ Case 3 Adding Z_b from existing bus (k) to the reference node

$$Z_{hi\,new} = Z_{hi\,old} - \frac{Z_{h(N+1)}\, Z_{(N+1)i}}{Z_{kk} + Z_b}$$

Case 4 Adding Z_b between two existing buses (j) & (k)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_j \\ V_k \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} Z_{orig.} & \begin{matrix}\text{col. } j - \\ \text{col } k) \\ \text{of } Z_{orig}\end{matrix} \\ \hline (\text{row } j - \text{row } k) \text{ of } Z_{orig} & Z_{bb} \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_j \\ I_k \\ \hline I_b \end{bmatrix}$$

$$Z_{bb} = Z_{jj} + Z_{kk} - 2Z_{jk} + Z_b$$

$$Z_{hi(new)} = Z_{hi(old)} - \frac{Z_{h(N+1)} \cdot Z_{(N+1)h}}{Z_{jj} + Z_{kk} - 2Z_{jk} + Z_b}$$

Substituting for I_B in the above eqn.

$$Z_B = \frac{(V_B)^2}{S_B}$$

$$Z_B = \frac{(kV_B)^2}{MVA_B}.$$

The phase and line quantities expressed in per unit are the same and the circuit laws are valid.

$$S_{p.u} = V_{pu} I_{pu}^*.$$

$$V_{pu} = Z_{pu} \cdot I_{pu}$$

For a 3ϕ complex load

$$S_{L(3\phi)} = 3 V_p I_p^*$$

The phase current in-terms of the ohmic load impedance is

$$I_p = \frac{V_p}{Z_p}$$

Substituting for I_p in the above equation results in the ohmic value of the load impedance.

$$Z_p = \frac{3|V_p|^2}{S^*_{L(3\phi)}}$$

$$= \frac{|V_{L-L}|^2}{S^*_{L(3\phi)}}$$

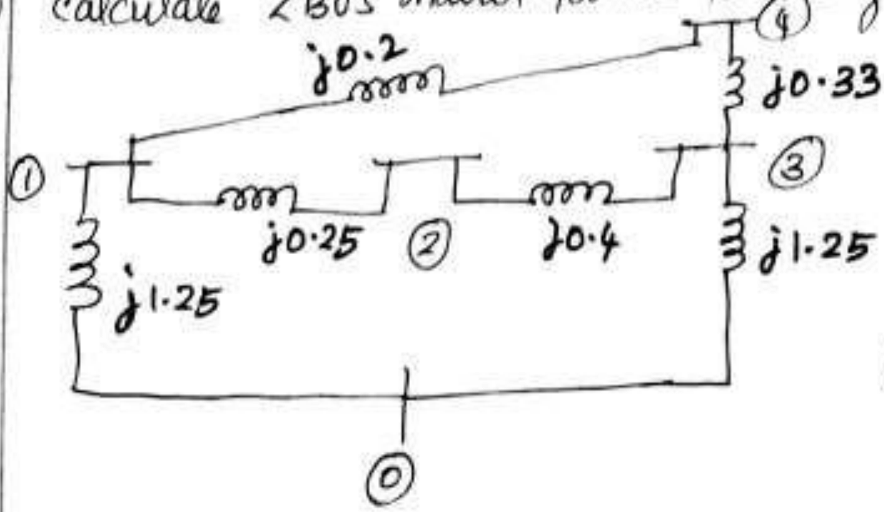
Load impedance in per-unit is

$$Z_{pu} = \frac{Z_p}{Z_B} = \left|\frac{V_{L-L}}{V_B}\right|^2 \cdot \frac{S_B}{S^*_{L(3\phi)}}$$

$$= (V_{p.u})^2 \cdot \frac{1}{\left(\frac{S^*_{L(3\phi)}}{S_B}\right)} = (V_{p.u.})^2 \cdot \frac{1}{S_{L(p.u.)}}$$

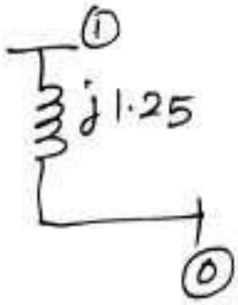
$$Z_{pu} = \frac{(V_{pu})^2}{S_{L(pu)}}.$$

9) calculate ZBUS matrix for the following network.



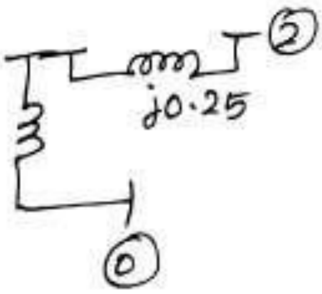
(A/M - 05)

Step-1



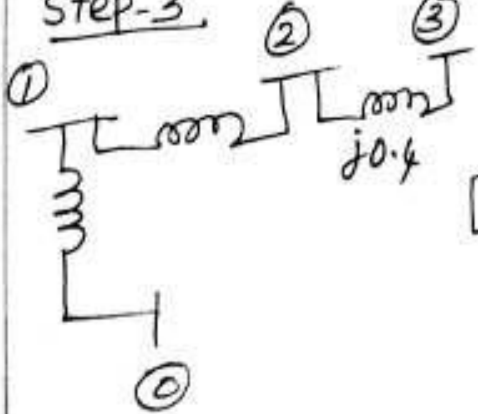
$$[Z] = \begin{array}{c c} & ① \\ ① & \begin{bmatrix} j1.25 \end{bmatrix} \end{array}$$

Step-2



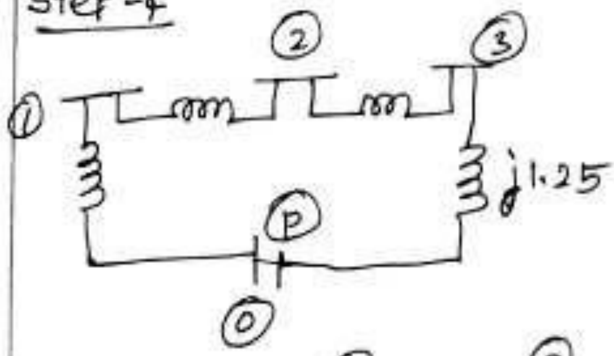
$$[Z] = \begin{array}{c c} & \begin{matrix} ① & \quad ② \end{matrix} \\ \begin{matrix} ① \\ ② \end{matrix} & \begin{bmatrix} j1.25 & j1.25 \\ j1.25 & j1.50 \end{bmatrix} \end{array}$$

Step-3.



$$[Z] = \begin{array}{c c} & \begin{matrix} ① & \quad ② & \quad ③ \end{matrix} \\ \begin{matrix} ① \\ ② \\ ③ \end{matrix} & \begin{bmatrix} j1.25 & j1.25 & j1.25 \\ j1.25 & j1.5 & j1.5 \\ j1.25 & j1.5 & j1.9 \end{bmatrix} \end{array}$$

Step-4



$$[Z] = \begin{array}{c|cccc} & ① & ② & ③ & ⓟ \\ \hline ① & j1.25 & j1.25 & j1.25 & j1.25 \\ ② & j1.25 & j1.5 & j1.5 & j1.5 \\ ③ & j1.25 & j1.5 & j1.9 & j1.9 \\ ⓟ & j1.25 & j1.5 & j1.9 & j3.15 \end{array}$$

using KRON reduction technique

$$Z_{11} = Z_{11} - \frac{Z_{1P} \cdot Z_{P1}}{j3.15}$$

$$= j1.25 - \frac{j1.25 \times j1.25}{j3.15}$$

$$= j0.7540$$

$$Z_{12} = Z_{12} - \frac{Z_{1P} \cdot Z_{P2}}{j3.15}$$

$$= j1.25 - \frac{j1.25 \times j1.5}{j3.15}$$

$$= j0.6548$$

$$Z_{13} = Z_{13} - \frac{Z_{1P} \cdot Z_{P3}}{j3.15}$$

$$= j1.25 - \frac{j1.25 \times j1.9}{j3.15} = j0.4960$$

$$Z_{22} = Z_{22} - \frac{Z_{2p} \cdot Z_{p2}}{j3.15}$$

$$= j1.5 - \frac{j1.5 \times j1.5}{j3.15}$$

$$= j0.7857$$

$$Z_{23} = Z_{23} - \frac{Z_{2p} \cdot Z_{p3}}{j3.15}$$

$$= j1.5 - \frac{j1.5 \times j1.9}{j3.15}$$

$$= j0.5952$$

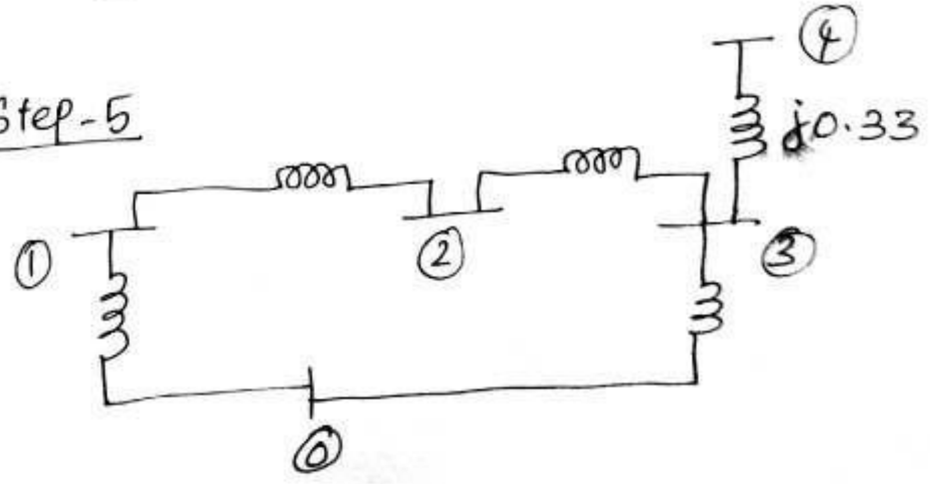
$$Z_{33} = Z_{33} - \frac{Z_{3p} \cdot Z_{p3}}{j3.15}$$

$$= j1.9 - \frac{j1.9 \times j1.9}{j3.15}$$

$$= j0.7540$$

$$[Z] = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j0.7540 & j0.6548 & j0.4960 \\ 2 & j0.6548 & j0.7857 & j0.5952 \\ 3 & j0.4960 & j0.5952 & j0.7540 \end{array}$$

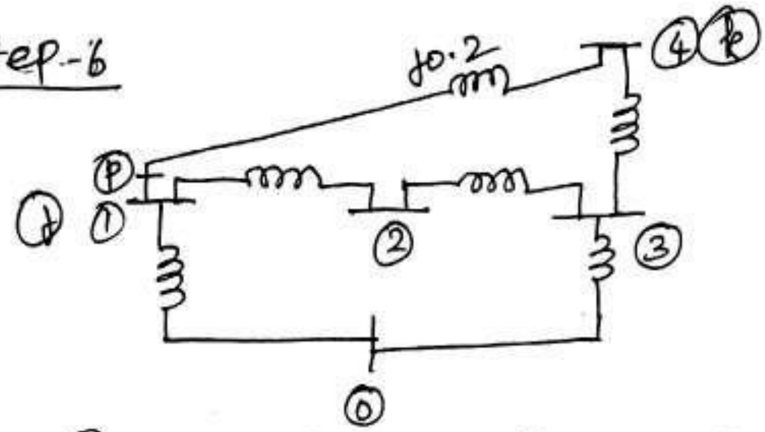
Step-5



✓

$$[Z] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\ \left[\begin{array}{ccc|c} j0.7540 & j0.6548 & j0.4960 & j0.4960 \\ j0.6548 & j0.7857 & j0.5952 & j0.5952 \\ j0.4960 & j0.5952 & j0.7540 & j0.7540 \\ \hline j0.4960 & j0.5952 & j0.7540 & j1.0840 \end{array}\right] \end{array}$$

Step-6



$j = 1$
$k = 4$

$$[Z] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ p \end{array}\begin{array}{c} \begin{array}{ccccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad & \quad p \quad \end{array} \\ \left[\begin{array}{cccc|c} j0.7540 & j0.6548 & j0.4960 & j0.4960 & j0.2580 \\ j0.6548 & j0.7857 & j0.5952 & j0.5952 & j0.0596 \\ j0.4960 & j0.5952 & j0.7540 & j0.7540 & -j0.2580 \\ j0.4960 & j0.5952 & j0.7540 & \cancel{j0.7540}\ j1.0840 & -j0.2580 \\ \hline j0.2580 & j0.0596 & -j0.2580 & -j0.2580 & j0.7160 \end{array}\right] \end{array}$$

$$z_{pp} = Z_{jj} + Z_{kk} - 2 Z_{jk} + Z_b$$

$$= Z_{11} + Z_{44} - 2 \cdot Z_{14} + j0.2$$

$$\cancel{= (j0.7540 + j0.4960 (2) \times}$$

$$= j0.7540 + j0.7540 - 2(j0.4960) + j0.2$$

$$= j0.7160$$

√

$$Z_{11} = Z_{11} - \frac{Z_{1P} \cdot Z_{P1}}{j0.7160}$$

$$= j0.7540 - \frac{j0.2580 \times j0.2580}{j0.7160}$$

$$= j0.6610$$

$$Z_{12} = Z_{12} - \frac{Z_{1P} \cdot Z_{P2}}{j0.7160}$$

$$= j0.6548 - \frac{j0.2580 \times j0.2580}{j0.7160}$$

$$= j0.5618$$

$$Z_{13} = Z_{13} - \frac{Z_{1P} \cdot Z_{P3}}{j0.7160}$$

$$= j0.4960 - \frac{j0.2580 \times (-j0.2580)}{j0.7160}$$

$$= j0.5890$$

$$Z_{14} = Z_{14} - \frac{Z_{1P} \cdot Z_{P4}}{j0.7160}$$

$$= j0.4960 - \frac{j0.2580 \times (-j0.2580)}{j0.7160}$$

$$= j0.5890$$

$$Z_{22} = Z_{22} - \frac{Z_{2P} \cdot Z_{P2}}{j0.7160}$$

$$= j0.7857 - \frac{j0.0596 \times j0.0596}{j0.7160}$$

$$= j0.7807$$

$$Z_{23} = Z_{23} - \frac{Z_{2p} \cdot Z_{p3}}{j0.7160}$$

$$= j0.5952 - \frac{j0.0596 \times (-j0.2580)}{j0.7160}$$

$$= j0.6167$$

$$Z_{24} = Z_{24} - \frac{Z_{2p} \cdot Z_{p4}}{j0.7160}$$

$$= j0.5952 - \frac{j0.0596 \times (-j0.2580)}{j0.7160}$$

$$= j0.6167$$

$$Z_{33} = Z_{33} - \frac{Z_{3p} \cdot Z_{p3}}{j0.7160}$$

$$= j0.7540 - \frac{(-j0.2580) \times (-j0.2580)}{j0.7160}$$

$$= j0.6610$$

$$Z_{34} = Z_{34} - \frac{Z_{3p} \cdot Z_{p4}}{j0.7160}$$

$$= j0.7540 - \frac{(-j0.2580) \times (-j0.2580)}{j0.7160}$$

$$= j0.6610.$$

$$Z_{44} = Z_{44} - \frac{Z_{4p} \cdot Z_{p4}}{j0.7160}$$

$$= \overset{j1.0840}{\cancel{j0.7840}} - \frac{(-j0.2580) \times (-j0.2580)}{j0.7160}$$

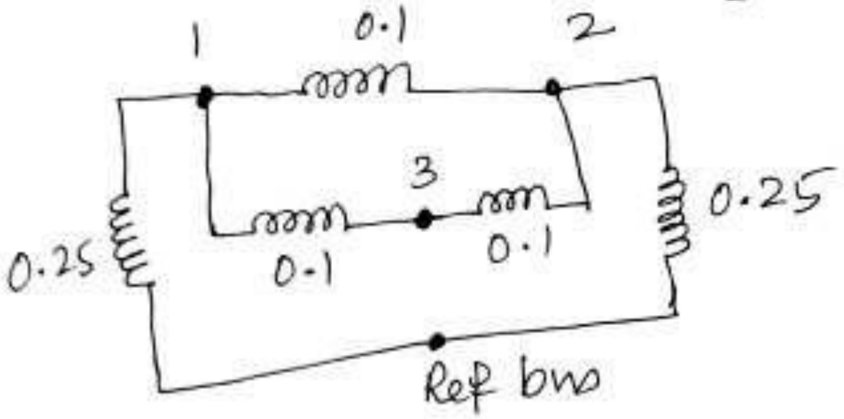
$$= j0.9910.$$

$$[Z] = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & j0.6610 & j0.5618 & j0.5890 & j0.5890 \\ 2 & j0.5618 & j0.7807 & j0.6167 & j0.6167 \\ 3 & j0.5890 & j0.6167 & j0.6610 & j0.6610 \\ 4 & j0.5890 & j0.6167 & j0.6610 & j0.9910 \end{array}$$

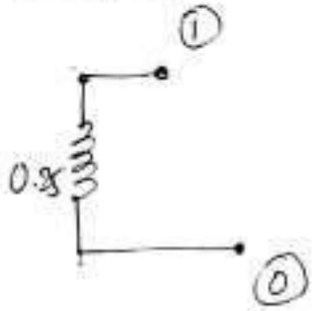
— x —

10) For the 3-bus network fig. shown below. Obtain Z bus by building algorithm.

[N/D - 14]

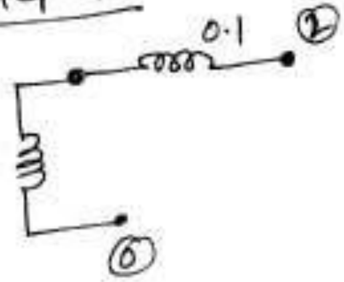


Step-1



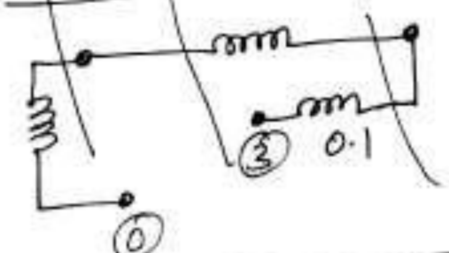
$$Z = \begin{array}{c|c} & ① \\ \hline ① & \begin{bmatrix} 0.25 \end{bmatrix} \end{array}$$

Step-2



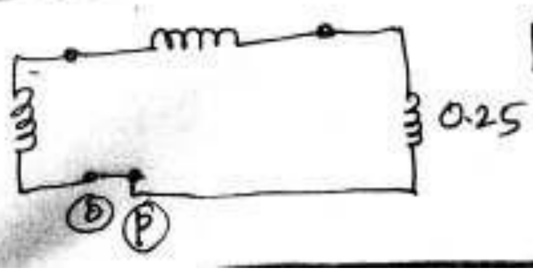
$$Z = \begin{array}{c|c} & ① \quad\quad ② \\ \hline \begin{matrix} ① \\ ② \end{matrix} & \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.35 \end{bmatrix} \end{array}$$

~~Step-3~~



$$Z = \begin{array}{c|c} & ① \quad\quad ② \quad\quad ③ \\ \hline \begin{matrix} ① \\ ② \\ ③ \end{matrix} & \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.35 \\ 0.25 & 0.35 & 0.45 \end{bmatrix} \end{array}$$

Step-3



$$[Z] = \begin{array}{c|c} & ① \quad\quad ② \quad\quad Ⓟ \\ \hline \begin{matrix} ① \\ ② \\ Ⓟ \end{matrix} & \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.35 \\ 0.25 & 0.35 & 0.6 \end{bmatrix} \end{array}$$

Using kron reduction reduce the matrix size to 2x2.

$$Z_{11} = Z_{11,old} - \frac{Z_{1p} \cdot Z_{p1}}{Z_{kk} + Z_b} \qquad k = 2$$

$$= Z_{11\,old} - \frac{Z_{1p} \cdot Z_{p1}}{Z_{22} + 0.35}$$

$$= 0.25 - \frac{0.25 \times 0.25}{0.6}$$

$$= 0.25 - 0.1041$$

$$= 0.1458$$

$$Z_{12} = Z_{12,old} - \frac{Z_{1p} \cdot Z_{p2}}{0.6}$$

$$= 0.25 - \frac{0.25 \times 0.35}{0.6}$$

$$= 0.25 - 1458$$

$$= 0.1042 = Z_{21}$$

$$Z_{22} = Z_{22\,old} - \frac{Z_{2p} \cdot Z_{p2}}{0.6}$$

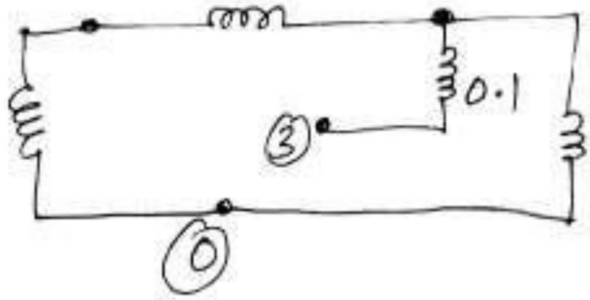
$$= 0.35 - \frac{0.35 \times 0.35}{0.6}$$

$$= 0.35 - 0.2042$$

$$= 0.1458$$

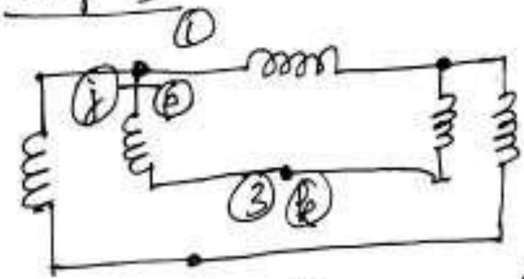
$$Z = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0.1458 & 0.1042 \\ 0.1042 & 0.1458 \end{bmatrix} \end{array}$$

Step-4



$$[Z] = \begin{bmatrix} 0.1458 & 0.1042 & 0.1042 \\ 0.1042 & 0.1458 & 0.1458 \\ 0.1042 & 0.1458 & 0.2458 \end{bmatrix}$$

Step-5



$$[Z] = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & \begin{array}{c} 1-3 \\ p \end{array} \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 1-3 \;\; p \end{array} & \left[\begin{array}{ccc|c} 0.1458 & 0.1042 & 0.1042 & 0.0416 \\ 0.1042 & 0.1458 & 0.1458 & -0.0416 \\ 0.1042 & 0.1458 & 0.2458 & -0.1416 \\ \hline 0.0416 & -0.0416 & -0.1416 & 0.429 \end{array}\right] \end{array}$$

$$Z_{pp} = Z_{jj} + Z_{kk} - 2Z_{jk} + Z_b \qquad j = 1, \; k = 3$$

$$= Z_{11} + Z_{33} - (2 \times Z_{13}) + 0.2458$$

$$= 0.1458 + 0.2458 - (2 \times 0.1042) + 0.2458$$

$$Z_{pp} = 0.429.$$

Using kron reduction technique reduce the matrix to 3X3.

$$Z_{11} = Z_{11} - \frac{Z_{1p} \cdot Z_{p1}}{0.429}$$

$$= 0.1458 - \frac{0.0416 \times 0.0416}{0.429}$$

$$= 0.1418$$

$$Z_{12} = Z_{12} - \frac{Z_{1p} \cdot Z_{p2}}{0.429}$$

$$= 0.1042 - \frac{0.0416 \times -0.0416}{0.429}$$

$$= 0.1082$$

$$Z_{13} = Z_{13} - \frac{Z_{1p} \cdot Z_{p3}}{0.429}$$

$$= 0.1042 - \frac{0.0416 \times (-0.1416)}{0.429}$$

$$= 0.1179.$$

$$Z_{22} = Z_{22} - \frac{Z_{2p} \cdot Z_{p2}}{0.429}$$

$$= 0.1458 - \frac{(-0.0416) \times (-0.0416)}{0.429}$$

$$0.1418$$

$$Z_{23} = Z_{23} - \frac{Z_{2p} \cdot Z_{p3}}{0.429}$$

$$= 0.1458 - \frac{(-0.0416) \times (-0.1416)}{0.429}$$

$$= 0.1321$$

$$Z_{33} = Z_{33} - \frac{Z_{3p} \cdot Z_{p3}}{0.429}$$

$$= 0.2458 - \frac{(-0.1416) \times (-0.1416)}{0.429}$$

$$Z_{33} = 0.1991$$

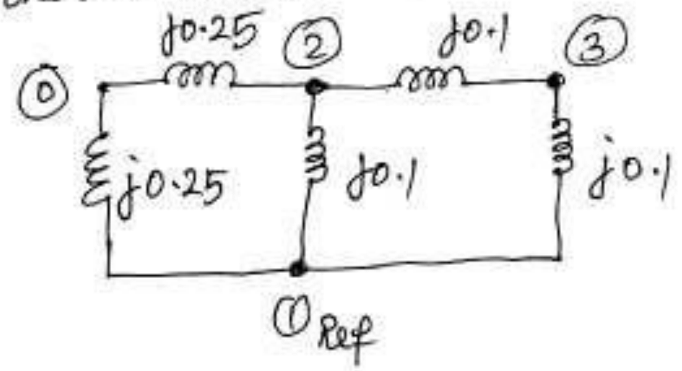
$$[Z] = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.1418 & 0.1082 & 0.1179 \\ 0.1082 & 0.1418 & 0.1321 \\ 0.1179 & 0.1321 & 0.1991 \end{bmatrix} \end{array}$$

11) For the three system given in the table, formulate the Z_{bus} using the Z_{bus} building algorithm. Take bus 1 as the reference bus. (A/M -10)

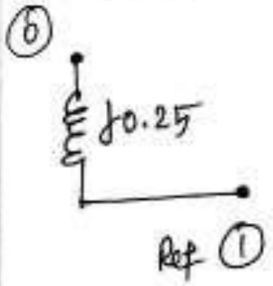
Element no.	From bus	To bus	Impedance (P.U.)
1	1	2	j0.1
2	2	3	j0.1
3	3	1	j0.1
4	1	0	j0.25
5	2	0	j0.25

solution

one line diagram for the given data as shown below

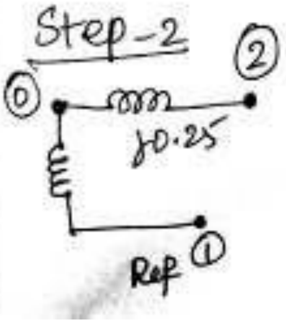


Step-1



$$[Z] = \begin{array}{c|c} & 0 \\ \hline 0 & \begin{bmatrix} j0.25 \end{bmatrix} \end{array}$$

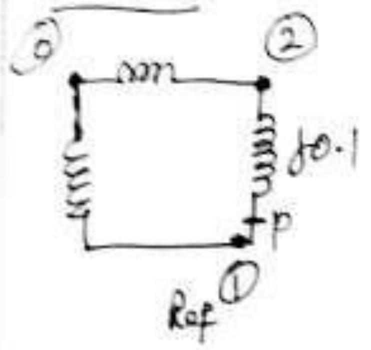
Step-2



$$[Z] = \begin{array}{c|c} & 0 \quad\quad 2 \\ \hline \begin{matrix} 0 \\ 2 \end{matrix} & \begin{bmatrix} j0.25 & j0.25 \\ j0.25 & j0.5 \end{bmatrix} \end{array}$$

Scanned by CamScanner

Step-3



$$
Z = \begin{array}{c} \\ 0 \\ 2 \\ p \end{array}
\begin{array}{c}
\begin{array}{ccc} 0 & 2 & p \end{array} \\
\left[\begin{array}{cc|c} j0.25 & j0.25 & j0.25 \\ j0.25 & j0.5 & j0.5 \\ \hline j0.25 & j0.5 & j0.6 \end{array}\right]
\end{array}
$$

using kron reduction technique reduce the matrix to 2x2.

$$Z_{00} = Z_{00} - \frac{Z_{0p} \cdot Z_{p0}}{j0.6}$$

$$= j0.25 - \frac{j0.25 \times j0.25}{j0.6}$$

$$= j0.1458$$

$$Z_{02} = Z_{02} - \frac{Z_{0p} \cdot Z_{p2}}{j0.6}$$

$$= j0.25 - \frac{j0.25 \times j0.5}{j0.6}$$

$$= j0.0417$$

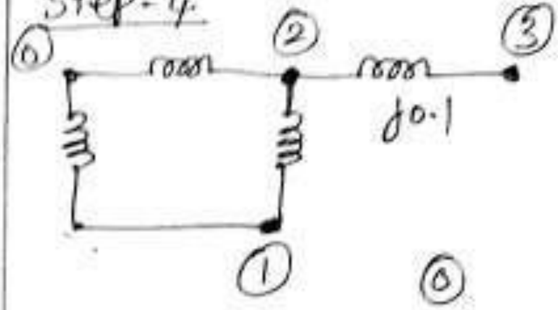
$$Z_{22} = Z_{22} - \frac{Z_{2p} \cdot Z_{p2}}{j0.6}$$

$$= j0.5 - \frac{j0.5 \times j0.5}{j0.6}$$

$$= j0.0833.$$

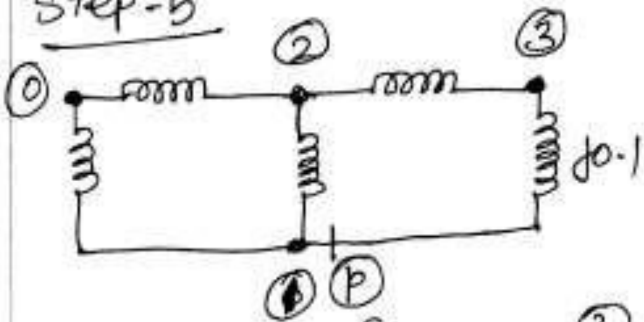
$$
Z = \begin{array}{c} \\ 0 \\ 2 \end{array}
\begin{array}{c}
\begin{array}{cc} 0 & 2 \end{array} \\
\begin{bmatrix} j0.1458 & j0.0417 \\ j0.0417 & j0.0833 \end{bmatrix}
\end{array}
$$

Step-4



$$[Z] = \begin{array}{c} \\ 0 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.1458 & j0.0417 & j0.0417 \\ j0.0417 & j0.0833 & j0.0833 \\ j0.0417 & j0.0833 & j0.1833 \end{bmatrix} \end{array}$$

Step-5



$$[Z] = \begin{array}{c} \\ 0 \\ 2 \\ 3 \\ P \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 2 & 3 & P \end{array} \\ \left[\begin{array}{ccc|c} j0.1458 & j0.0417 & j0.0417 & j0.0417 \\ j0.0417 & j0.0833 & j0.0833 & j0.0833 \\ j0.0417 & j0.0833 & j0.1833 & j0.1833 \\ \hline j0.0417 & j0.0833 & j0.1833 & j0.2833 \end{array}\right] \end{array}$$

Using kron reduction technique reduce the above matrix to 3x3.

$$Z_{00} = Z_{00} - \frac{Z_{0P} \cdot Z_{P0}}{j0.2833}$$

$$= j0.1458 - \frac{j0.0417 \times j0.0417}{j0.2833}$$

$$= j0.1397$$

$$Z_{02} = Z_{02} - \frac{Z_{0p} \cdot Z_{p2}}{j0.2833}$$

$$= j0.0417 - \frac{j0.0417 \times j0.0833}{j0.2833}$$

$$= j0.0294$$

$$Z_{03} = Z_{03} - \frac{Z_{0p} \cdot Z_{p3}}{j0.2833}$$

$$= j0.0417 - \frac{j0.0417 \times j0.1833}{j0.2833}$$

$$= j0.0147$$

$$Z_{22} = Z_{22} - \frac{Z_{2p} \cdot Z_{p2}}{j0.2833}$$

$$= j0.0833 - \frac{j0.0833 \times j0.0833}{j0.2833}$$

$$= j0.0588$$

$$Z_{23} = Z_{23} - \frac{Z_{2p} \cdot Z_{p3}}{j0.2833}$$

$$= j0.0833 - \frac{j0.0833 \times j0.1833}{j0.2833}$$

$$= j0.0294$$

$$Z_{33} = Z_{33} - \frac{Z_{3p} \cdot Z_{p3}}{j0.2833}$$

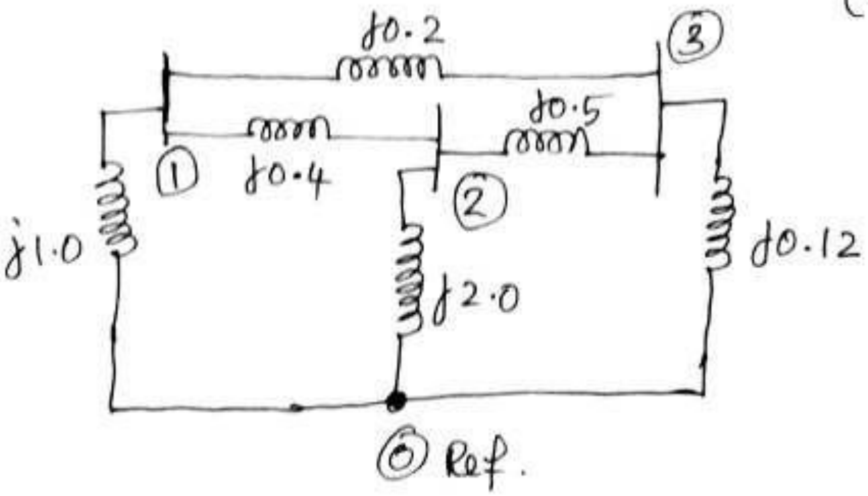
$$= j0.1833 - \frac{j0.1833 \times j0.1833}{j0.2833}$$

$$= j0.0647.$$

$$[Z] = \begin{bmatrix} j0.1397 & j0.0294 & j0.0147 \\ j0.0294 & j0.0588 & j0.0294 \\ j0.0147 & j0.0294 & j0.0647 \end{bmatrix}$$

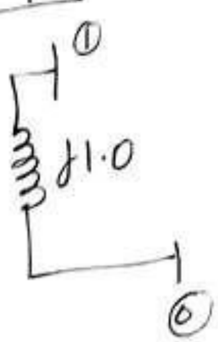
12. Using building algorithm method, determine Z_{bus} for the network shown in fig. where the impedances labeled are shown in p.u.

(N/D -07)



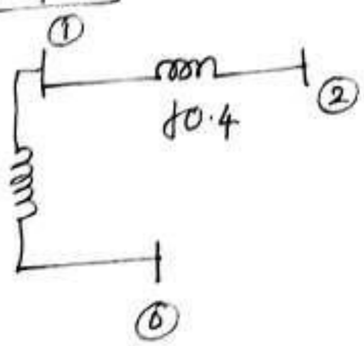
Solution

Step-1



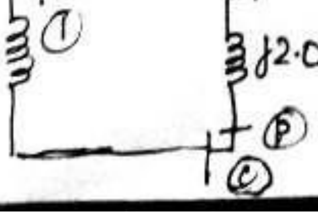
$$[Z] = \begin{array}{c|c} & ① \\ \hline ① & [j1.0] \end{array}$$

Step-2



$$[Z] = \begin{array}{c|c} & \begin{array}{cc} ① & ② \end{array} \\ \hline \begin{array}{c} ① \\ ② \end{array} & \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.4 \end{bmatrix} \end{array}$$

Step-3



$$[Z] = \begin{array}{c|c} & \begin{array}{ccc} ① & ② & ⓟ \end{array} \\ \hline \begin{array}{c} ① \\ ② \\ ⓟ \end{array} & \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.4 & j1.4 \\ j1.0 & j1.4 & j3.4 \end{bmatrix} \end{array}$$

using kron reduction reduce the above matrix to 2x2.

$$Z_{11} = Z_{11} - \frac{Z_{1p} \cdot Z_{p1}}{j3.4}$$

$$= j1.0 - \frac{j1 \times j1}{j3.4}$$

$$= j0.7059$$

$$Z_{12} = Z_{12} - \frac{Z_{1p} \cdot Z_{p2}}{j3.4}$$

$$= j1.0 - \frac{j1.0 \times j1.4}{j3.4}$$

$$= j0.5882$$

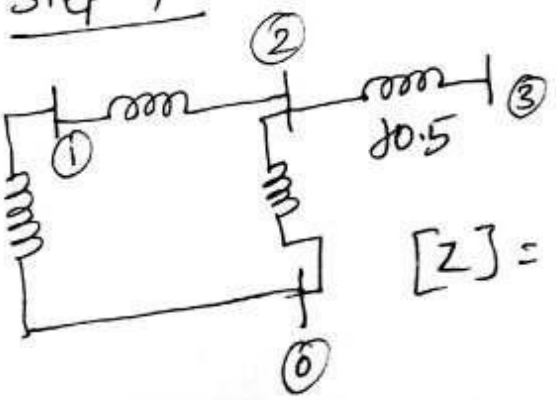
$$Z_{22} = Z_{22} - \frac{Z_{2p} \cdot Z_{p2}}{j3.4}$$

$$= j1.4 - \frac{j1.4 \times j1.4}{j3.4}$$

$$= j0.8235$$

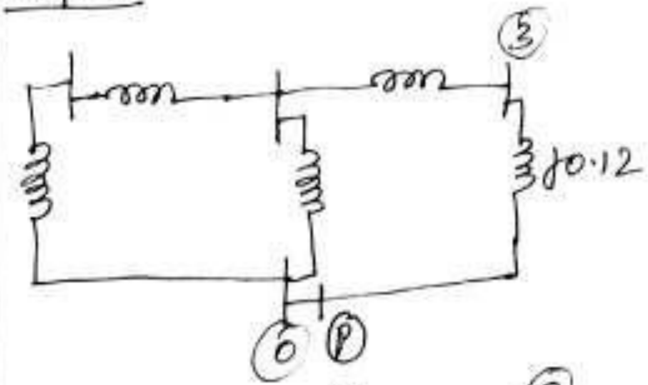
$$[Z] = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} j0.7059 & j0.5882 \\ j0.5882 & j0.8235 \end{bmatrix} \end{array}$$

Step-4



$$[Z] = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} j0.7059 & j0.5882 & j0.5882 \\ j0.5882 & j0.8235 & j0.8235 \\ j0.5882 & j0.8235 & j1.3235 \end{bmatrix} \end{array}$$

Step-5



$$[Z] = \begin{array}{c|cccc} & ① & ② & ③ & Ⓟ \\ \hline ① & j0.7059 & j0.5882 & j0.5882 & j0.5882 \\ ② & j0.5882 & j0.8235 & j0.8235 & j0.8235 \\ ③ & j0.5882 & j0.8235 & j1.3235 & j1.3235 \\ Ⓟ & j0.5882 & j0.8235 & j1.3235 & j1.4435 \end{array}$$

Reduce the above matrix to 3×3.

$$Z_{11} = Z_{11} - \frac{Z_{1P} \cdot Z_{P1}}{j1.4435}$$

$$= j0.7059 - \frac{j0.5882 \times j0.5882}{j1.4435}$$

$$= j0.4662$$

$$Z_{12} = Z_{12} - \frac{Z_{1P} \cdot Z_{P2}}{j1.4435}$$

$$= j0.5882 - \frac{j0.5882 \times j0.8235}{j1.4435}$$

$$= j0.2526$$

$$Z_{13} = Z_{13} - \frac{Z_{1P} \cdot Z_{P3}}{j1.4435}$$

$$= j0.5882 - \frac{j0.5882 \times j1.3235}{j1.4435}$$

$$= j0.0489$$

$$Z_{22} = Z_{22} - \frac{Z_{2p} \cdot Z_{p2}}{j1.4435}$$

$$= j0.8235 - \frac{j0.8235 \times j0.8235}{j1.4435}$$

$$= j0.3537$$

$$Z_{23} = Z_{23} - \frac{Z_{2p} \cdot Z_{p3}}{j1.4435}$$

$$= j0.8235 - \frac{j0.8235 \times j1.3235}{j1.4435}$$

$$= j0.4698$$

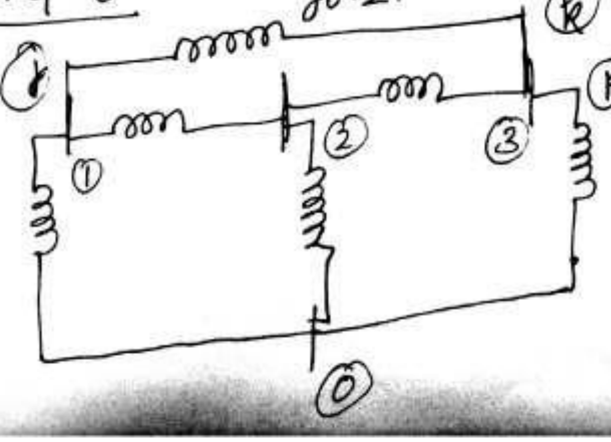
$$Z_{33} = Z_{33} - \frac{Z_{3p} \cdot Z_{p3}}{j1.4435}$$

$$= j1.3235 - \frac{j1.3235 \times j1.3235}{j1.4435}$$

$$= j0.1100.$$

$$[Z] = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & j0.4662 & j0.2526 & j0.0489 \\ 2 & j0.2526 & j0.3537 & j0.4698 \\ 3 & j0.0489 & j0.4698 & j0.1100 \end{array}$$

Step-6



$j = 1$
$k = 3$

$$[Z] = \begin{array}{c} \\ ① \\ ② \\ ③ \\ p \end{array}\begin{array}{c} \begin{array}{cccc} ① & ② & ③ & p \end{array} \\ \left[\begin{array}{ccc|c} j0.4662 & j0.2526 & j0.0489 & j0.4173 \\ j0.2526 & j0.3537 & j0.4698 & -j0.2172 \\ j0.0489 & j0.4698 & j0.1100 & -j0.0611 \\ \hline j0.4173 & -j0.2172 & -j0.0611 & j0.6784 \end{array}\right] \end{array}$$

$$Z_{1p} = j0.4662 - j0.0489 = j0.4173$$

$$Z_{2p} = j0.2526 - j0.4698 = -j0.2172$$

$$Z_{3p} = j0.0489 - j0.1100 = -j0.0611$$

$$Z_{pp} = Z_{jj} + Z_{kk} - 2Z_{jk} + Z_b$$

$$= Z_{11} + Z_{33} - 2.Z_{13} + Z_b$$

$$= j0.4662 + j0.1100 - (2 \times j0.0489) + j0.2$$

$$= j0.6784.$$

Reduce the above matrix to 3x3.

$$Z_{11} = Z_{11} - \frac{Z_{1p} \cdot Z_{p1}}{j0.6784}$$

$$= j0.4662 - \frac{j0.4173 \times j0.4173}{j0.6784}$$

$$= j0.2095$$

$$Z_{12} = Z_{12} - \frac{Z_{1p} \cdot Z_{p2}}{j0.6784}$$

$$= j0.2526 - \frac{j0.4173 \times (-j0.2172)}{j0.6784}$$

$$= j0.3862$$

$$Z_{13} = Z_{13} - \frac{Z_{1p} \cdot Z_{p3}}{j0.6784}$$

$$= j0.0489 - \frac{j0.4173 \times (-j0.0611)}{j0.6784}$$

$$= j0.0865$$

$$Z_{22} = Z_{22} - \frac{Z_{2p} \cdot Z_{p2}}{j0.6784}$$

$$= j0.3587 - \frac{(-j0.2172) \times (-j0.2172)}{j0.6784}$$

$$= j0.2842$$

$$Z_{23} = Z_{23} - \frac{Z_{2p} \cdot Z_{p3}}{j0.6784}$$

$$= j0.4698 - \frac{(-j0.2172) \times (-j0.0611)}{j0.6784}$$

$$Z_{23} = j0.4894$$

$$Z_{33} = Z_{33} - \frac{Z_{3p} \cdot Z_{p3}}{j0.6784}$$

$$= j0.1100 - \frac{(-j0.0611)(-j0.0611)}{j0.6784}$$

$$Z_{33} = j0.1045.$$

$$[Z] = \begin{bmatrix} j0.2095 & j0.3862 & j0.0865 \\ j0.3862 & j0.2842 & j0.4894 \\ j0.0865 & j0.4894 & j0.1045 \end{bmatrix}$$

DHANALAKSHMI COLLEGE OF ENGINEERING

EE6501 - POWER SYSTEM ANALYSIS

UNIT - II

4. Write the need for Slack bus in load flow analysis. EE-N/D-16

[N/D -07], [M/J -14]

Power balance equation is

$$\underbrace{P_L}_{\text{Real power loss}} = \sum_{i=1}^{N} P_i = \underbrace{\sum_{i=1}^{N} P_{Gi}}_{\text{Total generation}} - \underbrace{\sum_{i=1}^{N} P_{Di}}_{\text{Total load}}$$

→ P_L depends on I^2R loss in the transmission line & transformers in the network.

→ The each line current magnitude & angle are unknown because the Voltage in the network is unknown.

→ Initially we don't know the line losses. After load flow we can calculate line current based on the Voltage calculated from load flow solution..

→ To make the load flow calculation it is necessary to assign slack bus to take care of the losses in the network.

Compare Gauss-Seidal and Newton-Raphson methods of load flow solutions. EG - M/J

[A/M-15], [M/J-12]

U-II

S.No	G-S. Method	N-R Method
1.	computation per interation is less	computation per interation is more
2.	It has linear convergence characteristics	It has quadratic convergence characteristics
3.	No. of iterations required for convergence increase with size of the system.	No. of iterations are independent of system size.

Define voltage controlled bus. EE - N/D -14

U-II

A bus is said to be voltage controlled if the magnitude of voltage |V| and real power P are specified. The magnitude of the voltage is treated as fixed value.

What is Jacobian matrix? EE - A/M-11

[N/D-16]

U-II

Jacobian matrix

$$[J] = \left[\begin{array}{ccc|ccc} \left(\frac{\partial P_2}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial P_2}{\partial \delta_N}\right)^0 & \left(\frac{\partial P_2}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial P_2}{\partial |V_N|}\right)^0 \\ \vdots & & \vdots & \vdots & & \vdots \\ \left(\frac{\partial P_N}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial P_N}{\partial \delta_N}\right)^0 & \left(\frac{\partial P_N}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial P_N}{\partial |V_N|}\right)^0 \\ \hline \left(\frac{\partial Q_2}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial Q_2}{\partial \delta_N}\right)^0 & \left(\frac{\partial Q_2}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial Q_2}{\partial |V_N|}\right)^0 \\ \vdots & & \vdots & \vdots & & \vdots \\ \left(\frac{\partial Q_N}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial Q_N}{\partial \delta_N}\right)^0 & \left(\frac{\partial Q_N}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial Q_N}{\partial |V_N|}\right)^0 \end{array}\right]$$

When will the generator bus be treated as load bus? EE - N/D - 13

[M/J - 14]

U-II

when ever the reactive power of the generator is violated, the generator bus will be treated as load bus .

What is a slack bus? EE - A/M - 11

U-II In slack bus, voltage magnitude and phase angle of voltages are specified pertaining to a generator bus. Usually a large capacity generation bus is chosen. We assumed voltage (V) as reference phasor,

i.e. $\delta = 0$

where δ - phase angle of voltage

Explain why one of the bus in the system is taken as slack bus in the load flow studies. EE - N/D - 09

V-II

Power balance equation is

$$P_L = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} P_{Gi} - \sum_{i=1}^{N} P_{Di}$$

P_L depends on I^2R loss in the transmission line and transformers of the network. The individual currents in the various lines of the network cannot be calculated at the initial part of calculation, because Voltage magnitude and angle are not known. There fore P_L is unknown quantity.

One generator bus is assigned as slack bus and power generation is not prespecified. The total power consumed by loads plus I^2R losses are assigned to the slack bus.

—

What are the different types of buses in power systems? What are the quantities specifi
each bus? EE-N/D-11

[N/D-08]

U-II

The types of buses are

1) Slack bus (or) Reference bus (or) Swing bus
2) Generator bus
3) Load bus.

Sl. No.	Bus	Quantities specified
1.	Slack bus	$\lvert V \rvert, \delta$
2.	Generator bus	$P, \lvert V \rvert$
3.	Load bus	P, Q.

What is load flow analysis? Give the significance in power system analysis. EE - M/J -09

U-II [M/J-12], [N/D-14]

Load flow analysis is determination of the voltage, current, power at various points in electrical network.

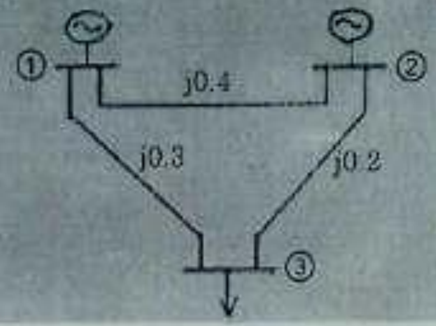
Load flow studies are important for planning, economic scheduling, control and operations of existing systems as well as planning its future expansion depends upon knowing the effect of interconnections, new loads, new generation, new transmission line, .. etc. before they are installed.

How are the disadvantages of Newton-Raphson method overcome? EE -N/D-11

The disadvantage of large memory requirement can be overcome by decoupling the weak coupling between P-δ & Q-V (i.e. using decoupled load flow algorithm). The disadvantage of large computational time per iteration can be reduced by simplifying the decoupled load flow equations.

12. (a) (i) Write a note on classification of buses. (6)

(ii) Fig shown below a three bus power system Bus.1: slack bus $V = 1.05\angle 0^\circ$ p.u. Bus 2: PV Bus $|V| = 1.0$ p.u., $P_g = 3$ p.u. Bus 3: PQ Bus $P_L = 4$ p.u, $Q_L = 2$ p.u. carry out one iteration of load flow solution by Gauss Seidel method. Neglect limits on reactive power generation? EE-N/D-14 (10)



Solution

Step-1 form Y Bus matrix

$$Y_{bus} = \begin{bmatrix} \frac{1}{j0.4} + \frac{1}{j0.3} & -\frac{1}{j0.4} & -\frac{1}{j0.3} \\ -\frac{1}{j0.4} & \frac{1}{j0.4} + \frac{1}{j0.2} & -\frac{1}{j0.2} \\ -\frac{1}{j0.3} & -\frac{1}{j0.2} & \frac{1}{j0.3} + \frac{1}{j0.2} \end{bmatrix}$$

$$= \begin{bmatrix} -j5.833 & j2.5 & j3.333 \\ j2.5 & -j7.5 & j5.0 \\ j3.333 & j5.0 & -j8.333 \end{bmatrix}$$

Step-2

Initialize the bus voltage

$V_1^{old} = 1.05\angle 0^\circ$ p.u.

$V_2^{old} = 1.02\angle 0^\circ$ p.u.

$V_3^{old} = 1.0\angle 0^\circ$ p.u.

For generation bus, calculate V_i^{new} using the formulae

$$V_i^{new} = V_{specified} \angle \delta_{\text{calculated value}}.$$

Step-3

$$Q_i^{cal} = -\text{Im}\left\{ V_i^{old*} \left[\sum_{j=1}^{i-1} Y_{ij} \cdot V_j^{new} + \sum_{j=1}^{n} Y_{ij} \cdot V_j^{old} \right] \right\}$$

$$Q_2^{cal} = -\text{Im}\left\{ V_2^{old*} \left[Y_{21} \cdot V_1^{new} + Y_{22} \cdot V_2^{old} + Y_{23} \cdot V_3^{old} \right] \right.$$

$$= -\text{Im}\left\{ 1.02\angle 0^\circ \left[j2.5 \times 1.05\angle 0^\circ + (-j7.5 \times 1.02\angle 0^\circ) + j5 \times 1\angle 0^\circ \right) \right\}$$

$$= -\text{Im}\left\{ 1.02\angle 0^\circ \left[j\,2.625 - j7.65 + j5 \right] \right.$$

$$Q_2^{cal} = 0.025 \text{ p.u.}$$

$$Q_{2\,min} \leq Q_2^{cal} \leq Q_{2\,max}.$$

Q_2^{cal} is within the specified limit.

Step-4 Calculate V_i^{new}

$$V_1^{new} = 1.05\angle 0^\circ \text{ p.u.}$$

$$V_i^{new} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^{old*}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{n} Y_{ij} \cdot V_j^{old} \right]$$

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21}V_1^{new} - Y_{23}\cdot V_3^{old}\right]$$

$P_2 = 0.3$ p.u. $\Big\}$ from data
$Q_2 = 0.025$ p.u. $\Big\}$

$$V_2 = \frac{1}{-j7.5}\left[\frac{0.3 - j0.025}{1.02\angle 0^\circ} - (j2.5 \times 1.05\angle 0^\circ) - (j5 \times 1\angle 0^\circ)\right]$$

$$= 1.0199 + j0.0392$$

$$= 1.0207\angle 2.2^\circ$$

$$V_2^{new} = V_{2(spec)}\angle \delta_2^{cal}$$

$$= 1.02\angle 2.2^\circ = 1.0192 + j0.0392$$

$$P_3 = P_{G3} - P_{L3} = 0 - 0.4 = -0.4 \text{ P.U.}$$

$$Q_3 = Q_{G3} - Q_{L3} = 0 - 0.2 = -0.2 \text{ P.U.}$$

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{old*}} - Y_{31}\cdot V_1^{new} - Y_{32}\cdot V_2^{new}\right]$$

$$= \frac{1}{-j8.333}\left[\frac{-0.4 + j0.2}{1.0\angle 0^\circ} - j3.333 \times 1.05\angle 0^\circ - j5 \times 1.02\angle 2.2^\circ\right]$$

$$= \frac{1}{-j8.333}\left[-0.4 + j0.2 \cancel{+j3.433} - j3.4999 - j5.096 + 0.196\right]$$

$$= 1.0075 - j0.0244 = 1.0078\angle -1.39^\circ$$

$$V_3^{new} = 1.0075 - j0.0244 = 1.0078\angle -1.39^\circ.$$

Step-5 Slack bus power

$$S_1 = P_1 - jQ_1 = V_i^* \sum_{j=1}^{N} y_{ij} \cdot V_j .$$

$$S_1 = V_1^* [y_{11} V_1 + y_{12} V_2 + y_{13} V_3]$$

$$= 1.05 [-j5.833 \times 1.05 \angle 0^\circ + j2.5 (1.0192 + j0.0312) + j3.333 (1.0075 - j0.0244)]$$

$$= -0.0175 - j0.2295 \text{ p.u.}$$

$P_1 = -0.0175$ p.u. $= -1.75$ MW

$Q_1 = 0.2295$ p.u. $= 22.95$ MVAr.

Step-6 Line flow

$$S_{ij} = P_{ij} + jQ_{ij}$$

$$= V_i [V_i^* - V_j^*] y_{ij\,series}^* + |V_i|^2 \cdot y_{pi}^* .$$

Line flow from bus 1 to 2

$$S_{12} = P_{12} + jQ_{12}$$

$$= V_1 [V_1^* - V_2^*] y_{12\,series}^* .$$

$$= 1.05 [(1.05 \angle -0^\circ - 1.0192 + j0.0392] \times j2.5 .$$

$$= -0.1029 + j0.0808 \text{ p.u.}$$

$S_{21} = P_{21} + jQ_{21}$

$= V_2 [V_2^* - V_1^*] Y_{21\,series}^*$

$= 1.0192 + j0.0392 [1.0192 - j0.0392 - 1.05] \times j2.5$

$= 1.0192 - j0.0746$ P. U.

$S_{23} = P_{23} + jQ_{23}$.

$S_{23} = V_2 [V_2^* - V_3^*] Y_{23\,series}^*$

$= 1.0192 + j0.0392 [1.0192 - j0.0392 - 1.0075 - j0.0244] j5$

$= 0.3218 + j0.072$ P.U.

$S_{32} = P_{32} + jQ_{32}$.

$= V_3 [V_3^* - V_2^*] Y_{32\,series}^*$

$= 1.0075 - j0.0244 [1.0075 - j0.0244 - 1.0192 + j0.0392] \times j5$

$= -0.3218 - j0.0512$ P. U.

$S_{13} = \cancel{6}P_{13} - jQ_{13}$.

$S_{13} = P_{13} + jQ_{13}$.

$= V_1 [V_1^* - V_3^*] Y_{13\,series}^*$.

$= 1.05 [1.05 \angle 0^\circ - 1.0075 - j0.0244] \times j3.333$.

$= 0.085 + j0.148 \text{ P.U.}$

$S_{31} = P_{31} + jQ_{31}$

$= V_3 [V_3^* - V_1^*] Y_{31 \text{ series}}^*$

$= 1.0075 - j0.0244 \times [1.0075 + j0.0244 - 1.05] \times j3.333$

$= -0.085 - j0.1407 \text{ P.U.}$

Transmission Loss

$S_{ij\,loss} = S_{ij} + S_{ji}$

~~to~~ For line 1-2,

$S_{12\,loss} = P_{12\,loss} + jQ_{12\,loss}$

$= S_{12} + S_{21}$

$S_{12\,loss} = -0.1029 + j0.0808 + 0.1029 - j0.0746$

$= 0 + j0.0061$

$P_{12\,loss} = 0$; $Q_{12\,loss} = 0.0061 \text{ P.U.}$

$= 0.61 \text{ MVAr.}$

For line 2-3

$S_{23\,loss} = P_{23\,loss} + jQ_{23\,loss} = S_{23} + S_{32}$

$= 0.3218 + j0.072 + (-0.3218 - j0.0512)$

$= 0 + j0.021$

$P_{23\,loss} = 0$; $Q_{23\,loss} = 0.021 \text{ P.U.} = 2.1 \text{ MVAr.}$

For line 1-3

$$S_{13\,loss} = P_{13\,loss} + jQ_{13\,loss} = S_{13} + S_{31}$$

$$= 0.085 + j0.148 + [-0.085 - j0.1407]$$

$$= 0 + j0.00726.$$

$P_{13\,loss} = 0$; $Q_{13\,loss} = 0.00726$ p.u. $= 0.726$ MVAr.

Describe step by step procedure to calculate the power flow analysis using Newton Raphson method. (N/D-14)

1) Calculate Y-bus matrix

2) Take flat start for starting voltage solution.

$$\delta_i^0 = 0, \text{ for } i = 1, \ldots, N \text{ for all buses except slack bus}$$

$$|V_i^0| = 1.0, \text{ for } i = M+1, M+2, \ldots, N \text{ (for all PQ buses)}$$

$$|V_i| = |V_i|_{(spec)} \text{ for all PV buses \& slack bus}$$

3) For load bus calculate P_i^{cal} & Q_i^{cal}

4) For Generator bus check for Q-limit violation

If $Q_{i(min)} < Q_i^{cal} < Q_{i(max)}$, the bus acts as generator bus.

~~If $Q_{i(min)} < Q_{i(max)}$; $Q_{i(spec)} = Q_{i(min)}$~~

If $Q_i^{cal} < Q_{i(min)}$; $Q_{i(spec)} = Q_{i(min)}$

If $Q_i^{cal} > Q_{i(max)}$; $Q_{i(spec)} = Q_{i(max)}$

the bus will be converted from generator bus to load bus.

5) Compute mismatch vector using

$$\Delta P_i = P_{i(spec)} - P_i^{cal}$$

$$\Delta Q_i = Q_{i(spec)} - Q_i^{cal.}$$

6) Compute $\Delta P_{i(max)} = max|\Delta P_i|$; $i = 1, 2, \cdots, N$ except slack bus

$\Delta Q_{i(max)} = max|\Delta Q_i|$, $i = M+1, \cdots, N$.

7) Calculate Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial P_i}{\partial \delta} & |V| \frac{\partial P_i}{\partial |V|} \\ \frac{\partial Q_i}{\partial \delta} & |V| \frac{\partial Q_i}{\partial |V|} \end{bmatrix}$$

8) Get the state correction vector

$$\begin{bmatrix} \Delta \delta \\ \frac{\Delta V}{|V|} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

9) Update state vector using

$$V^{new} = V^{old} + \Delta V = V^{old} + \frac{\Delta V}{|V^{old}|} = V^{old} \left[1 + \frac{\Delta V}{|V^{old}|} \right]$$

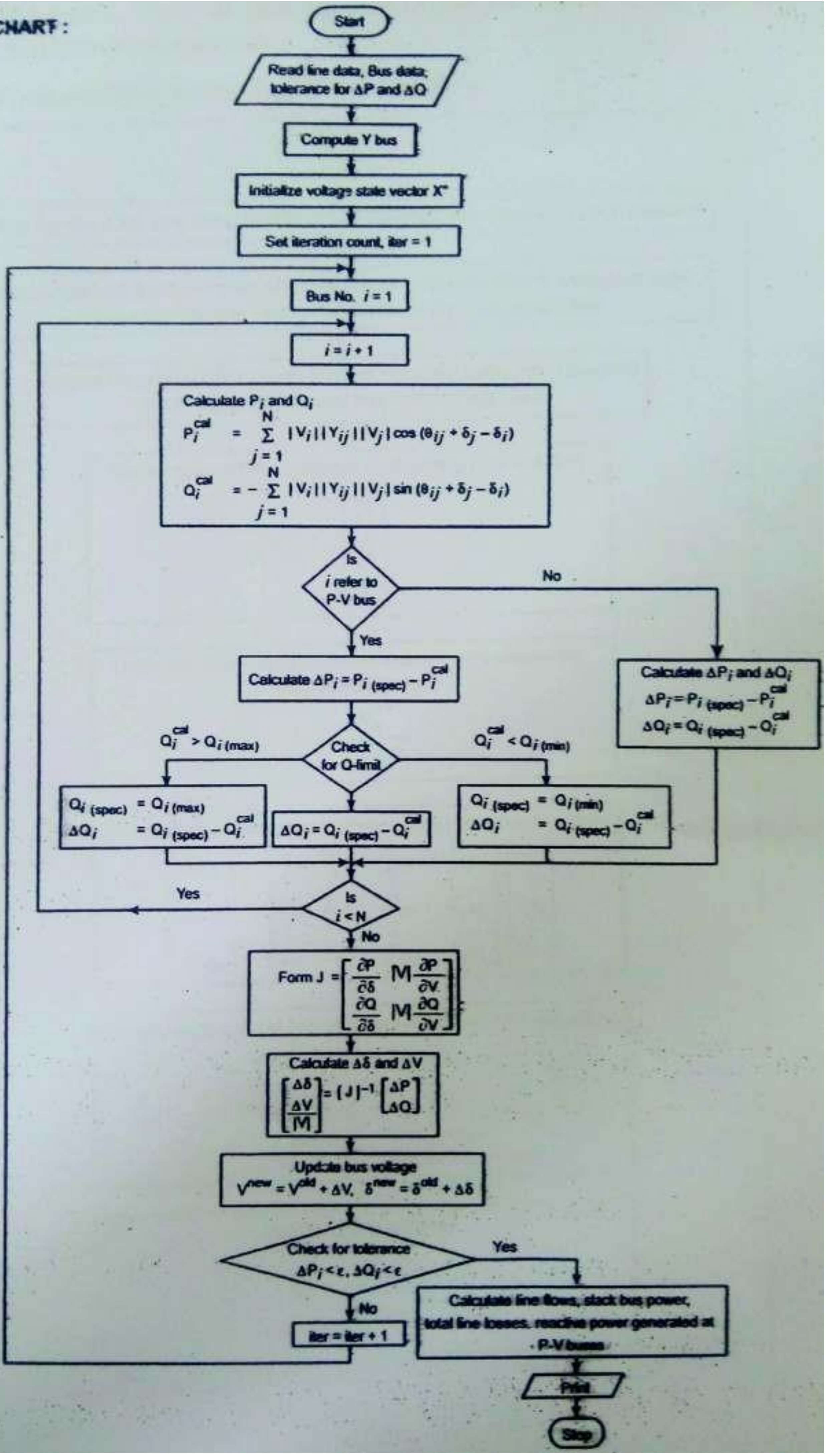
$$\delta^{new} = \delta^{old} + \Delta \delta$$

10) above procedure is executed till

$|\Delta P_i| < \varepsilon$ and $|\Delta Q_i| < \varepsilon$, other wise goto (3).

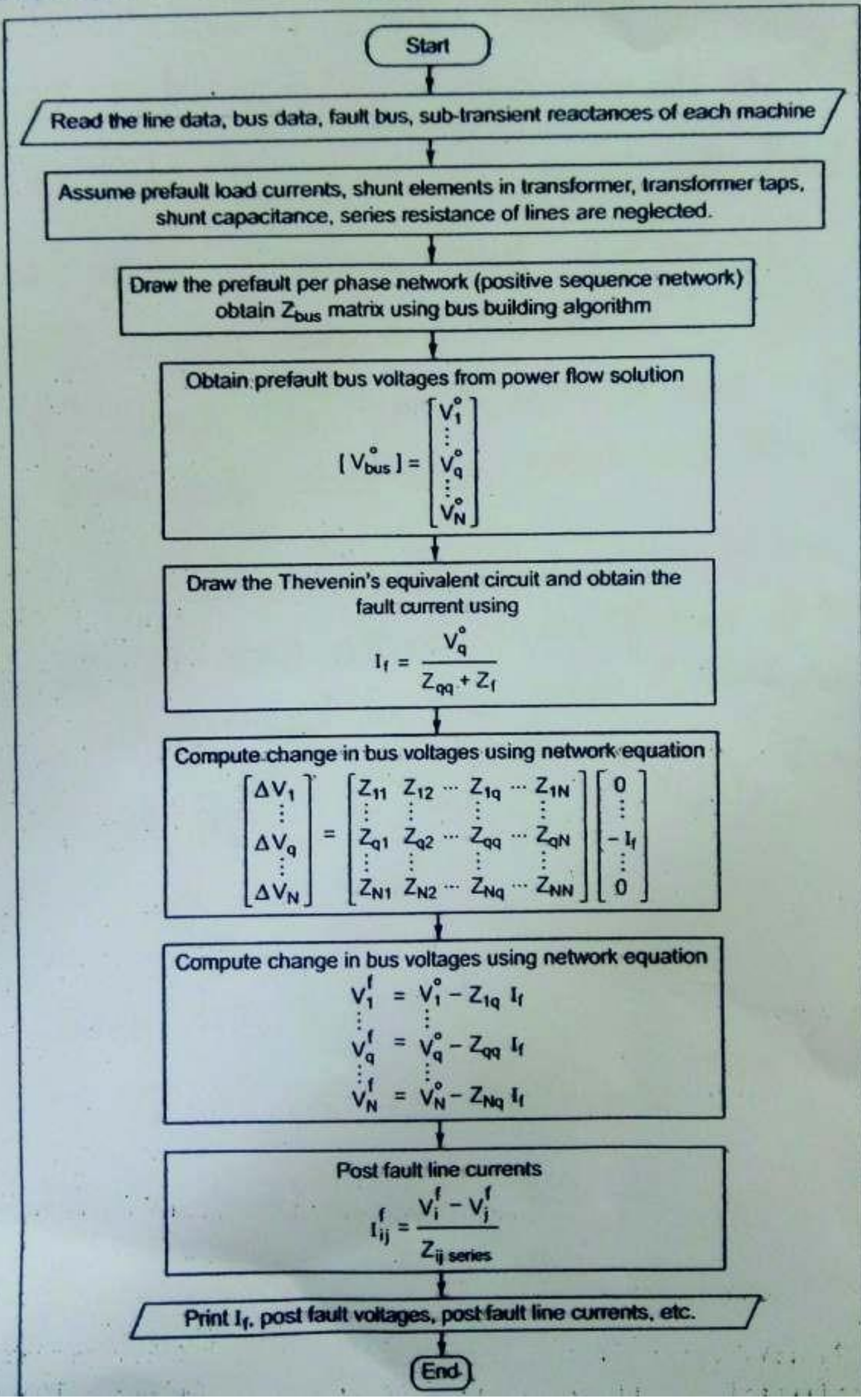
—

FLOW CHART :


Start
Read line data, Bus data; tolerance for ΔP and ΔQ
Compute Y bus
Initialize voltage state vector X°
Set iteration count, iter = 1
Bus No. i = 1
i = i + 1
Calculate P_i and Q_i
$P_i^{cal} = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$
$Q_i^{cal} = -\sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$
Is i refer to P-V bus
No
Yes
Calculate $\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
Calculate ΔP_i and ΔQ_i
$\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$Q_i^{cal} > Q_{i\,(max)}$
Check for Q-limit
$Q_i^{cal} < Q_{i\,(min)}$
$Q_{i\,(spec)} = Q_{i\,(max)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$Q_{i\,(spec)} = Q_{i\,(min)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
Is i < N
Yes
No
Form $J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & |V|\frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & |V|\frac{\partial Q}{\partial V} \end{bmatrix}$
Calculate Δδ and ΔV
$\begin{bmatrix} \Delta\delta \\ \frac{\Delta V}{|V|} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$
Update bus voltage
$V^{new} = V^{old} + \Delta V, \ \delta^{new} = \delta^{old} + \Delta\delta$
Check for tolerance
$\Delta P_i < \varepsilon, \Delta Q_i < \varepsilon$
Yes
No
iter = iter + 1
Calculate line flows, slack bus power, total line losses, reactive power generated at P-V buses
Print
Stop

very simple and practical. Thus all fault calculations are formulated in the bus frame of reference using bus impedance matrix Z_{bus}.

Symmetrical Fault Analysis using Z_{bus} (Flow chart)



Describe the step by step procedure to calculate the power flow analysis using Gauss-seidal method. (M/J-14) (~~N/D-14~~)

Gauss-seidal algorithm for iteration

1) Calculate Y-bus matrix
2) Take $V_k = V_{k(spec)} \angle 0^\circ$ for all generator buses.
3) Take $V_k = 1\angle 0^\circ = 1 + j0$ at all load buses.
4) Make iteration count = 1 (iter=1)
5) Assume ~~buse~~ bus number $i = 1$.
6) Check the type of bus. If bus is generator bus goto next step. otherwise goto ~~step~~ (8)
7) check the bus for slack bus. If bus is slack bus goto (9) ~~step~~. other wise (if bus is generator bus proceed in this point further.

Calculate $Q_i^{cal} = -\text{Im}\left[\sum_{j=1}^{n} V_i^* \cdot Y_{ij} \cdot V_j\right]$

$$Q_{Gi} = Q_i^{cal} + Q_{Li}$$

Check for Q limit

If $Q_{i(min)} < Q_{Gi} < Q_{i\,max}$, then $Q_{i(spec)}^{*} = Q_i^{cal}$

If $Q_{i(min)} < Q_{Gi}$; then $Q_{i(spec)} = Q_{i(min)} - Q_{Li}$

If $Q_{i(max)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(max)} - Q_{Li}$

If Q limit is violated, treat the particular bus as load bus till the convergence.

8) Calculate voltage at all the buses except slack bus using the eqn. shown below

$$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_{i(spec)} - jQ_{i(spec)}}{V_i^{old*}} - \sum_{i=1}^{j-1} Y_{ij} V_j^{new} - \sum_{i=j+1}^{n} Y_{ij} V_j^{old}\right]$$

9) If i is less than number of buses increase the i by 1 and goto (6)

10) Compare successive iteration values of V_i

If $V_i^{new} - V_i^{old} <$ tolerance, goto (12) (or) Proceed with (11)

11) Update the new voltages as

$$V^{new} = V^{old} + \alpha\left(V^{new} - V^{old}\right)$$

$$V^{old} = V^{new}.$$

12) Compute relevent quantities.

Slack bus power, $S_1 = P_i - jQ_i = V^* I$

$$= V_i^* \sum_{j=1}^{n} Y_{ij} \cdot V_j$$

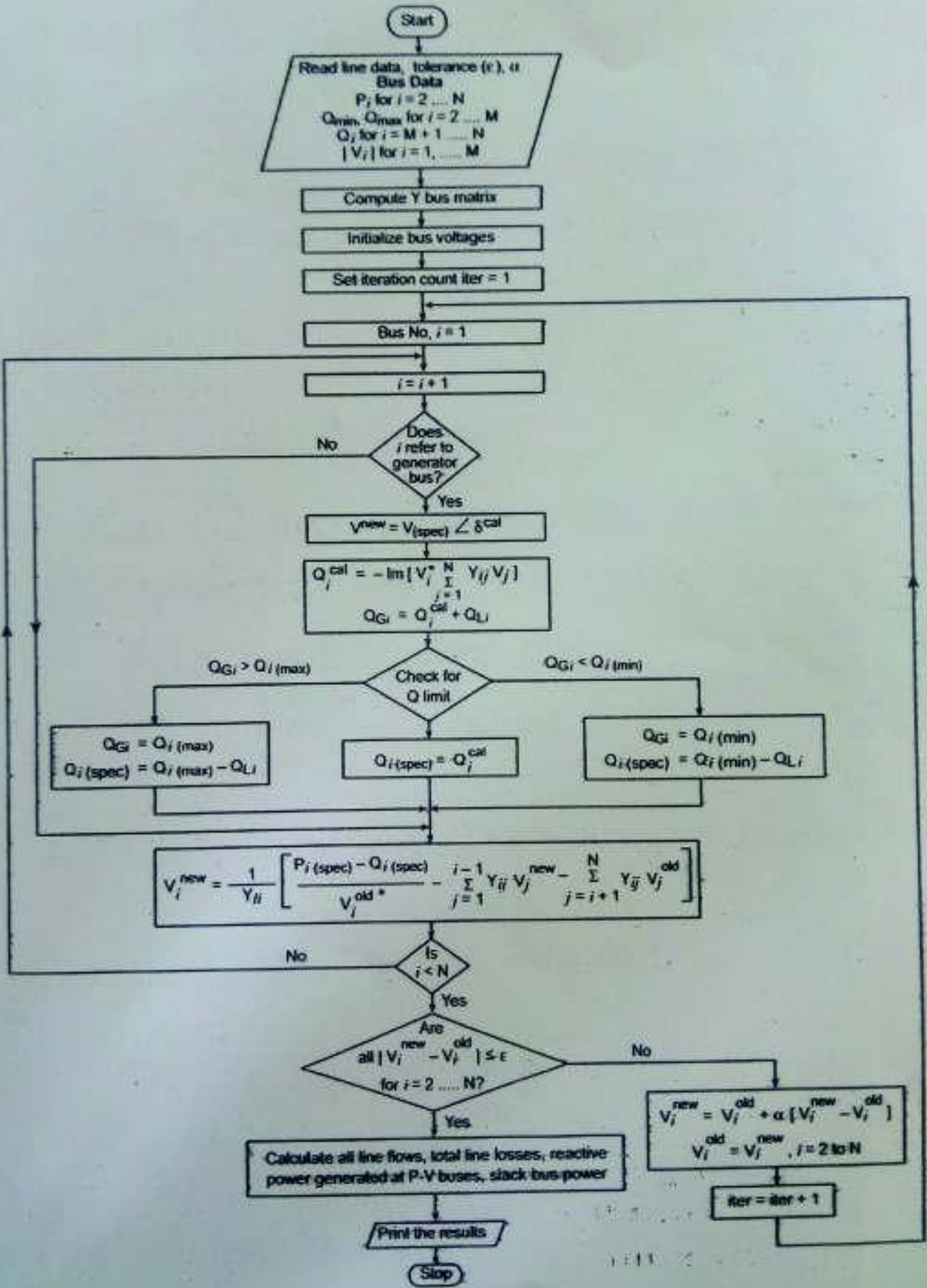
Line flows, $S_{ij} = P_{ij} + jQ_{ij}$

$$= V_i\left[V_i^* - V_j^*\right] Y_{ij\,series}^* + |V_i|^2 \cdot Y_{ii}^*$$

$$P_{Loss} = P_{ij} + P_{ji}\,;\; Q_{Loss} = Q_{ij} + Q_{ji}.$$

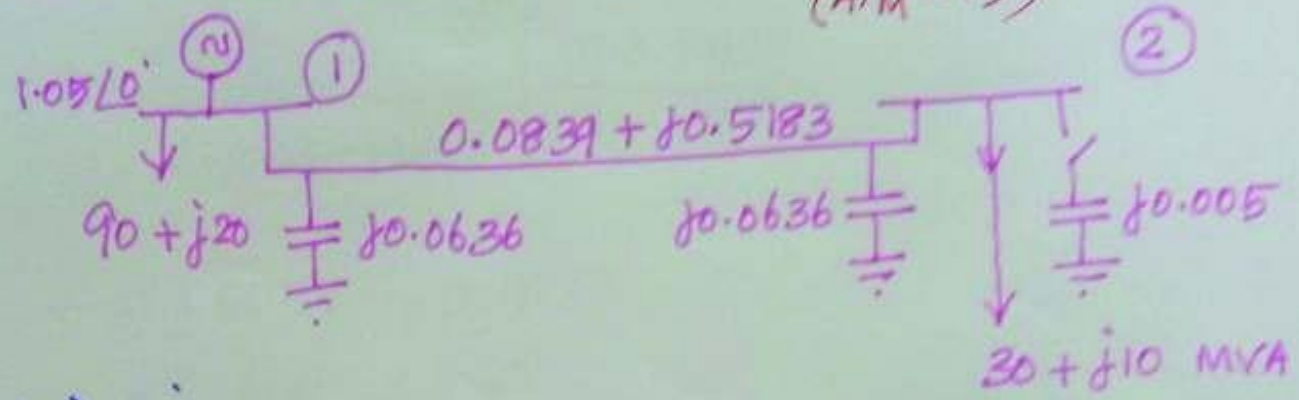
13) Stop the calculation process.

5.7.6. FLOW CHART FOR GAUSS-SEIDEL METHOD INCLUDING PV BUS ADJUSTMENT



Perform power flow of one iteration for the system as shown in fig. using Gauss-Seidal method. Determine slack bus power, line flows and line losses. Take base MVA as 100 ($\alpha = 1.1$)

(AIM-15)



Solution

Calculation of Y_{bus}

when the switch is open, there is no connection of capacitor at bus 2.

Take bus 2 as load bus

$$Y_{bus} = \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$$

Initialization of bus Voltages

$$V_1^{old} = 1.05 \angle 0° \text{ p.u.}$$

$$V_2^{old} = 1.0 \angle 0° \text{ p.u.}$$

Calculation of V_2^{new}

$$P_2 = -30\text{ MW} = \frac{-30}{100}\text{ p.u.} = -0.3\text{ p.u.}$$

$$Q_2 = -10 \text{ MVAr} = \frac{-10}{100} \text{ p.u.} = -0.1 \text{ p.u.}$$

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21} V_1^{new}\right]$$

$$= \frac{1}{0.3044 - j1.816}\left[\frac{-0.3 + j0.1}{1.0 \angle -0^\circ} - (-0.3044 + j1.88)\, 1.05\right]$$

$$= 1.0054 - j0.1577$$

$$= 1.018 \angle -8.915^\circ$$

Calculation of V_2^{new} using acceleration factor.

$$V_{2acc}^{new} = V_2^{old} + \alpha\left[V_2^{new} - V_2^{old}\right]$$

$$= 1.0 + 1.1\left[1.0054 - j0.1577 - 1\right]$$

$$= 1.0059 - j0.173$$

$$= 1.0207 \angle -9.78^\circ$$

Calculation of slack bus power

$$S_1 = P_1 - jQ_1 = V_1^*\left[Y_{11} V_1 + Y_{12} V_2\right]$$

$$= 1.05 \angle -0^\circ \left[(0.3044 - j1.816)1.05 + (-0.3044 + j1.88)(1.0207 \angle -9.78^\circ)\right]$$

$$= 0.3556 + j0.0388 \text{ p.u.}$$

$$= 35.56 + j3.88 \text{ MVA}$$

$P_1 = 35.56$ MW, $Q_1 = -3.88$ MVAr.

Real power generation, $P_{G1} = P_1 + P_{L1}$

$= 35.56 + 90 = 125.56$ MW

Reactive power generation, $Q_{G1} = Q_1 + Q_{L1}$

$= -3.88 + 20$

$= 16.12$ MVAr.

Line flows

$$S_{12} = V_1 [V_1^* - V_2^*] Y_{12\,series}^* + |V_1|^* y_{10}^*$$

$$= 1.05 [1.05 \angle -0^\circ - (1.0059 + j0.173)] \times (0.3044 + j1.88) + 1.05^2 \times (-j0.0636)$$

$$= 0.3556 - j0.0383 \text{ p.u.}$$

$P_{12} = 0.3556$ p.u. $= 35.56$ MW

$Q_{12} = -0.0383$ p.u. $= -3.83$ MVAr.

$$S_{21} = V_2 [V_2^* - V_1^*] y_{21\,series}^* + |V_2|^2 . y_{20}^*$$

$$= (0.973 + j0.32) [0.973 - j0.32 - 1.05] \times (0.3044 + j1.88) + 1.02^2 \times (-j0.0636)$$

$$= 0.64 - j0.1167 \text{ p.u.}$$

$P_{21} = 64$ MW ; $Q_{21} = -0.1167$ p.u. $= -11.67$ MVAr

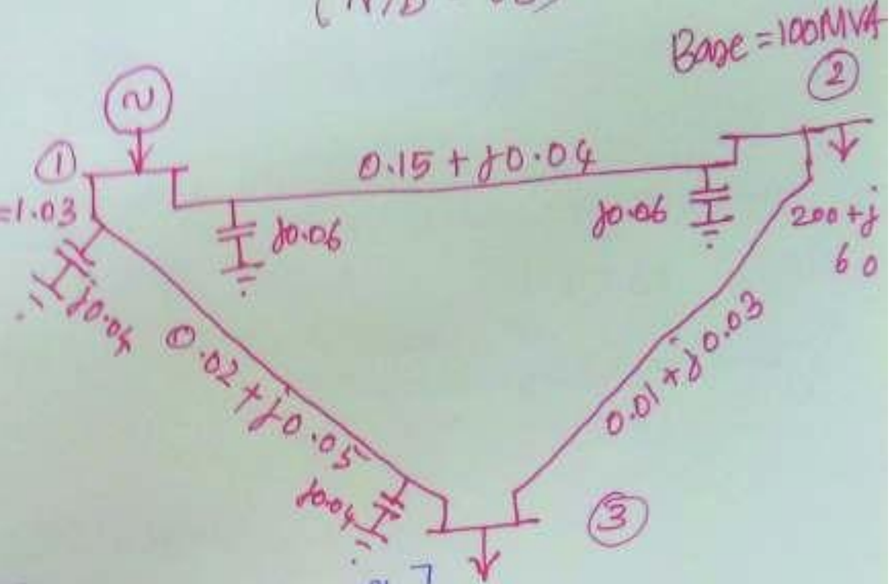
Transmission line loss

$$S_{ij\,loss} = S_{ij} + S_{ji}$$

$$P_{12\,loss} = P_{12} + P_{21} = -60.7 + 64 = 3.3\ MW$$

$$Q_{12\,loss} = Q_{12} + Q_{21} = 18 - 11.67 = 6.33\ MVAr.$$

Perform Gauss-Seidal load flow for the system shown in fig. and the bus data given. Determine bus voltages, slack bus power, line flows and transmission line losses. (N/D − 08)



Solution

Step-1

$$Y_{bus} = \begin{bmatrix} y_{12}+y_{13}+y_{10} & -y_{12} & -y_{13} \\ -y_{12} & y_{12}+y_{23}+y_{20} & -y_{23} \\ -y_{13} & -y_{23} & -y_{31}+y_{32}+y_{30} \end{bmatrix}$$

$$= \begin{bmatrix} 13.1206 - j18.8011 & -6.2241 + j1.6598 & -6.8966 + j17.2414 \\ -6.2241 + j1.6598 & 16.2241 - j31.5998 & -10 + j30 \\ -6.8966 + j17.2414 & -10 + j30 & 16.8966 - j47.2014 \end{bmatrix}$$

Initialize bus voltages

$V_1^{old} = 1.03 \angle 0$

$V_2^{old} = 1.0 \angle 0$

$V_3^{old} = 1.0 \angle 0$

Calculation of V_2^{new}

$P_2 = -P_{L2} = \frac{-200}{100} = -2$ p.u.;

$Q_2 = -Q_{L2} = \frac{-60}{100} = -0.6$ P.U.;

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{-2+j0.6}{1.0\angle -0^\circ} - (-6.2241+j1.6598)\,1.03\angle 0^\circ - (-10+j30)\times 1.0\angle 0^\circ\right]$$

$Y_{22} = 16.2241 - j31.5998$

$$= \frac{14.4108 - j31.1096}{16.2241 - j31.5998}$$

$= 0.9644 - j0.0391.$

$= 0.965 \angle -2.32^\circ$

Given $\alpha = 1.2$,

$V_2^{new} = 1.0 + 1.2[0.9644 - j0.0391 - 1.0]$

$= 0.9573 - j0.047$

$= 0.9584 \angle -2.81^\circ$

$\underline{V_3^{new}}$

$P_3 = P_{G3} - P_{L3} = \frac{160 - 60}{100} = -1$ p.u.

$Q_3 = 0 - Q_{L3} = \frac{-50}{100} = -0.5$ P.u.

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{old*}} - Y_{31}V_1^{new} - Y_{32}V_2^{new}\right]$$

$$= \frac{1}{16.8966 - j47.2014}\left[\frac{-1+j0.5}{1\angle -0^\circ} - (-6.8966+j17.2414)\times 1.03 - (-10+j30)(0.9573 - j0.047)\right]$$

$= 0.9682 - j0.0443$

$= 0.969 \angle -2.61^\circ$ p.u.

$\alpha = 1.2$

$V_3^{new} = V_3^{old} + \alpha \left[V_3^{new} - V_3^{old} \right]$

$= 1 + j0 + 1.2 \left[0.9682 - j0.0443 - 1 - j0 \right]$

$= 0.96184 - j0.05316$

$= 0.9633 \angle -3.16^\circ$

<u>slack bus power</u>

$S_1 = P_1 - jQ_1$

$= V_1^* \left[Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3 \right]$

$= 1.03 \angle -0^\circ \left[(13.1206 - j18.8011)\, 1.03 \angle 0^\circ \right.$

$+ (-6.2241 + j1.6598) \times 0.9584 \angle -2.81^\circ$

$\left. + (-6.8966 + j17.2414)(0.9633 \angle -3.16^\circ) \right]$

$= 1.9731 - j0.5516$ p.u.

$= 197.31 - j55.16$ MVA

$P_1 = 197.31$ MVA

$Q_1 = 55.16$ MVAR.

<u>Calculation of line flows</u>

$S_{12} = 1.03 \left[1.03 - (0.9573 + j0.047) \right] (6.22 + j1.66)$

$+ 1.03^2 (-j0.06)$

$= 0.5461 - j0.2405$ p.u.

$P_{12} = 54.61$ MW; $Q_{12} = -24.05$ MVAR.

$$S_{21} = (0.9573 - j0.047)[0.9573 + j0.047 - 1.30]$$
$$(6.22 + j1.66) + 0.9584^2(-j0.06)$$

$$= -0.4995 + j0.1341 \text{ P.u.}$$

$P_{21} = -49.95$ MW

$Q_{21} = 13.41$ MVAr.

$$S_{23} = (0.9573 - j0.047)[0.9573 + j0.047 -$$
$$(0.9618 + j0.053)] \times (10 + j30) + 0.9584^2 \times 0$$

$$= 0.1201 - j0.1930 \text{ P.u.}$$

$P_{23} = 12.01$ MW

$Q_{23} = -19.3$ MVAr.

$$S_{32} = (0.9618 - j0.053)[0.9618 + j0.053 -$$
$$(0.9573 + j0.047)] \times (10 + j30) + 0.$$

$$= -0.1195 + j0.1947 \text{ P.u.}$$

$P_{32} = -11.95$ MW

$Q_{32} = 19.47$ MVAr.

$$S_{13} = (0.9618 - j0.053)(0.9618 + j0.053 - 1.03)$$
$$(6.9 + j17.24) + 0.9633^2 \times (-j0.04)$$

$$= -1.374 - j0.7429 \text{ P.u.}$$

$P_{13} = -1.374$ p.u. $= -137.4$ MW

$Q_{13} = -0.7429 = -74.29$ MVAr.

Transmission line losses

$P_{12\,loss} = P_{12} + P_{21} = 54.61 - 49.95$

$= 4.66$ MW

$Q_{12\,loss} = Q_{21} + Q_{12} = -24.05 + 13.41$

$= -10.64$ MVAr.

$Q_{23\,loss} = Q_{23} + Q_{32}$

$= -19.3 + 19.47$

$= 0.17$ MVAr.

$P_{23\,loss} = P_{23} + P_{32}$

$= 12.01 - 11.95$

$= 0.06$ MW

$P_{13\,loss} = P_{13} + P_{31}$

$= 142.6 - 137.4 = 5.2$ MW

$Q_{13\,loss} = Q_{13} + Q_{31}$

$= 79.19 - 74.29$

$= 4.9$ MVAr.

EE6501 - Power System Analysis

Part - A Unit - III

1. **Define short circuit capacity.** EE - N/D - 16

U-III (A/M -15), (M/J - 09)

Short circuit capacity is defined as the product of the magnitudes of the prefault bus voltage and post fault current.

2 **What is the need for short circuit study?** EE - N/D -16

(A/M-11), (N-11), (N/D -14)

The need for short circuit study are

1) ~~Bo~~ To select the proper circuit breaker
2) To design the protective relays

3 List the symmetrical and unsymmetrical faults that occur in a power system. EE - M/J -12

(A/M - 11) ;

The symmetrical fault is

→ LLLG fault

The unsymmetrical faults are

→ L-G fault

→ L-L fault

→ L-L-G fault.

4.What is meant by a fault? EE - M/J -12

(N/D -07)

U-III

Fault is indication of heavy current flow due to failure of insulation and voltage falls below rated value in the electrical system.
This may develop damage to the equipments (or) apparatuses connected to electrical system.

5. **How the shunt and series faults are classified?** EE - N/D - 16

(N/D -11)

Shunt faults are classified as follows

1) Three phase fault (LLLG)
2) Line to ground fault (L-G)
3) Line to Line fault (L-L)
4) Double line to ground fault (L-L-G

Series faults are classified as follows.

1) open conductor fault
2) Two open conductor fault.

6. Define bolted fault. EE - M/J - 14

U - III

The bolted fault is defined as the fault with zero fault impedance.

7. What is symmetrical fault? EE - N/D -14

V-III

During the fault on a 3 phase line, the short circuit current magnitude on all the three phases is same. This type of fault is called symmetrical fault.

Or

(b) With the help of a detailed algorithm, explain how a symmetrical fault can be analysed using Z_{Bus}. (16)

EE-M/J-14

(N/D-11), (A/M-11), (N/D-09)

1) Draw the prefault per phase network. Assume 3φ fault occurs at a particular Point.

The prefault bus voltages are taken from load flow solution.

Initial bus Voltages, $V_{bus}^{0} = \begin{bmatrix} V_1^0 \\ \vdots \\ V_q^0 \\ \vdots \\ V_N^0 \end{bmatrix}$

represent the load by a constant impedance evaluated at the prefault bus Voltages.

$$Z_{iL} = \frac{|V_i^0|^2}{S_L^*}$$

2) Obtain Z_{bus} matrix by bus building algorithm.

3) Obtain the fault current by the

following eqn.

$$I_f \text{ (or) } I_{sc} = \frac{V_q^0}{Z_{qq} + Z_f}$$

Z_{qq} - diagonal element of the Z_{bus} matrix.

4) Obtain thevenin's network with Z_f as V_{th}. The current entering every bus is zero except the faulted bus.

$$I_1 = I_2, \ldots, I_N = 0 \quad \text{except } I_q.$$

$$I_q = -I_f$$

$$I_{bus} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_q \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

$$I_{bus(F)} = Y_{bus} \cdot \Delta V_{bus}.$$

Solving for ΔV_{bus}

$$\Delta V_{bus} = Z_{bus} \cdot I_{bus(F)}$$

5) calculation of post fault bus voltages.

$$V_{bus(f)} = V_{bus}^{0} + \Delta V_{bus}.$$

$$V_1^f = V_1^0 - Z_{1q} \cdot I_f$$

$$V_2^f = V_2^0 - Z_{2q} \cdot I_f.$$

$$\vdots$$

$$V_q^f = V_q^0 - Z_{qq} \cdot I_f$$

$$V_N^f = V_N^0 - Z_{Nq} \cdot I_f.$$

In general

$$V_i^f = V_i^0 - Z_{iq} \cdot I_f$$

Bus voltage during the fault

$$V_i^f = V_i^0 - \frac{Z_{iq} \times V_q^0}{Z_{qq} + Z_f} \; ; \; i \neq q$$

$$V_q^f = V_q^0 \cdot \frac{Z_f}{Z_{qq} + Z_f} \; ; \; i = q$$

In bolted fault.

$$I_f = \frac{V_q^0}{Z_{qq}} \; ; \; \cancel{V_q^f \neq 0} \; V_q^f = 0.$$

6) calculation of post fault line currents.

$$I_{ij}^{f} = \frac{V_i^{f} - V_j^{f}}{Z_{ij\ series}}.$$

with the bus impedance matrix, the fault current and bus voltages during the fault and post fault line currents are obtained for any faulted bus. This method is simple.

Thus all the fault calculations are formulated in the bus frame of reference using bus impedance matrix Z_{bus}.

—

Symmetrical Fault Analysis using Z_{bus} (Flow chart)

Start

Read the line data, bus data, fault bus, sub-transient reactances of each machine

Assume prefault load currents, shunt elements in transformer, transformer taps, shunt capacitance, series resistance of lines are neglected.

Draw the prefault per phase network (positive sequence network) obtain Z_{bus} matrix using bus building algorithm

Obtain prefault bus voltages from power flow solution

$$[V_{bus}^{\circ}] = \begin{bmatrix} V_1^{\circ} \\ \vdots \\ V_q^{\circ} \\ \vdots \\ V_N^{\circ} \end{bmatrix}$$

Draw the Thevenin's equivalent circuit and obtain the fault current using

$$I_f = \frac{V_q^{\circ}}{Z_{qq} + Z_f}$$

Compute change in bus voltages using network equation

$$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_q \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1q} & \cdots & Z_{1N} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{q1} & Z_{q2} & \cdots & Z_{qq} & \cdots & Z_{qN} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nq} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

Compute change in bus voltages using network equation

$$\begin{aligned} V_1^f &= V_1^{\circ} - Z_{1q}\, I_f \\ &\vdots \\ V_q^f &= V_q^{\circ} - Z_{qq}\, I_f \\ &\vdots \\ V_N^f &= V_N^{\circ} - Z_{Nq}\, I_f \end{aligned}$$

Post fault line currents

$$I_{ij}^f = \frac{V_i^f - V_j^f}{Z_{ij\ series}}$$

Print I_f, post fault voltages, post fault line currents, etc.

End

(b) A 25 MVA, 11 KV generator with $X_d'' = 0.2$ per unit is connected through a transformer, line and a transformer to a bus that supplies three identical motors as shown in Fig. 13 (b). Each motor has $X_d'' = 0.25$ per unit and $X_d' = 0.3$ per unit on a base of 5 MVA, 6.6 kV. The three-phase rating of the step-up transformer is 25 MVA, 11 / 66 KV with a leakage reactance of 0.1 per unit and that of the step-down transformer is 25 MVA, 66 / 6.6 KV with a leakage reactance of 0.1 per unit. The bus voltage at the motors is 6.6 KV when a three-phase fault occurs at the point F. For the specified fault, calculate

(i) the subtransient current in the fault,

(ii) the subtransient current in the breaker B,

(iii) the momentary current in breaker B   EE-N(b)-07

Fig. 13 (b)

Solution

The base MVA = 25

For a generator Voltage base of 11 kV,
line voltage base is 66 kV and
motor Voltage base is 6.6 kV

(i) For each motor

$$X''_{dm} = j0.25 \times \frac{25}{5}$$

$$= j1.25 \text{ p.u.}$$

Line, transformers and generators reactances are already given on proper base values.

The circuit model of the system for fault calculations is given ~~to~~ in fig-1. The system is initially on no load, the generator and motor induced emf s are identical.

The circuit can be reduced to new architecture as shown in fig-2.

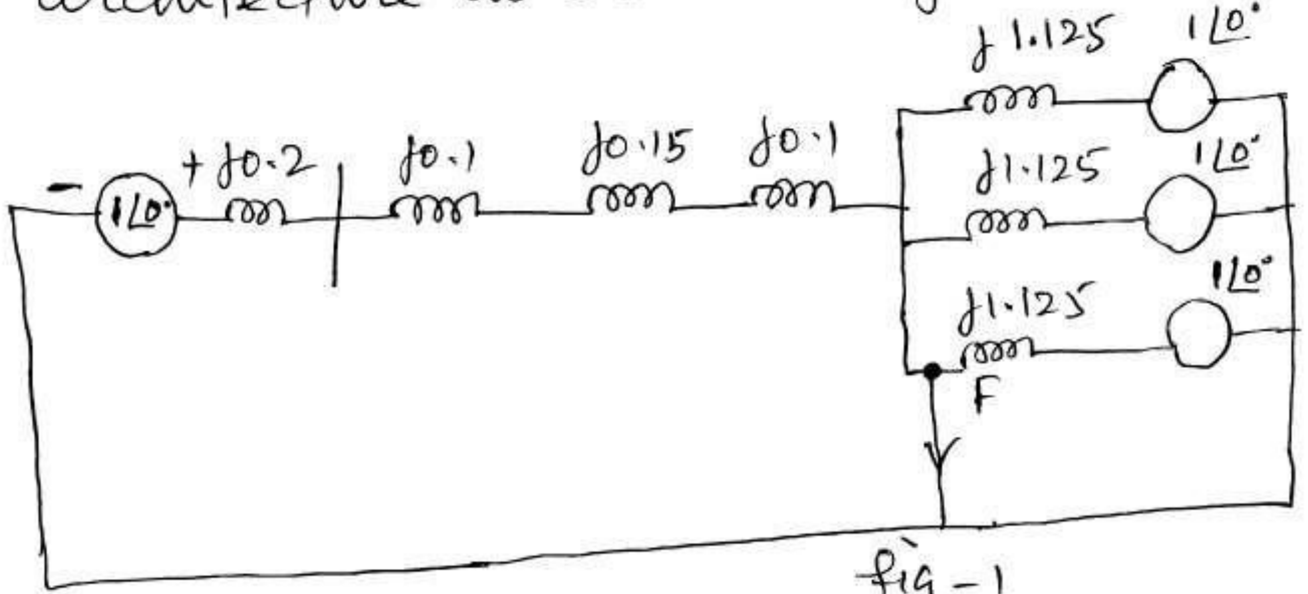


fig-1

⇓

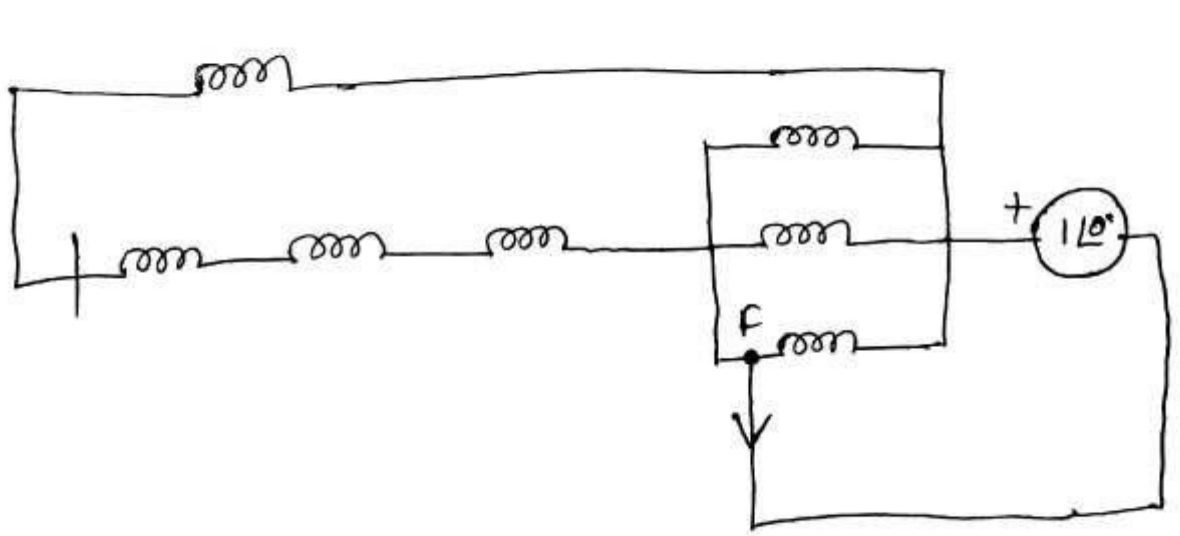


fig-2

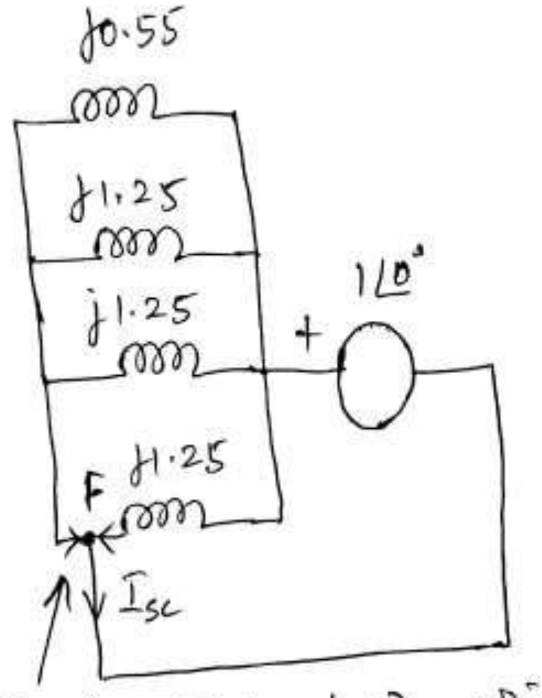


fig -3

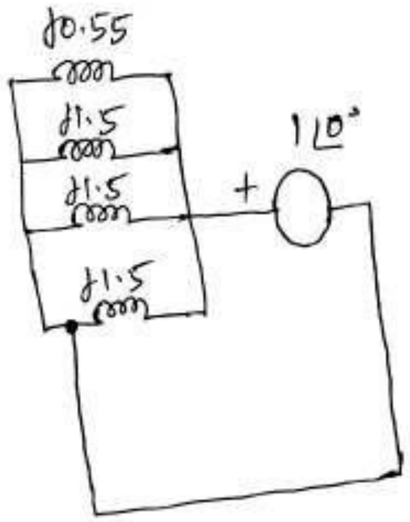


fig -4

$$I_{sc} = 3 \times \frac{1}{j1.25} + \frac{1}{j0.55}$$

$$= -j4.22 \text{ p.u.}$$

Base current in 6.6 kV circuit

$$= \frac{25 \times 1000}{\sqrt{3} \times 6.6} = 2187 \text{ A}$$

$$I_{sc} = 4.22 \times 2187$$

$$= 9229 \text{ A}$$

From fig-3

b)

Current through circuit breaker

$$I_{sc} = 2 \times \frac{1}{j1.25} + \frac{1}{j0.55}$$

$$= -j3.42$$

$$= 3.42 \times 2187$$

$$= 7{,}479.5 \text{ A}.$$

c) For finding momentary current through the breaker, DC off-set current should be added with the subtransient current obtained in the section (b).

The momentary current can be calculated by considering empirical value.

momentary current through breaker, B.

$$= 1.6 \times 7{,}479.5$$

$$= 11967 \text{ A}$$

(b) Figure 13. (b) Shows a generating station feeding a 132 KV system. Determine the total fault current, fault level and fault current supplied by each alternator for a 3 — phase fault at the receiving end bus. The line is 200 km long. Take a base of 100 MVA, 11 KV for LV side and 132 KV for HT side.

EE-N/D-13 (16)

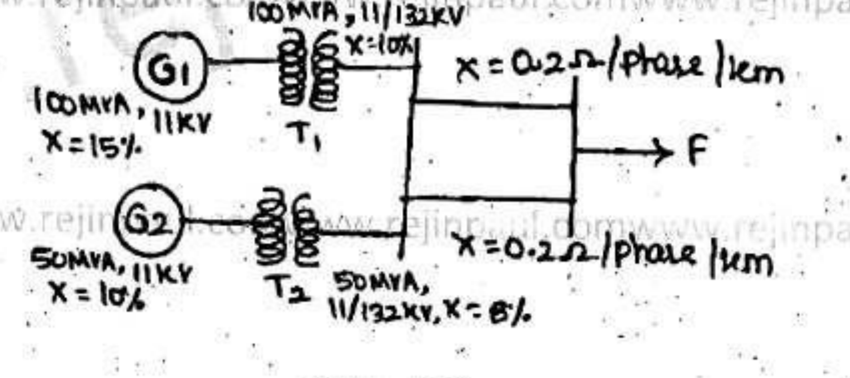


Figure. 13(b)

Solution

The base MVA is 100 of G_1.

base kV for LV side is 11 kV

base kV for HV side is 132 kV

P.U. reactance of $G_1 = j0.15$

P.U. reactance of $G_2 = j0.1 \times \frac{100}{50} = j0.2$

P.U. reactance of $T_1 = j0.1$

P.U. reactance of $T_2 = j0.08 \times \frac{100}{50} = j0.16$

P.U. reactance of each line $= \frac{j0.2 \times 200 \times 100}{132 \times 132}$

$= j0.23$.

The single line reactance diagram has been shown in fig-1.

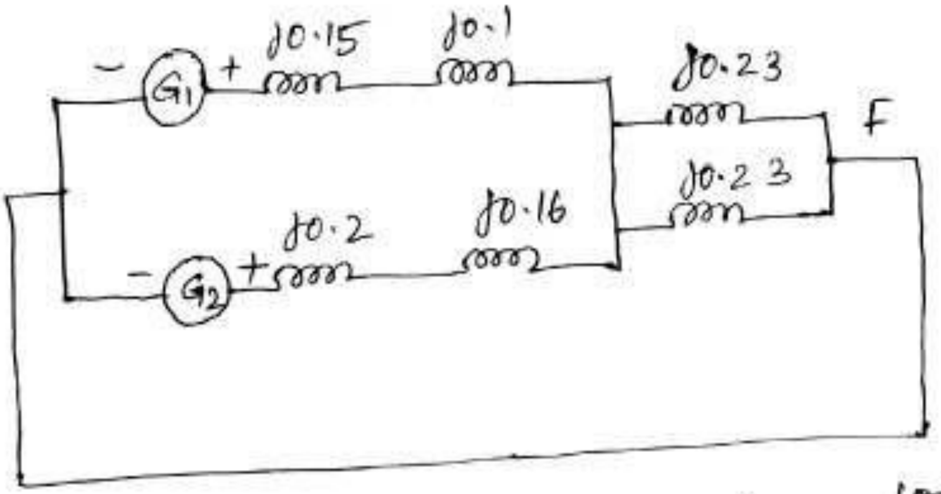


fig-1

Base current for 132 kV side $= \dfrac{100 \times 1000}{\sqrt{3} \times 132}$

$= 437.4\ A$

The above diagram can be reduced to the following

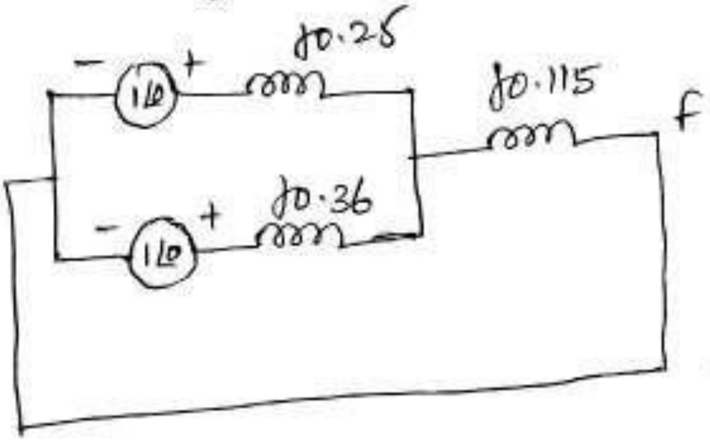


fig-2

⇓

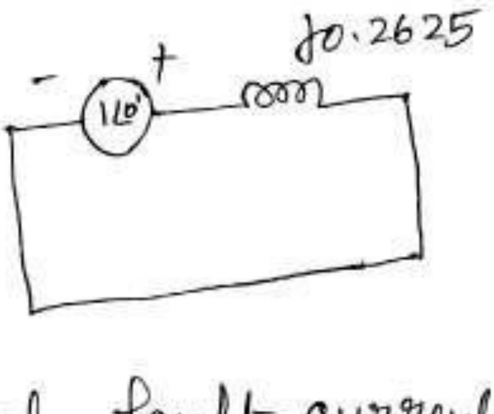


fig-3.

Total fault current $= \dfrac{1\angle 0}{j0.2625} = -j3.8095$ p.u.

$= 3.8095 \times 437.4 \angle -90^\circ$ A

$= 1666.27 \angle -90^\circ$ A

Fault level $= 3.8095$ p.u. (or)

$3.8095 \times 100 = 380.95$ MVA.

Base current for 11 kV side of transformer

$$= \frac{100 \times 1000}{\sqrt{3} \times 11}$$

$= 5248.8$ A

Total fault current supplied by the two generators

$= -j\,3.8095 \times 5248.8$

$= 19995 \angle -90^\circ$ A

From fig-2

Fault current supplied by Generator, G_1

$$= \frac{(19995 \angle -90^\circ)(j0.36)}{j0.25 + j0.36}$$

$= 11800.3 \angle -90^\circ$ A.

Fault current supplied by generator G_2

$$= (19995 \angle -90^\circ) - (11800.3 \angle -90^\circ).$$

$$= 8194.7 \angle -90^\circ \text{ A}.$$

—.

EE6501 - Power System Analysis

UNIT – IV FAULT ANALYSIS –UNBALANCED FAULT

PART – A

1. Name the faults involving ground.

The faults involving ground are:single line to ground fault ii)double line to ground faultiii)Three phase fault

2. Define positive sequence impedance.

The negative sequence impedance of equipment is the impedance offered by the equipment to the flow of positive sequence currents.

3. In what type of fault the +ve sequence component of current is equal in magnitude but opposite in phase to negative sequence components of current?

Line to line fault.

4. In which fault the negative and zero sequence currents are absent?

In three phase fault the negative and zero sequence currents are absent.

5. What are the boundary condition in line-to-line fault?

$$I_a=0;\ I_a+I_c=0;\ V_b=V_c$$

6. Write down the boundary condition in double line to ground fault?

$$I_a=0;\ V_b=0;\ V_c=0$$

7. Give the boundary condition for the 3-phase fault.

$$I_a + I_b = I_c=0;\ V_a=V_b=V_c=0$$

8. Name the fault in which positive, -ve and zero sequence component currents are equal.(May 2012)

In single line to ground fault the +ve, -ve and zero sequence component currents are equal.

9.Name the various unsymmetrical faults in a power system.

i) single line to ground fault ii) line to line fault iii) double line to ground fault iv) open conductor fault.

10.Write a short notes on Zero sequence network.

While drawing the zero sequence network of a given power system, the following points may be kept in view.The zero sequence currents will flow only if there is a return path i.e.path from neutral to ground or to another point in the circuit.In the case of a system with no return path for zero sequence currents, these currents cannot exist.

11.Write a short notes on negative sequence network.

The negative sequence network can be readiley obtained from positive sequence network with the following modifications:i)Omit the emfs of 3 – phase generators and motors in the positive sequence network. It is because these devices have only positive sequence generated voltages.ii)Change, if neccesary, the impedances between the generators neutral and ground pass no negative sequence current and hence are not included in the negative seqeunce network.iii)For static devices such as transmission lines and transformers, the negative sequence impedances have the same value as the corresponding positive sequence impedances.

12.Write a short notes on positive sequence network.

While drawing the positive sequence network of a given power system, the following points may be kept in view: i) Each generator in the system is represented by the generated voltage in series with appropriate reactance and resistance. ii) Current limiting impedances between the generators neutral and ground pass no positive sequence current and hence are not included in the positive sequence network. Iii) All resistance and magnetizing currents for each transformer are neglected as a matter of simplicity.For transmission lines, the shunt capacitances and resistances are generally neglected.

13. Which is the most frequently occurring fault?
Single line to ground fault is the most frequently occurring fault

14. Define unsymmetrical fault.
The fault is called unsymmetrical fault if the fault current isnot same in all the three phases.

15. Which is the most severe fault in power system?
Three phase fault is the most severe and rarely occurring fault in the power system.

16. What is sequence network?(May 2011) (Nov 2014)
The network which is used to represent the positive, negative and zero sequence components of unbalanced system is called as sequence network

17. What are the symmetrical components of a three phase system? (May 2011)(Nov 2012) (Nov 2014) (Nov 2015)(May 2016)
1) Positive sequence 2) negative sequence 3) Zero sequence

18. What is meant by a Fault?(May 2012)
A fault in acircuit is any failure which interferes with the normal flow of current .The faults are associated with abnormal change in current, voltage and frequency of the power system. The faults may cause damage to the equipment if it is allowed to persist for a long time.

19. List the various symmetrical and unsymmetrical faults in a power system.(May 2012)
Symmetricalfault:3 phase short circuit fault.
Unsymmetrical fault: i) single line to ground fault ii) line to line fault iii) double line to ground fault iv)open conductor fault

20. Define negative sequence impedance?(May 2013)
The negative sequence impedance of an equipment is the impedance offered by the equipment to the flow of negative sequence current.

21. What are the observations made from the analysis of various faults?(Nov 2013)
i) To check the MVA ratings of the existing circuit breakers, when new generation are added into a system; ii) To select the rating for fuses, circuit breaker and switch gear in addition to setting up of protective relays; iii) To determine the magnitudes of currents flowing throughout the power system at various time intervals after a fault occurs.

22. Write the boundary conditions for single line to ground fault.(Nov 2013)
The boundary conditions are Va = 0; I_b=I_c=0

23.What are the features of zero sequence current?(May 2014)
As zero sequence currents in three phases are equal and of same phase, three systems operate like single phase as regards zero sequence currents.Zero sequence currents flow only if return path is available through which circuit is completed.

24.Write the symmetrical component current of phase 'a' in terms of 3ϕ currents.(May 2016).

25. State the reason why, the negative sequence impedance of a transmission line is taken as equal to positive sequence impedance of the line. (May 2015).
A transmission line is a passive and bilateral device. By passive, we mean there are no voltage or current sources present in the equivalent model of a transmission line. Bilateral means the line behaves the same way regardless of the direction of the current. Note that although a single transmission line is bilateral. Because of a transmission line's passive and bilateral properties, the phase sequence of the applied voltage makes no difference, as a-b-c (positive-sequence) voltages produce the same voltage drops as a-c-b (negative-sequence) voltages. This means that the positive- and negative-sequence impedances of a transmission line are identical, provided that the line is transposed. Transposition means physically exchanging the position of each phase conductor along the length of the line such that conductor #1 occupies: position #1 for 1/3 of the line length, position #2 for 1/3 of the line length, and position #3 for 1/3 of the line length. Conductors #2 and #3 are rotated similarly.

PART - B

1. A 50Hz, 13.2 KV, 15MVA alternator has X1 = X2 = 20% and X0 = 8% and the neutral is grounded through a reactor of 0.5 ohm. Determine the initial symmetrical rms current in the ground reactor when a double line to ground fault occurs at the generator terminals at a time when the generator voltage was 12 kV.

Solution:

Base MVA=15MVA,Base KV=13.2KV

E_a(p.u)=0.909p.u

Z_1=j0.2 p.u,Z_2=j0.2p.u,Z_{g0}=j0.08p.u

X_n=0.5Ω,X_n(p.u)=j0.043 p.u

$Z_0=Z_{g0}+3Z_n$=j0.08+3xj0.043=j0.21p.u

I_{a1}= -j3p.u I_{a2}=j1.545p.u

I_{a0}=j1.47p.u I_b=-3.938+j2.197

I_c=3.938+j2.197 $I_n=I_b+I_c=4.394\angle 90^o$

I_n=4.394 X base current

Base current=656 A

There fore I_n=2882.46A

2. A 3-phase, 10 MVA, 11KV, generator with solidity earthed neutral point supplies a feeder. The relevant impedances of the generator and feeder in ohm are as below:

	Generator	Feeder
(a) +ve sequence	**j1.2**	**j1.0**
(b)-ve sequence	**j0.8**	**j1.0**
(c) zero sequence	**j0.4**	**j3.0**

If the line to line fault occurs at the far end of the feeder, calculate the fault current.

Solution:

E_a=6350V

The total impedances are

Z_1=j2.2Ω

Z_2=j1.8Ω

Z_3=j3.4Ω

For line to ground fault,

$I_1=I_2=I_0$=-j858.10A

Fault Current,$I_a=3I_0$=-j2574.32A.

The line-to-neutral Voltage of a-Phase V_a=4319.6V

3. **A salient pole generator is rated 20 MVA, 13.8 kV and has X1=0.25p.u X2=0.35p.u and X0=0.1p.u. The neutral of the generator is solidly grounded. Compute fault current in the generator and line to line to ground fault at its terminals. Neglect initial load on the generator. The reactance of a generator are X''=X2=0.15p.u and X0=0.05p.u. The generator ratings are 10 MVA, 6KV. The generator is star connected with neutral point grounded through a reactor of 0.5ohm reactance. Compute fault current in amps when a single line to ground fault occurs at the generator terminals.**

<u>Solution:</u>

I_{a1}= -j1.667p.u

I_{a2}= -I_{a1}= j1.667p.u

I_{a0}=0 $I_a=I_{a0}+I_{a1}+I_{a2}=0$

I_b= -2.866 p.u

I_c= -I_b= 2.866 p.u

Base Current =837 A

Therefore
I_a=0

I_b= 2416∟180°

I_c= 2416∟0°

Line to ground voltages are
V_a=1.166
V_b=-0.5983p.u

Line to line voltages are
V_{ab}=1.749∟0°p.u
V_{bc}=0p.u
V_{ca}=1.749∟180°p.u

4. **A11 kV, 30MVA alternator has Z1=Z2=-j0.2 pu and Z0=-j0.05 pu. A line to ground fault occurs on the generator terminals. Determine the fault current and line to line voltages during faulted conditions. Assume that the generator neutral is solidly grounded and the generator is operating at no load and at the rated voltage during the occurrence of the fault. (May 2012)**

<u>Solution:</u>
Base MVA=30MVA,Base Voltage=11KV

Base Current=1574.6A

Z_f=0

I_{a1}= I_{a2}= I_{a0}=-j2.222pu

Fault Current=3I_{a1}=-j6.666p.u=10496.3∟-90^{0}A

Line to line voltages are

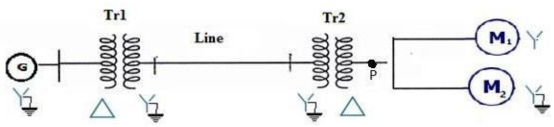
$V_{ab}=5.6\angle 79.1^{o}KV$

$V_{bc}=11\angle 270^{o}KV$

$V_{ca}=5.6\angle 100.9^{o}KV$

Actual value of line current$=7873\angle 90^{o}A$

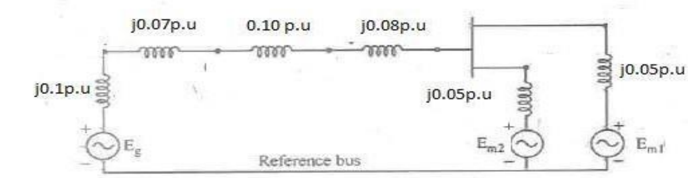
5. A single line diagram of a power network is shown in the figure. (May 2013)



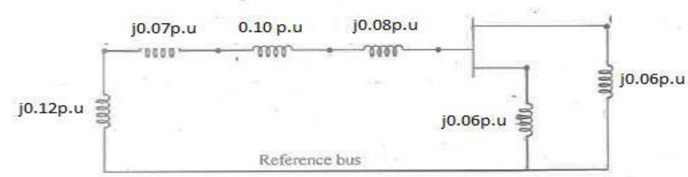
The system data is given in the tables below:

Element	**Positive sequence reactance**	**Negative sequence reactance**	**Zero sequence reactance**
Generator G	**0.1**	**0.12**	**0.05**
Motor M_1	**0.05**	**0.06**	**0.025**
Motor M_2	**0.05**	**0.06**	**0.025**
Transformer T_{r1}	**0.07**	**0.07**	**0.07**
Transformer T_{r2}	**0.08**	**0.08**	**0.08**
Line	**0.10**	**0.10**	**0.10**

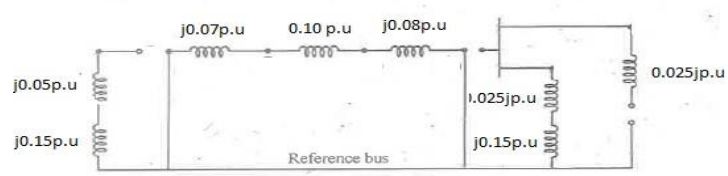
Generator grounding reactance is 0.5 pu. Draw sequence networks and calculate the fault for a line to line fault on phase b and c at point P. Assume 1.0 pu pre fault voltage throughout.

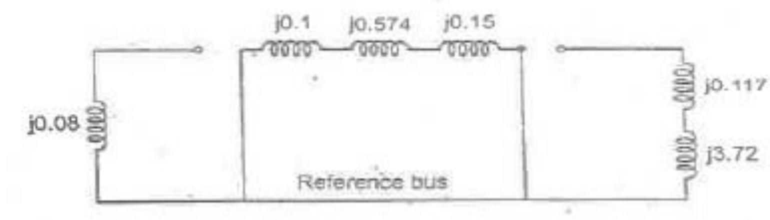


Negative Sequence Network



Zero Sequence Network





6. A 25 MVA , 13.2 KV alternator with solidly grounded neutral has a sub transient reactance of 0.25 p.u. The negative and zero sequence reactance are 0.35 and 0.01 p.u. respectively. If a double line-to-ground fault occurs at the terminals of the alternator, determine the fault current and line to line voltage at the fault. (May 2014)

Solution:

Base MVA=25MVA,Base KV=13.2KV

E_a(p.u)=0.909p.u

Z_1=j0.25 p.u,Z_2=j0.35p.u,Z_{g0}=j0.1p.u

X_n=0.1p.u,

I_{a1}= -j2.29p.u

I_{a2}=j1.225p.u

I_{a0}=j1.072p.u

I_b=-3.04+j1.6045

I_c=3.04+j1.6045

I_n=I_b+I_c=j3.208 p.u

I_n=4209A

7. Obtain the expression for fault current for a line to line fault taken place through an impedance Z_b in a power system.(Nov 2013)(May 2014)

$$V_b - V_c = Z_f I_b \tag{10.64}$$
$$I_b + I_c = 0 \tag{10.65}$$
$$I_a = 0 \tag{10.66}$$

Substituting for $I_a = 0$, and $I_c = -I_b$, the symmetrical components of currents from (10.14) are

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \tag{10.67}$$

From the above equation, we find that

$$I_a^0 = 0 \tag{10.68}$$
$$I_a^1 = \frac{1}{3}(a - a^2)I_b \tag{10.69}$$
$$I_a^2 = \frac{1}{3}(a^2 - a)I_b \tag{10.70}$$

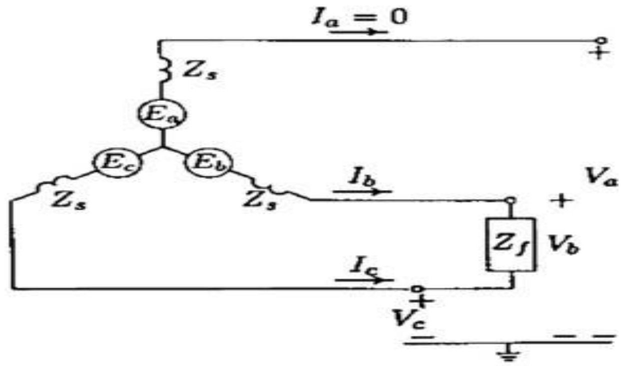


FIGURE 10.12
Line-to-line fault between phase b and c.

Also, from (10.69) and (10.70), we note that

$$I_a^1 = -I_a^2$$

From (10.16), we have

$$\begin{aligned} V_b - V_c &= (a^2 - a)(V_a^1 - V_a^2) \\ &= Z_f I_b \end{aligned}$$

Substituting for V_a^1 and V_a^2 from (10.54) and noting $I_a^2 = -I_a^1$, we get

$$(a^2 - a)[E_a - (Z^1 + Z^2)I_a^1] = Z_f I_b$$

Substituting for I_b from (10.69), we get

$$E_a - (Z^1 + Z^2)I_a^1 = Z_f \frac{3I_a^1}{(a - a^2)(a^2 - a)}$$

Since $(a - a^2)(a^2 - a) = 3$, solving for I_a^1 results in

$$I_a^1 = \frac{E_a}{Z^1 + Z^2 + Z_f}$$

The phase currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_a^1 \\ -I_a^1 \end{bmatrix}$$

The fault current is

$$I_b = -I_c = (a^2 - a)I_a^1$$

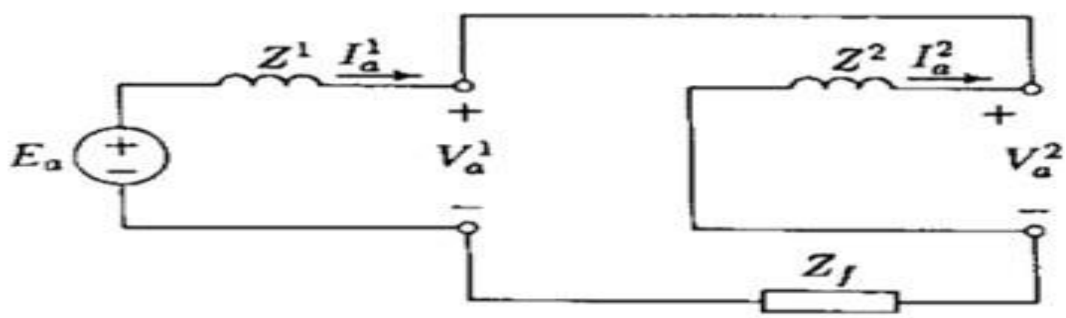


FIGURE 10.13
Sequence network connection for line-to-line fault.

8. A 30 MVA, 11Kv, 3ϕ synchronous generator has a direct sub transient reactance of 0.25 pu. The negative and zero sequence reactance are 0.35 and 0.1pu respectively. The neutral of the generator is solidly grounded. Determine the sub transient current in the generator and the line to line voltages for sub transient conditions when a single line to ground fault occurs at the generator terminals with the generator operating unloaded at rated voltages. (Nov 2015)

<u>Solution:</u>

Z^1 = j0.25 p.u Z^2 = j0.35 p.u Z^0 = j0.1 p.u Prefault voltage $V^0 = 1\angle 0^0$

Symmetrical components of Fault Current $I_a^1 = I_a^2 = I_a^0$ = -j1.4286 p.u

Fault Current in p.u = 3 I_a^1 = -j4.2857 p.u

Base Current = 1574.59 Amp

Subtransient Current or Phase Currrent:

I_a= -j4.2858 x 1574.59 =-j6748.37 Amp I_b=0 I_c=0

Symmetrical Component of Bus Voltages for Phase a:

V_a^1 = -0.1429 p.u

V_a^2 = 0.6429 p.u

V_a^0 = -0.5 p.u

Subtransient phase voltages:

V_a =0

V_b = -0.2144 – j0.9898

V_c = -0.2144 + j0.9898

Line to Line Volage :

V_{ab}=0.2144 +j0.9898p.u

V_{bc} = -j1.9796p.u

V_{ac} = -0.2144 –j 0.9898 p.u

9. Derive the expression to calculate the fault current for single-line-to-ground fault.

Let a 1LG fault has occurred at node *k* of a network. The faulted segment is then as shown in Fig. 8.2 where it is assumed that phase-a has touched the ground through an impedance Z_f. Since the system is unloaded before the occurrence of the fault we have

$$I_{fb} = I_{fc} = 0 \tag{8.1}$$

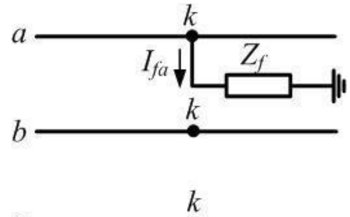


Fig. 8.2 Representation of 1LG fault.

Also the phase-a voltage at the fault point is given by

$$V_{ka} = Z_f I_{fa} \tag{8.2}$$

From (8.1) we can write

$$I_{fa012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix} \tag{8.3}$$

Solving (8.3) we get

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{I_{fa}}{3} \tag{8.4}$$

This implies that the three sequence currents are in series for the 1LG fault. Let us denote the zero, positive and negative sequence Thevenin impedance at the faulted point as Z_{kk0}, Z_{kk1} and Z_{kk2} respectively. Also since the Thevenin voltage at the faulted phase is V_f we get three sequence circuits that are similar to the ones shown in Fig. 7.7. We can then write

$$\begin{aligned} V_{ka0} &= -Z_{kk0} I_{fa0} \\ V_{ka1} &= V_f - Z_{kk1} I_{fa1} \\ V_{ka2} &= -Z_{kk2} I_{fa2} \end{aligned} \tag{8.5}$$

Then from (8.4) and (8.5) we can write

$$\begin{aligned} V_{ka} &= V_{ka0} + V_{ka1} + V_{ka2} \\ &= V_f - \left(Z_{kk0} + Z_{kk1} + Z_{kk2}\right) I_{fa0} \end{aligned} \tag{8.6}$$

Again since

$$V_{ka} = Z_f I_{fa} = Z_f \left(I_{fa0} + I_{fa1} + I_{fa2}\right) = 3Z_f I_{fa0}$$

we get from (8.6)

$$I_{fa0} = \frac{V_f}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f} \tag{8.7}$$

The Thevenin equivalent of the sequence network is shown in Fig. 8.3.

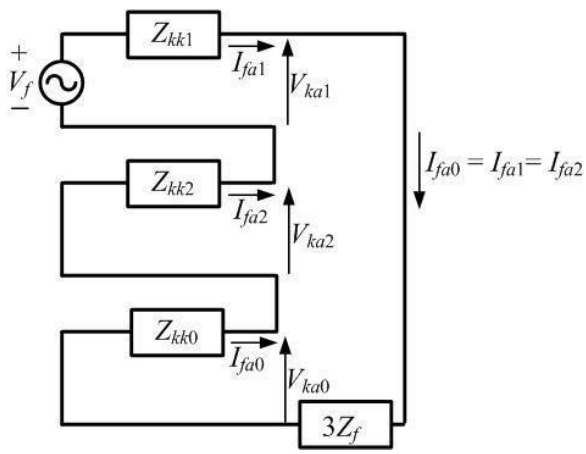


Fig. 8.3 Thevenin equivalent of a 1LG fault.

10. A three-phase Y-connected synchronous generator is running unloaded with rated voltage when a 1LG fault occurs at its terminals. The generator is rated 20 kV, 220 MVA, with subsynchronous reactance of 0.2 per unit. Assume that the subtransient mutual reactance between the windings is 0.025 per unit. The neutral of the generator is grounded through a 0.05 per unit reactance. The equivalent circuit of the generator is shown in Fig. 8.4. We have to find out the negative and zero sequence reactances.

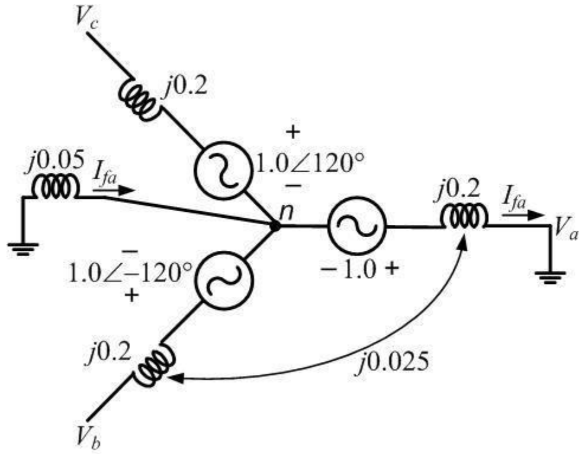


Fig. 8.4 Unloaded generator of Example 8.1.

Since the generator is unloaded the internal emfs are

$$E_{an} = 1.0 \quad E_{bn} = 1.0\angle -120° \quad E_{cn} = 1.0\angle 120°$$

Since no current flows in phases b and c, once the fault occurs, we have from Fig. 8.4

$$I_{fa} = \frac{1}{j(0.2 + 0.05)} = -j4.0$$

Then we also have

$$V_n = -X_n I_{fa} = -0.2$$

From Fig. 8.4 and (7.34) we get

$$\begin{aligned} V_a &= 0 \\ V_b &= E_{bn} + V_n + j0.025 I_{fa} = -0.6 - j0.866 = 1.0536\angle -124.72° \\ V_c &= E_{cn} + V_n + j0.025 I_{fa} = -0.6 + j0.866 = 1.0536\angle 124.72° \end{aligned}$$

Therefore

$$V_{a012} = C \begin{bmatrix} 0 \\ 1.0536\angle -124.72° \\ 1.0536\angle 124.72° \end{bmatrix} = \begin{bmatrix} -0.4 \\ 0.7 \\ -0.3 \end{bmatrix}$$

From (7.38) we can write $Z_1 = j\omega(L_s + M_s) = j0.225$. Then from Fig. 7.7 we have

$$I_{fa1} = \frac{E_{an} - V_{a1}}{Z_1} = \frac{1 - 0.7}{j0.225} = -j1.3333$$

Also note from (8.4) that

$$I_{fa0} = I_{fa1} = I_{fa2}$$

Therefore from Fig. 7.7 we get

$$Z_{g0} = -\frac{V_{a0}}{I_{a0}} - 3Z_n = j(0.3 - 0.15) = j0.15$$

$$Z_2 = -\frac{V_{a2}}{I_{a2}} = j0.225$$

Comparing the above two values with (7.37) and (7.39) we find that Z_0 indeed is equal to $j\omega(L_s - 2M_s)$ and Z_2 is equal to $j\omega(L_s + M_s)$. Note that we can also calculate the fault current from (8.7) as

$$I_{fa0} = \frac{1}{j(0.225 + 0.225 + 0.15 + 3\times 0.05)} = -j1.3333$$

ΔΔΔ

11. Derive the expression to calculate the fault current for line-to-line fault.

The faulted segment for an L-L fault is shown in Fig. 8.5 where it is assumed that the fault has occurred at node k of the network. In this the phases b and c got shorted through the impedance Z_f. Since the system is unloaded before the occurrence of the fault we have

$$I_{fa} = 0 \tag{8.8}$$

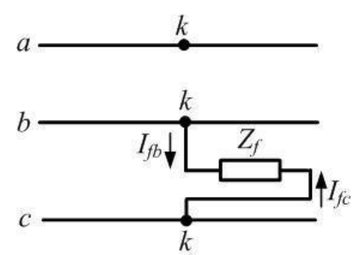


Fig. 8.5 Representation of L-L fault.

Also since phases b and c are shorted we have

$$I_{fb} = -I_{fc} \tag{8.9}$$

Therefore from (8.8) and (8.9) we have

$$I_{fa012} = C\begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ (a - a^2)I_{fb} \\ (a^2 - a)I_{fb} \end{bmatrix} \tag{8.10}$$

We can then summarize from (8.10)

$$
\begin{aligned}
I_{fa0} &= 0 \\
I_{fa1} &= -I_{fa2}
\end{aligned}
\tag{8.11}
$$

Therefore no zero sequence current is injected into the network at bus k and hence the zero sequence remains a dead network for an L-L fault. The positive and negative sequence currents are negative of each other.

Now from Fig. 8.5 we get the following expression for the voltage at the faulted point

$$
V_{kb} - V_{kc} = Z_f I_{fb} \tag{8.12}
$$

Again

$$
\begin{aligned}
V_{kb} - V_{kc} &= V_{kb0} + V_{kb1} + V_{kb2} - V_{kc0} - V_{kc1} - V_{kc2} \\
&= \left(V_{kb1} - V_{kc1}\right) + \left(V_{kb2} - V_{kc2}\right) \\
&= \left(a^2 - a\right)V_{ka1} + \left(a - a^2\right)V_{ka2} \\
&= \left(a^2 - a\right)\left(V_{ka1} - V_{ka2}\right)
\end{aligned}
\tag{8.13}
$$

Moreover since $I_{fa0} = I_{fb0} = 0$ and $I_{fa1} = -I_{fb2}$, we can write

$$
I_{fb} = I_{fb1} + I_{fb2} = a^2 I_{fa1} + aI_{fb2} = \left(a^2 - a\right)I_{fa1} \tag{8.14}
$$

Therefore combining (8.12)-(8.14) we get

$$
V_{ka1} - V_{ka2} = Z_f I_{fa1} \tag{8.15}
$$

Equations (8.12) and (8.15) indicate that the positive and negative sequence networks are in parallel. The sequence network is then as shown in Fig. 8.6. From this network we get

$$
I_{fa1} = -I_{fa2} = \frac{V_f}{Z_{kk1} + Z_{kk2} + Z_f} \tag{8.16}
$$

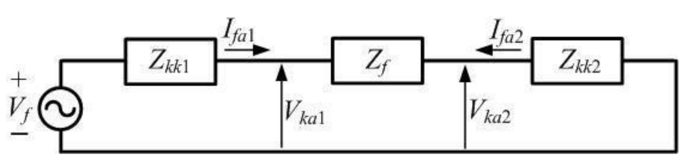


Fig. 8.6 Thevenin equivalent of an LL fault.

12. Let us consider the same generator as given in Example 8.1. Assume that the generator is unloaded when a bolted (Z_f = 0) short circuit occurs between phases b and c. Then we get from (8.9) $I_{fb} = -I_{fc}$. Also since the generator is unloaded, we have I_{fa} = 0. Therefore from (7.34) we get

$$
\begin{aligned}
V_{an} &= E_{an} = 1.0 \\
V_{bn} &= E_{bn} - j0.225 I_{fb} = 1\angle -120^\circ - j0.225 I_{fb} \\
V_{cn} &= E_{cn} - j0.225 I_{fc} = 1\angle 120^\circ + j0.225 I_{fb}
\end{aligned}
$$

Also since $V_{bn} = V_{cn}$, we can combine the above two equations to get

$$
I_{fb} = -I_{fc} = \frac{1\angle -120^\circ - 1\angle 120^\circ}{j0.45} = -3.849
$$

Then

$$I_{fa012} = C\begin{bmatrix} 0 \\ -3.849 \\ 3.849 \end{bmatrix} = \begin{bmatrix} 0 \\ -j2.2222 \\ j2.2222 \end{bmatrix}$$

We can also obtain the above equation from (8.16) as

$$I_{fa1} = -I_{fa2} = \frac{1}{j0.225 + j0.225} = -j2.2222$$

Also since the neutral current I_n is zero, we can write $V_a = 1.0$ and

$$V_b = V_c = V_{bn} = -0.5$$

Hence the sequence components of the line voltages are

$$V_{a012} = C\begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

Also note that

$$V_{a1} = 1.0 - j0.225 I_{fa1} = 0.5$$
$$V_{a2} = -j0.225 I_{fa2} = 0.5$$

which are the same as obtained before.

ΔΔΔ

13. Derive the expression to calculate the fault current for double-line-to-ground fault.

The faulted segment for a 2LG fault is shown in Fig. 8.7 where it is assumed that the fault has occurred at node k of the network. In this the phases b and c got shorted through the impedance Z_f to the ground. Since the system is unloaded before the occurrence of the fault we have the same condition as (8.8) for the phase-a current. Therefore

$$I_{fa0} = \frac{1}{3}\left(I_{fa} + I_{fb} + I_{fc}\right) = \frac{1}{3}\left(I_{fb} + I_{fc}\right)$$
$$\Rightarrow 3I_{fa0} = I_{fb} + I_{fc} \tag{8.17}$$

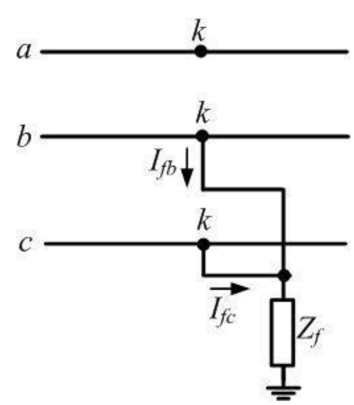


Fig. 8.7 Representation of 2LG fault.

Also the voltages of phases b and c are given by

$$V_{kb} = V_{kc} = Z_f \left(I_b + I_c\right) = 3Z_f I_{fa0} \tag{8.18}$$

Therefore

$$V_{ka012} = C\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_{ka} + 2V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \end{bmatrix} \tag{8.19}$$

We thus get the following two equations from (8.19)

$$V_{ka1} = V_{ka2} \tag{8.20}$$

$$3V_{ka0} = V_{ka} + 2V_{kb} = V_{ka0} + V_{ka1} + V_{ka2} + 2V_{kb} \tag{8.21}$$

Substituting (8.18) and (8.20) in (8.21) and rearranging we get

$$V_{ka1} = V_{ka2} = V_{ka0} - 3Z_f I_{fa0} \tag{8.22}$$

Also since $I_{fa} = 0$ we have

$$I_{fa0} + I_{fa1} + I_{fa2} = 0 \tag{8.23}$$

The Thevenin equivalent circuit for 2LG fault is shown in Fig. 8.8. From this figure we get

$$I_{fa1} = \frac{V_f}{Z_{kk1} + Z_{kk2} \| \left(Z_{kk0} + 3Z_f\right)} = \frac{V_f}{Z_{kk1} + \dfrac{Z_{kk2}\left(Z_{kk0} + 3Z_f\right)}{Z_{kk2} + Z_{kk0} + 3Z_f}} \tag{8.24}$$

The zero and negative sequence currents can be obtained using the current divider principle as

$$I_{fa0} = -I_{fa1}\left(\frac{Z_{kk2}}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \tag{8.25}$$

$$I_{fa2} = -I_{fa1}\left(\frac{Z_{kk0} + 3Z_f}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \tag{8.26}$$

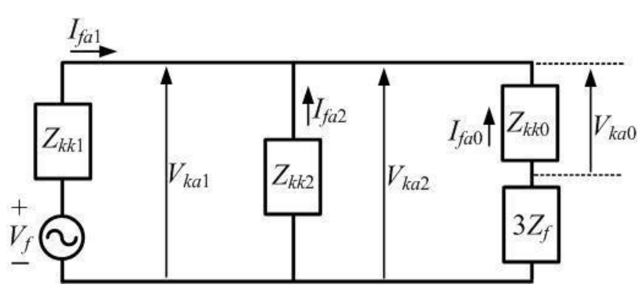


Fig. 8.8 Thevenin equivalent of a 2LG fault.

14. Let us consider the same generator as given in Examples 8.1 and 8.2. Let us assume that the generator is operating without any load when a bolted 2LG fault occurs in phases b and c. The equivalent circuit for this fault is shown in Fig. 8.9. From this figure we can write

$$E_{bn} + V_n = 1\angle -120° + V_n = j0.2 I_{fb} - j0.025 I_{fc}$$
$$E_{cn} + V_n = 1\angle 120° + V_n = j0.2 I_{fc} - j0.025 I_{fb}$$
$$V_n = -j0.05\left(I_{fb} + I_{fc}\right)$$

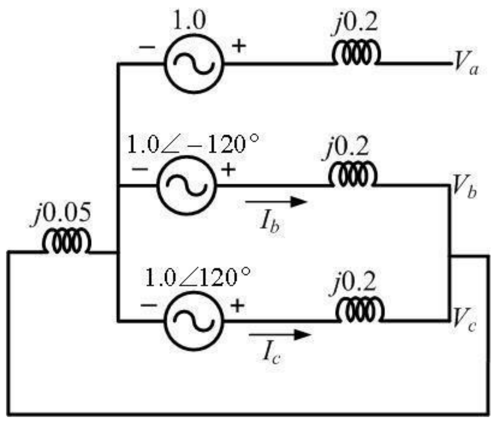


Fig. 8.9 Equivalent circuit of the generator in Fig. 8.4 for a 2LG fault in phases b and c.

Combining the above three equations we can write the following vector-matrix form

$$j\begin{bmatrix} 0.25 & 0.025 \\ 0.025 & 0.25 \end{bmatrix}\begin{bmatrix} I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1\angle -120° \\ 1\angle 120° \end{bmatrix}$$

Solving the above equation we get

$$I_b = -3.849 + j1.8182$$
$$I_c = 3.849 + j1.8182$$

Hence

$$i_{fa012} = C\begin{bmatrix} 0 \\ -3.849 + j1.8182 \\ -3.849 + j1.8182 \end{bmatrix} = \begin{bmatrix} j1.2121 \\ -j2.8283 \\ j1.6162 \end{bmatrix}$$

We can also obtain the above values using (8.24)-(8.26). Note from Example 8.1 that

$$Z_1 = Z_2 = j0.225,\ Z_0 = j\left(0.15 + 3\times 0.05\right) = j0.3 \text{ and } Z_f = 0$$

Then

$$I_{fa1} = \frac{1.0}{j0.225 + \left(\dfrac{j0.225\times j0.3}{j0.525}\right)} = -j2.8283$$

$$I_{fa2} = -I_{fa1}\frac{j0.3}{j0.525} = j1.6162$$

$$I_{fa0} = -I_{fa1}\frac{j0.225}{j0.525} = j1.2121$$

Now the sequence components of the voltages are

$$V_{a1} = 1.0 - j0.225 \times I_{fa1} = 0.3636$$
$$V_{a2} = -j0.225 \times I_{fa2} = 0.3636$$
$$V_{a0} = -j0.3 \times I_{fa0} = 0.3636$$

Also note from Fig. 8.9 that

$$V_a = E_{an} + V_n + j0.0225\left(I_{fb} + I_{fc}\right) = 1.0909$$

and $V_b = V_c = 0$. Therefore

$$V_{a012} = C\begin{bmatrix} 1.0909 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.3636 \\ 0.3636 \\ 0.3636 \end{bmatrix}$$

which are the same as obtained before.

ΔΔΔ

15. Consider the network shown in Fig. 8.10. The system parameters are given below:

Generator G: 50 MVA, 20 kV, $X'' = X_1 = X_2 = 20\%$, $X_0 = 7.5\%$
Motor M: 40 MVA, 20 kV, $X'' = X_1 = X_2 = 20\%$, $X_0 = 10\%$, $X_n = 5\%$
Transformer T_1: 50 MVA, 20 kVΔ/110 kVY, $X = 10\%$
Transformer T_2: 50 MVA, 20 kVΔ/110 kVY, $X = 10\%$
Transmission line: $X_1 = X_2 = 24.2\ \Omega$, $X_0 = 60.5\ \Omega$

We shall find the fault current for when a (a) 1LG, (b) LL and (c) 2LG fault occurs at bus-2.

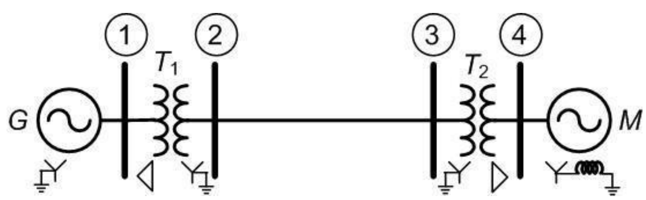


Fig. 8.10 Radial power system of Example 8.4.

Let us choose a base in the circuit of the generator. Then the per unit impedances of the generator are:

$$X_{G1} = X_{G2} = 0.2, \quad X_{G0} = 0.075$$

The per unit impedances of the two transformers are

$$X_{T1} = X_{T2} = 0.1$$

The MVA base of the motor is 40, while the base MVA of the total circuit is 50. Therefore the per unit impedances of the motor are

$$X_{M1} = X_{M2} = 0.2 \times \frac{50}{40} = 0.25, \quad X_{M0} = 0.1 \times \frac{50}{40} = 0.125, \quad X_n = 0.05 \times \frac{50}{40} = 0.0625$$

For the transmission line

$$Z_{base} = \frac{110^2}{50} = 242\Omega$$

Therefore

$$X_{L1} = X_{L2} = \frac{24.2}{242} = 0.1, \quad X_{L0} = \frac{60.5}{242} = 0.25$$

Let us neglect the phase shift associated with the Y/Δ transformers. Then the positive, negative and zero sequence networks are as shown in Figs. 8.11-8.13.

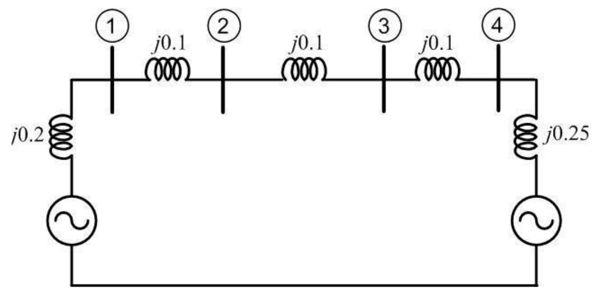


Fig. 8.11 Positive sequence network of the power system of Fig. 8.10.

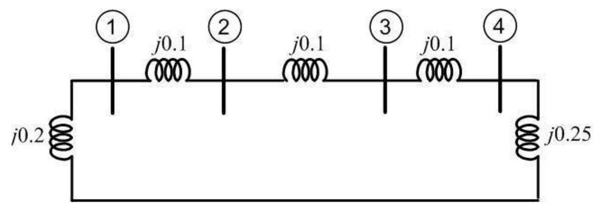


Fig. 8.12 Negative sequence network of the power system of Fig. 8.10.

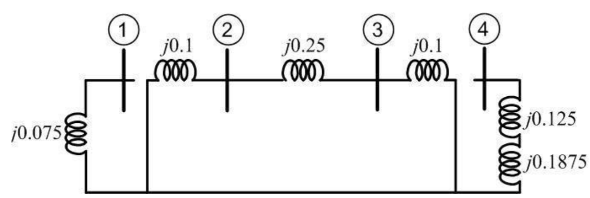


Fig. 8.13 Zero sequence network of the power system of Fig. 8.10.

From Figs. 8.11 and 8.12 we get the following Y_{bus} matrix for both positive and negative sequences

$$Y_{bus1} = Y_{bus2} = j\begin{bmatrix} -15 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 \\ 0 & 10 & -20 & 10 \\ 0 & 0 & 10 & 14 \end{bmatrix}$$

Inverting the above matrix we get the following Z_{bus} matrix

$$Z_{bus1} = Z_{bus2} = j\begin{bmatrix} 0.1467 & 0.1200 & 0.0933 & 0.0667 \\ 0.1200 & 0.1800 & 0.1400 & 0.1000 \\ 0.0933 & 0.1400 & 0.1867 & 0.1333 \\ 0.0667 & 0.1000 & 0.1333 & 0.1667 \end{bmatrix}$$

Again from Fig. 8.13 we get the following Y_{bus} matrix for the zero sequence

$$Y_{bus0} = j\begin{bmatrix} -13.3333 & 0 & 0 & 0 \\ 0 & -14 & 4 & 0 \\ 0 & 4 & -14 & 0 \\ 0 & 0 & 0 & -3.2 \end{bmatrix}$$

Inverting the above matrix we get

$$Z_{bus0} = j\begin{bmatrix} 0.075 & 0 & 0 & 0 \\ 0 & 0.0778 & 0.0222 & 0 \\ 0 & 0.0222 & 0.0778 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$$

Hence for a fault in bus-2, we have the following Thevenin impedances

$$Z_1 = Z_2 = j0.18, \quad Z_0 = j0.0778$$

Alternatively we find from Figs. 8.11 and 8.12 that

$$Z_1 = Z_2 = j0.3 \| j0.45 = j0.18$$
$$Z_0 = j0.1 \| j0.35 = j0.0778$$

***(a) Single-Line-to-Ground Fault*:** Let a bolted 1LG fault occurs at bus-2 when the system is unloaded with bus voltages being 1.0 per unit. Then from (8.7) we get

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{1}{j(2 \times 0.18 + 0.0778)} = -j2.2841 \text{ per unit}$$

Also from (8.4) we get

$$I_{fa} = 3I_{fa0} = -j6.8524 \text{ per unit}$$

Also $I_{fb} = I_{fc} = 0$. From (8.5) we get the sequence components of the voltages as

$$V_{2a0} = -j0.0778 I_{fa0} = -0.1777$$
$$V_{2a1} = 1 - j0.18 I_{fa1} = 0.5889$$
$$V_{2a2} = -j0.18 I_{fa2} = -0.4111$$

Therefore the voltages at the faulted bus are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = C^{-1}\begin{bmatrix} V_{2a0} \\ V_{2a1} \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9061\angle -107.11° \\ 0.9061\angle 107.11° \end{bmatrix}$$

***(b) Line-to-Line Fault*:** For a bolted LL fault, we can write from (8.16)

$$I_{fa1} = -I_{fa2} = \frac{1}{j2 \times 0.18} = -j2.7778 \text{ per unit}$$

Then the fault currents are

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = C^{-1}\begin{bmatrix} 0 \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \begin{bmatrix} 0 \\ -4.8113 \\ 4.8113 \end{bmatrix}$$

Finally the sequence components of bus-2 voltages are

$$V_{2a0} = 0$$
$$V_{2a1} = 1 - j0.18 I_{fa1} = 0.5$$
$$V_{2a2} = -j0.18 I_{fa2} = 0.5$$

Hence faulted bus voltages are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = C^{-1}\begin{bmatrix} V_{2a0} \\ V_{2a1} \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix}$$

***(c)Double-Line-to-Ground Fault*:** Let us assumes that a bolted 2LG fault occurs at bus-2. Then

$$Z_{eq} = j0.18 \| j0.0778 = j0.0543$$

Hence from (8.24) we get the positive sequence current as

$$I_{fa1} = \frac{1}{j0.18 + Z_{eq}} = -j4.2676 \text{ per unit}$$

The zero and negative sequence currents are then computed from (8.25) and (8.26) as

$$I_{fa0} = -I_{fa1}\frac{j0.18}{j(0.18 + 0.0778)} = j2.9797 \text{ per unit}$$
$$I_{fa2} = -I_{fa1}\frac{j0.0778}{j(0.18 + 0.0778)} = j1.2879 \text{ per unit}$$

Therefore the fault currents flowing in the line are

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{fa0} \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \begin{bmatrix} 0 \\ 6.657\angle 137.11° \\ 6.657\angle 42.89° \end{bmatrix}$$

Furthermore the sequence components of bus-2 voltages are

$$V_{2a0} = -j0.0778 I_{fa0} = 0.2318$$
$$V_{2a1} = 1 - j0.18 I_{fa1} = 0.2318$$
$$V_{2a2} = -j0.18 I_{fa2} = 0.2318$$

Therefore voltages at the faulted bus are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = C^{-1} \begin{bmatrix} V_{2a0} \\ V_{2a1} \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 0.6954 \\ 0 \\ 0 \end{bmatrix}$$

ΔΔΔ

16. Let us now assume that a 2LG fault has occurred in bus-4 instead of the one in bus-2.

Therefore

$$X_1 = X_2 = j0.1667, \qquad X_0 = j0.3125$$

Also we have

$$Z_{eq} = j0.1667 \| j0.3125 = j0.1087$$

Hence

$$I_{fa1} = \frac{1}{j0.1667 + Z_{eq}} = -j3.631 \text{ per unit}$$

Also

$$I_{fa0} = -I_{fa1} \frac{j0.1667}{j(0.1667 + 0.3125)} = j1.2631 \text{ per unit}$$
$$I_{fa2} = -I_{fa1} \frac{j0.3125}{j(0.1667 + 0.3125)} = j2.3678 \text{ per unit}$$

Therefore the fault currents flowing in the line are

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{fa0} \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \begin{bmatrix} 0 \\ 5.5298\angle 159.96° \\ 5.5298\angle 20.04° \end{bmatrix}$$

We shall now compute the currents contributed by the generator and the motor to the fault. Let us denoted the current flowing to the fault from the generator side by I_g, while that flowing from the motor by I_m. Then from Fig. 8.11 using the current divider principle, the positive sequence currents contributed by the two buses are

$$I_{ga1} = I_{fa1} \times \frac{j0.25}{j0.75} = -j1.2103 \text{ per unit}$$

$$I_{ma1} = I_{fa1} \times \frac{j0.5}{j0.75} = -j2.4206 \text{ per unit}$$

Similarly from Fig. 8.12, the negative sequence currents are given as

$$I_{ga2} = I_{fa2} \times \frac{j0.25}{j0.75} = j0.7893 \text{ per unit}$$

$$I_{ma2} = I_{fa2} \times \frac{j0.5}{j0.75} = j1.5786 \text{ per unit}$$

Finally notice from Fig. 8.13 that the zero sequence current flowing from the generator to the fault is 0. Then we have

$$I_{ga0} = 0$$

$$I_{ma0} = j1.2631 \text{ per unit}$$

Therefore the fault currents flowing from the generator side are

$$\begin{bmatrix} I_{ga} \\ I_{gb} \\ I_{gc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{ga0} \\ I_{ga1} \\ I_{ga2} \end{bmatrix} = \begin{bmatrix} 0.4210\angle -90^\circ \\ 1.7445\angle 173.07^\circ \\ 1.7445\angle 6.93^\circ \end{bmatrix}$$

and those flowing from the motor are

$$\begin{bmatrix} I_{ma} \\ I_{mb} \\ I_{mc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{ma0} \\ I_{ma1} \\ I_{ma2} \end{bmatrix} = \begin{bmatrix} 0.4210\angle 90^\circ \\ 3.8512\angle 154.07^\circ \\ 3.8512\angle 25.93^\circ \end{bmatrix}$$

It can be easily verified that adding I_g and I_m we get I_f given above.

ΔΔΔ

In the above two examples we have neglected the phase shifts of the Y/Δ transformers. However according to the American standard, the positive sequence components of the high tension side lead those of the low tension side by 30°, while the negative sequence behavior is reverse of the positive sequence behavior. Usually the high tension side of a Y/Δ transformer is Y-connected. Therefore as we have seen in Fig. 7.16, the positive sequence component of Y side leads the positive sequence component of the Δ side by 30° while the negative sequence component of Y side lags that of the Δ side by 30°. We shall now use this principle to compute the fault current for an unsymmetrical fault.

16. Let us consider the same system as given in Example 8.5. Since the phase shift does not alter the zero sequence, the circuit of Fig. 8.13 remains unchanged. The positive and the negative sequence circuits must however include the respective phase shifts. These circuits are redrawn as shown in Figs. 8.14 and 8.15.

Note from Figs. 8.14 and 8.15 that we have dropped the $\sqrt{3}\alpha$ vis-à-vis that of Fig. 7.16. This is because the per unit impedances remain unchanged when referred to the either high tension or low tension side of an ideal transformer. Therefore the per unit impedances will also not be altered.

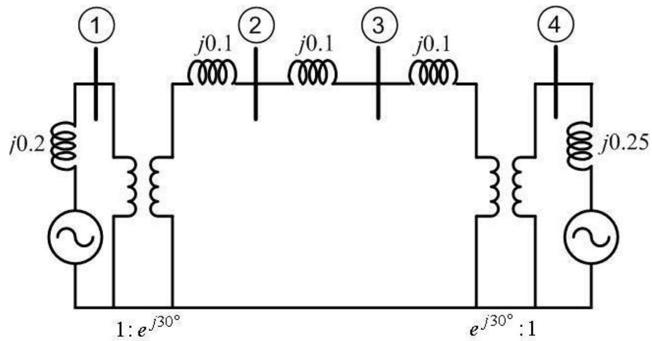


Fig. 8.14 Positive sequence network of the power system of Fig. 8.10 including transformer phase shift.

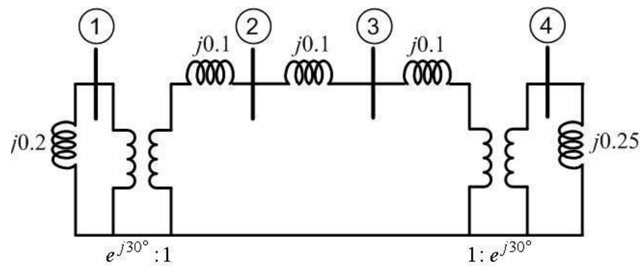


Fig. 8.15 Negative sequence network of the power system of Fig. 8.10 including transformer phase shift.

Since the zero sequence remains unaltered, these currents will not change from those computed in Example 8.6. Thus

$$I_{ga0} = 0 \text{ and } I_{ma0} = j1.2631 \text{ per unit}$$

Now the positive sequence fault current from the generator I_{ga1}, being on the Y-side of the Y/Δ transformer will lead I_{ma1} by 30°. Therefore

$$I_{ga1} = -j1.2103 \times 1\angle 30° = 1.2103\angle -60° \text{ per unit}$$

$$I_{ma1} = -j2.4206 \text{ per unit}$$

Finally the negative sequence current I_{ga2} will lag I_{ma2} by 30°. Hence we have

$$I_{ga2} = j0.7893 \times 1\angle -30° = j0.7893\angle 60° \text{ per unit}$$

$$I_{ma2} = j1.5786 \text{ per unit}$$

Therefore

$$\begin{bmatrix} I_{ga} \\ I_{gb} \\ I_{gc} \end{bmatrix} = C^{-1}\begin{bmatrix} I_{ga0} \\ I_{ga1} \\ I_{ga2} \end{bmatrix} = \begin{bmatrix} 1.0642\angle -20.04° \\ 1.9996\angle -180° \\ 1.0642\angle 20.04° \end{bmatrix}$$

Also the fault currents flowing from the motor remain unaltered. Also note that the currents flowing into the fault remain unchanged. This implies that the phase shift of the Y/Δ transformers does not affect the fault currents.

ΔΔΔ

17. Let us consider the same power system as given in Example 1.2, the sequence diagrams of which are given in Figs. 7.18 to 7.20. With respect to Fig. 7.17, let us define the system parameters as:

Generator G_1:	**200 MVA, 20 kV, $X'' = 20\%$, $X_0 = 10\%$**
Generator G_2:	**300 MVA, 18 kV, $X'' = 20\%$, $X_0 = 10\%$**
Generator G_3:	**300 MVA, 20 kV, $X'' = 25\%$, $X_0 = 15\%$**
Transformer T_1:	**300 MVA, 220Y/22 kV, $X = 10\%$**
Transformer T_2:	**Three single-phase units each rated 100 MVA, 130Y/25 kV, $X = 10\%$**
Transformer T_3:	**300 MVA, 220/22 kV, $X = 10\%$**
Line B-C:	**$X_1 = X_2 = 75\ \Omega$, $X_0 = 100\ \Omega$**
Line C-D:	**$X_1 = X_2 = 75\ \Omega$, $X_0 = 100\ \Omega$**
Line C-F:	**$X_1 = X_2 = 50\ \Omega$, $X_0 = 75\ \Omega$**

Let us choose the circuit of Generator 3 as the base, the base MVA for the circuit is 300. The base voltages are then same as those shown in Fig. 1.23. Per unit reactances are then computed as shown below.

Generator G_1: $$X' = 0.2 \times \frac{300}{200} = 0.3\,,\ X_0 = 0.15$$

Generator G_2: $$X' = 0.2 \times \left(\frac{18}{22.22}\right)^2 = 0.1312\,,\ X_0 = 0.0656$$

Generator G_3: $$X' = 0.2\,,\ X_0 = 0.15$$

Transformer T_1: $$X = 0.1 \times \left(\frac{220}{200}\right)^2 = 0.121$$

Transformer T_2: $$X = 0.1 \times \left(\frac{25}{22.22}\right)^2 = 0.1266$$

Transformer T_3: $$X = 0.1 \times \left(\frac{22}{20}\right)^2 = 0.121$$

Line B-C: $$X_1 = X_2 = \frac{75}{133.33} = 0.5625\,,\ X_0 = \frac{100}{133.33} = 0.75$$

Line C-D: $$X_1 = X_2 = \frac{75}{133.33} = 0.5625\,,\ X_0 = \frac{100}{133.33} = 0.75$$

Line C-F: $$X_1 = X_2 = \frac{50}{133.33} = 0.375\,,\ X_0 = \frac{75}{133.33} = 0.5625$$

Neglecting the phase shifts of Y/Δ connected transformers and assuming that the system is unloaded, we shall find the fault current for a 1LG fault at bus-1 (point C of Fig. 7.17).

From Figs. 7.18 and 7.19, we can obtain the positive and negative sequence Thevenin impedance at point C as (verify)

$$X_1 = X_2 = j0.2723 \text{ per unit}$$

Similarly from Fig. 7.20, the Thevenin equivalent of the zero sequence impedance is

$$X_0 = j0.4369 \text{ per unit}$$

Therefore from (8.7) we get

$$I_{fa0} = \frac{1}{j(2 \times 0.2723 + 0.4369)} = -j1.0188 \text{ per unit}$$

Then the fault current is $I_{fa} = 3I_{fa0} = 3.0565$ per unit.

EE6501 - Power System Analysis

UNIT–V STABILITYANALYSIS

PART – A

1. Define Dynamic stability of a power system.

Dynamic stability is the stability given to an inherently unstable system by automatic control devices and this dynamic stability is concerned with small disturbances lasting for times of the order of 10 to 30 seconds.

2. Define the inertia constants M & H.

Angular momentum (M) about a fixed axis is defined as the product of moment of inertia about that axis and the associated angular velocity. M = I. ω watt/rad/Sec^2.Inertia constant (H) is the K.E in Mega joules to the three phase MVA rating of the machine.

3. Define load angle of a generator.

Load angle:- This is the angle between the generated e.m.f or the supply voltage (E) and the terminal voltage. This angle is also called as torque or power angle of the machine.

4. State equal area criterion of stability.

The system is stable if the area under accelerating power (Pa) - δ curve reduces to zero at some value ofδ. In other words positive area under Pa -δ curve must be equal to the negative area and hence the name equal area criterion of stability.

5. What are limitations of equal area criterion?

The limitations of equal area criterion are: i) one drawback of equal area criterion approach is that critical clearing time cannot be calculated even though the critical clearing angle is known. Hence numerical methods such as Runge-kutta method, point by point or Euler‟s method are employed.
ii) It‟s a more simplified approach.

6. If two machines with inertia's H_1, H_2 are swinging together, what will be the inertia of the equivalent machine?

H_1 and H_2 is the Inertia constant of M_1 and M_2; G_1 and G_2 is the capacity of M_1 and M_2.
H_s is the equivalent inertia of M_1 and M_2 ; G_s is the equivalent capacity of M_1 and M_2.

7. On what basis do you conclude that the given synchronous machine has lost stability?

Following a sudden disturbance on a power system rotor speeds, rotor angular differences and power transfer undergo fast changes whose magnitude is dependent on the severity of the disturbance. If these disturbances leads to growing oscillations in the power system even after some period of time say more than 30 seconds then system said are in asynchronous state and it has lost synchronism.

8. On what a factor does the critical clearing angle depends.

The critical clearing angle depends upon the clearing time, which depends upon auto closing/reclosing and opening of circuit breakers.

9. Define steady state stability limit.(Nov 2014)

It is the maximum power that can be transferred without the system becoming unstable when the system is subjected to small disturbances.

10. Mention methods of improving stability limit. (Nov 2016)

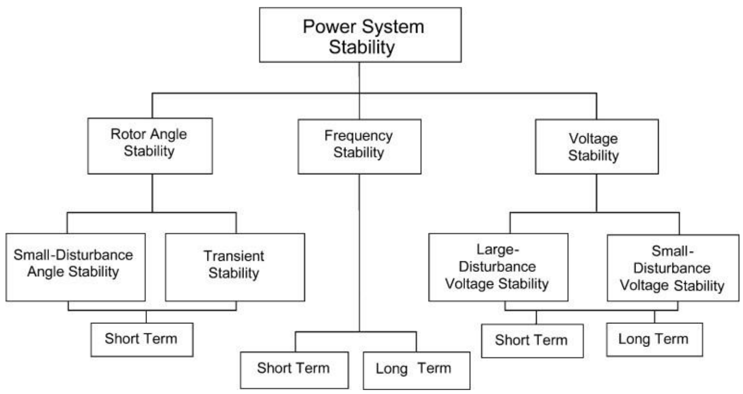
Pmax= (E.V / X). The steady state stability limit can be increased by i) Reducing the X, in case of transmission lines by using double circuit lines. ii) Use of series capacitors to get better voltage. iii) Higher excitation systems and quick excitation system are employed.
The following methods are employed to increase the transient stability limit of the power system-

(i)Increase of system voltages,(ii) use of AVR.(iii)Use of High speed excitation systems.(iv)Reduction in transfer reactance.(v)Useof high speed reclosing breakers.

11. A 50Hz, 4 pole turbo alternator rated at 20 MVA, 13.2 KV has as inertia constant H = 4 KW – sec/ KVA. Find the K.E stored in the rotor at synchronous speed.
F = 50Hz. P = 4, G = 20 MVA, H = 4 KW – Sec/ KVA. Stored K.E = 4 x 20 = 80MJ.

12. Mention the methods used for the solution of swing equation.
Methods used for solution of swing equation are: Point by point method, Modified Euler‟s method andRunge-kutta method.

13. How is the power system stability classified? (May 2015)



14. Define the term synchronizing power coefficient of a synchronous machine?
The rate(dp/dδ), ie, the differential power increase obtained per differential load angle increase is called the synchronizing power coefficient or electrical stiffness of a synchronous machine.

15. What are the applications of equal areacriterion?
(i)Switching operation. (ii)Fault and subsequent circuit isolation. (iii) Fault, circuit isolation and reclosing

16. What are the classifications of angle stability?
Small signal stability (steady state) and transient stability (large signal).Small signal is further classified as Oscillatory and Non oscillatory stability.Oscillatory includes Inter area mode, control mode and Torsional mode

17. Define critical clearing angle and time? (May 2011)(May 2012)(Nov 2012) (Nov 2014) (May15)
Critical clearing angle „δ_c "corresponds to critical clearing timet_c,in which the fault in the line is cleared by the circuit breaker above which the system goes out of synchronism.

18. Write swing equation(May 2011)
P_m-P_e= Md^2 /dt^2. P_{m-} Input Mechanical power: P_{e-} outputelectricalpower;M- Angular momentum

19. Define transient stability and stability limit.(May 2012)
The maximum power that can be transferred through the system during a very large disturbance without loss of synchronism is called transient stability limit.

20. Distinguish between steady state and transient state stability.(Nov 2012)
Steady state stability is basically concerned with the ability of the system to restore back to its stable state upon a small disturbance whereas the transient stability is concerned with large disturbances.

21. What is meant by power angle curve?(May 2013) (Nov 2015)
The graphical plot of real power versus power/torque angle is called as power angle curve.
$P_e = P_m \sin \delta$. $P_m = E_1E_2 / X$.

22. Define Infinite bus in power system. (Nov 2012)(May 2013)
The capacity of a system comprising of many machines is so large, that its voltage & frequency may be taken as constant. The connection or disconnection of a single machine does not change the |V| and frequency. Such a constant voltage and frequency system is called as Infinite bus.

23. Differentiate between voltage stability and rotor angle stability.(Nov 2013, Nov 2016)
Voltage stability is the ability of a power system to maintain steady acceptable voltage at all buses in the system under normal operating conditions and after being subjected to a disturbance.
Rotor angle stability is the ability of interconnected synchronous machines of a power system to remain in synchronism.

PART – B

1. Explain the equal area criterion for the stability studies in the power system.

Equal Area Criterion

1.0 Development of equal area criterion

As in previous notes, all powers are in per-unit.
I want to show you the equal area criterion a little differently than the book does it.
Let"s start from Eq. (2.43) in the book.

$$\frac{2H}{\omega_{\text{Re}}}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{1}$$

Note in (1) that the book calls ω_{Re} as ω_R; however, we need to use 377 (for a 60 Hz system).
We can also write (1) as

$$\frac{2H}{\omega_{\text{Re}}}\frac{d\omega}{dt} = P_m - P_e = P_a \tag{2}$$

Now multiply the left-hand-side by ω and the right-hand side by dδ/dt (recall ω= dδ/dt) to get:

$$\frac{H}{\omega_{\text{Re}}}\left\{2\omega\frac{d\omega}{dt}\right\} = \left[P_m - P_e\right]\frac{d\delta}{dt} \tag{3}$$

Note:

$$\frac{d\left(\omega(t)^2\right)}{dt} = 2\omega(t)\frac{d\omega(t)}{dt} \tag{4}$$

Substitution of (4) into the left-hand-side of (3) yields:

$$\frac{H}{\omega_{\text{Re}}}\left\{\frac{d\omega^2}{dt}\right\} = \left[P_m - P_e\right]\frac{d\delta}{dt} \tag{5}$$

Multiply by dt to obtain:

$$\frac{H}{\omega_{Re}} d\omega^2 = \left[P_m - P_e\right] d\delta \tag{6}$$

Now consider a change in the state such that the angle goes from δ_1 to δ_2 while the speed goes from ω_1 to ω_2. Integrate (6) to obtain:

$$\frac{H}{\omega_{Re}} \int_{\omega_1^2}^{\omega_2^2} d\omega^2 = \int_{\delta_1}^{\delta_2} \left[P_m - P_e\right] d\delta \tag{7}$$

Note the variable of integration on the left is ω^2. This results in

$$\frac{H}{\omega_{Re}} \left[\omega_2^2 - \omega_1^2\right] = \int_{\delta_1}^{\delta_2} \left[P_m - P_e\right] d\delta \tag{8}$$

The left-hand-side of (8) is proportional to the change in kinetic energy between the two states, which can be shown more explicitly by substituting $H=W_k/S_B=(1/2)J\omega_R^2/S_B$ into (8), for H:

$$\frac{1}{2}\frac{J\omega_R^2}{S_B\omega_{Re}} \left[\omega_2^2 - \omega_1^2\right] = \int_{\delta_1}^{\delta_2} \left[P_m - P_e\right] d\delta \tag{8a}$$

$$\frac{\omega_R^2}{S_B\omega_{Re}} \left[\frac{1}{2}J\omega_2^2 - \frac{1}{2}J\omega_1^2\right] = \int_{\delta_1}^{\delta_2} \left[P_m - P_e\right] d\delta \tag{8b}$$

Returning to (8), let ω_1 be the speed at the initial moment of the fault ($t=0^+$, $\delta=\delta_1$), and ω_2 be the speed at the maximum angle ($\delta=\delta_r$), as shown in Fig. 1 below.

Note that the fact that we identify a maximum angle $\delta=\delta_r$ indicates an implicit assumption that the performance is stable. Therefore the following development *assumes stable performance*.

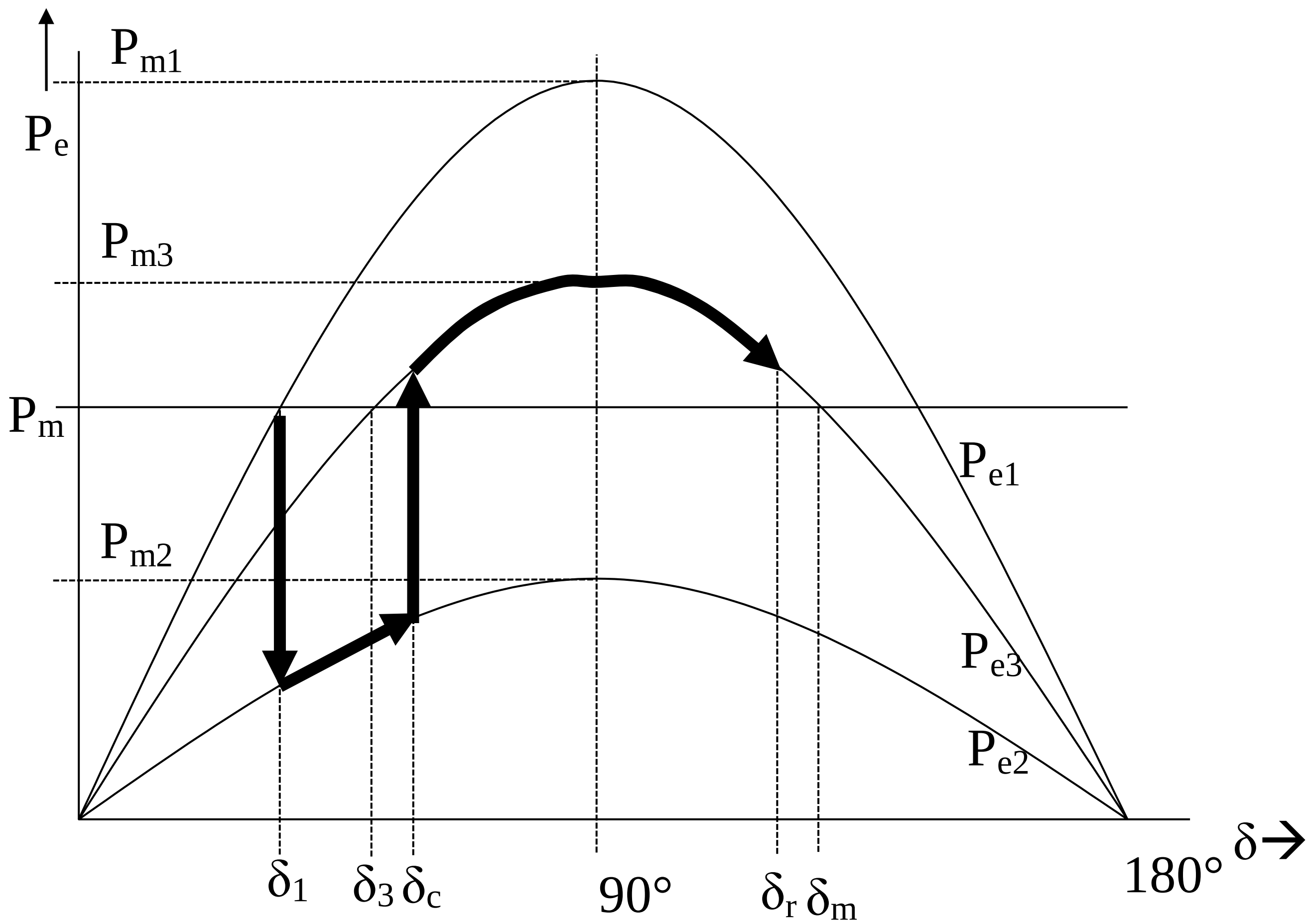


Fig. 1

Since speed is zero at t=0, it remains zero at $t=0^+$. Also, since δ_r is the maximum angle, the speed is zero at this point as well. Therefore, the angle and speed for the two points of interest to us are (note the dual meaning of δ_1: it is lower variable of integration; it is initial angle):

$\delta_1=\delta_1$ $\qquad$ $\delta=\delta_r$

$\omega_1=0$ $\qquad$ $\omega_2=0$

Therefore, (8) becomes:

$$\frac{H}{\omega_{\text{Re}}}\left[\omega_2^2-\omega_1^2\right]=0=\int_{\delta_1}^{\delta_r}\left[P_m-P_e\right]d\delta \tag{9a}$$

We have developed a criterion under the assumption of stable performance, and that criterion is:

$$\int_{\delta_1}^{\delta_r}\left[P_m-P_e\right]d\delta=0 \tag{9b}$$

Recalling that $P_a=P_m-P_e$, we see that (9b) says that for stable performance, the integration of the accelerating power from initial angle to maximum angle must be zero. Recalling again (8b), which indicated the left-hand-side was proportional to the change in the kinetic energy between the two states, we can say that (9b) indicates that the accelerating energy must exactly counterbalance the decelerating energy.

Inspection of Fig. 1 indicates that the integration of (9b) includes a discontinuity at the moment when the fault is cleared, at angle $\delta=\delta_c$. Therefore we need to break up the integration of (9b) as follows:

$$\int_{\delta_1}^{\delta_c}\left[P_m - P_{e2}\right]d\delta + \int_{\delta_c}^{\delta_r}\left[P_m - P_{e3}\right]d\delta = 0 \tag{10}$$

Taking the second term to the right-hand-side:

$$\int_{\delta_1}^{\delta_c}\left[P_m - P_{e2}\right]d\delta = -\int_{\delta_c}^{\delta_r}\left[P_m - P_{e3}\right]d\delta \tag{11}$$

Carrying the negative inside the right integral:

$$\int_{\delta_1}^{\delta_c}\left[P_m - P_{e2}\right]d\delta = \int_{\delta_c}^{\delta_r}\left[P_{e3} - P_m\right]d\delta \tag{12}$$

Observing that these two terms each represent areas on the power-angle curve, we see that we have developed the so-called *equal-area criterion* for stability. This criterion says that stable performance requires that the accelerating area be equal to the decelerating area, i.e.,

$$A_1 = A_2 \tag{13}$$

where

$$A_1 = \int_{\delta_1}^{\delta_c}\left[P_m - P_{e2}\right]d\delta \tag{13a}$$

$$A_2 = \int_{\delta_c}^{\delta_r}\left[P_{e3} - P_m\right]d\delta \tag{13b}$$

Figure 2 illustrates.

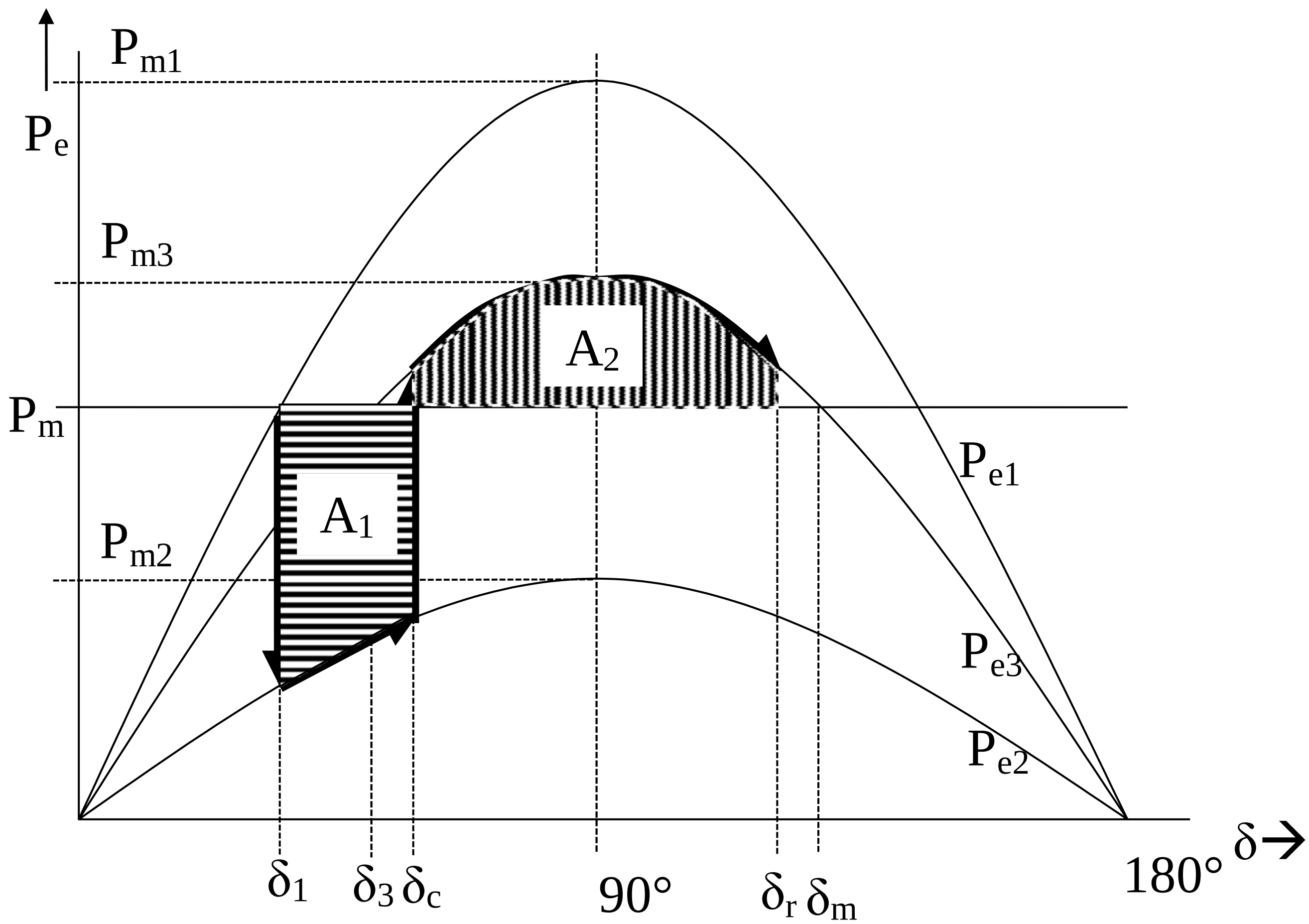


Fig. 2

Figure 2 indicates a way to identify the maximum swing angle, δ_r. Given a particular clearing angle δ_c, which in turn fixes A_1, the machine angle will continue to increase until it reaches an angle δ_r such that A_2=A_1.

2. Derive the swing equation for a synchronous generator.

Let us consider a three-phase synchronous alternator that is driven by a prime mover. The equation of motion of the machine rotor is given by

$$J\frac{d^2\theta}{dt^2} = T_m - T_e = T_a \tag{9.6}$$

where

J	is the total moment of inertia of the rotor mass in kgm^2
T_m	is the mechanical torque supplied by the prime mover in N-m
T_e	is the electrical torque output of the alternator in N-m
θ	is the angular position of the rotor in rad

Neglecting the losses, the difference between the mechanical and electrical torque gives the net accelerating torque T_a. In the steady state, the electrical torque is equal to the mechanical torque, and hence the accelerating power will be zero. During this period the rotor will move at *synchronous speed* ω_s in rad/s.

The angular position θ is measured with a stationary reference frame. To represent it with respect to the synchronously rotating frame, we define

$$\theta = \omega_s t + \delta \tag{9.7}$$

where δ is the angular position in rad with respect to the synchronously rotating reference frame. Taking the time derivative of the above equation we get

$$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt} \tag{9.8}$$

Defining the angular speed of the rotor as

$$\omega_r = \frac{d\theta}{dt}$$

we can write (9.8) as

$$\omega_r - \omega_s = \frac{d\delta}{dt} \tag{9.9}$$

We can therefore conclude that the rotor angular speed is equal to the synchronous speed only when $d\delta/dt$ is equal to zero. We can therefore term $d\delta/dt$ as the error in speed. Taking derivative of (9.8), we can then rewrite (9.6) as

$$J\frac{d^2\delta}{dt^2} = T_m - T_e = T_a \tag{9.10}$$

Multiplying both side of (9.11) by ω_m we get

$$J\omega_r \frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{9.11}$$

where P_m, P_e and P_a respectively are the mechanical, electrical and accelerating power in MW.

We now define a normalized inertia constant as

$$H = \frac{\text{Stored kinetic energy at synchronous speed in mega - joules}}{\text{Generator MVA rating}} \quad \frac{J\omega_s^2}{2S_{rated}} \tag{9.12}$$

Substituting (9.12) in (9.10) we get

$$2H\frac{S_{rated}}{\omega_s^2}\omega_r \frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{9.13}$$

In steady state, the machine angular speed is equal to the synchronous speed and hence we can replace ω_r in the above equation by ω_s. Note that in (9.13) P_m, P_e and P_a are given in MW. Therefore dividing them by the generator MVA rating S_{rated} we can get these quantities in per unit. Hence dividing both sides of (9.13) by S_{rated} we get

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \text{ per unit} \tag{9.14}$$

Equation (7.14) describes the behaviour of the rotor dynamics and hence is known as the swing equation. The angle δ is the angle of the internal emf of the generator and it dictates the amount of power that can be transferred. This angle is therefore called the *load angle*.

3. A 50 Hz, 4-pole turbogenerator is rated 500 MVA, 22 kV and has an inertia constant (*H*) of 7.5. Assume that the generator is synchronized with a large power system and has a zero accelerating power while delivering a power of 450 MW. Suddenly its input power is changed to 475 MW. We have to find the speed of the generator in rpm at the end of a period of 10 cycles. The rotational losses are assumed to be zero.

We then have

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}\left(P_m - P_e\right) = \frac{100\pi}{15} \times 25 = 523.6 \text{ electrical deg/s}^2$$

$$= \frac{523.6\pi}{180} = 9.1385 \text{ electrical rad/s}^2$$

Noting that the generator has four poles, we can rewrite the above equation as

$$\frac{d^2\delta}{dt^2} = \frac{9.1385}{2} = 4.5693 \text{ mechanical rad/s}^2$$

$$= 60 \times \frac{4.5693}{2\pi} = 43.6332 \text{ rpm/s}$$

The machines accelerates for 10 cycles, i.e., $20 \times 10 = 200$ ms $= 0.2$ s, starting with a synchronous speed of 1500 rpm. Therefore at the end of 10 cycles

$$\text{Speed} = 1500 + 43.6332 \times 0.2 = 1508.7266 \text{ rpm.}$$

4. Derive the expression for the power-angle relationship.

The power-angle relationship has been discussed in Section 2.4.3. In this section we shall consider this relation for a lumped parameter lossless transmission line. Consider the single-machine-infinite-bus (SMIB) system shown in Fig. 9.1. In this the reactance *X* includes the reactance of the transmission line and the synchronous reactance or the transient reactance of the generator. The sending end voltage is then the internal emf of the generator. Let the sending and receiving end voltages be given by

$$V_S = V_1\angle\delta, \quad V_R = V_2\angle 0° \tag{9.1}$$

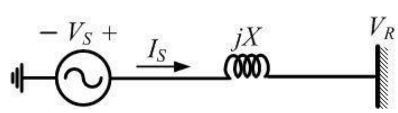


Fig. 9.1 An SMIB system.

We then have

$$I_S = \frac{V_1\angle\delta - V_2}{jX} = \frac{V_1\cos\delta - V_2 + jV_1\sin\delta}{jX} \tag{9.2}$$

The sending end real power and reactive power are then given by

$$P_S + jQ_S = V_S I_S^* = V_1\left(\cos\delta + j\sin\delta\right)\frac{V_1\cos\delta - V_2 - jV_1\sin\delta}{-jX}$$

This is simplified to

$$P_S + jQ_S = \frac{V_1V_2\sin\delta + j\left(V_1^2 - V_1V_2\cos\delta\right)}{X} \tag{9.3}$$

Since the line is loss less, the real power dispatched from the sending end is equal to the real power received at the receiving end. We can therefore write

$$P_e = P_S = P_R = \frac{V_1V_2}{X}\sin\delta = P_{max}\sin\delta \tag{9.4}$$

where $P_{max} = V_1V_2/X$ is the maximum power that can be transmitted over the transmission line. The power-angle curve is shown in Fig. 9.2. From this figure we can see that for a given power P_0. There are two possible values of the angle $\delta - \delta_0$ and δ_{max}. The angles are given by

$$\begin{aligned}\delta_0 &= \sin^{-1}\left(\frac{P_0}{P_{max}}\right)\\ \delta_{max} &= 180° - \delta_0\end{aligned} \tag{9.5}$$

5. A generator is connected to a constant voltage bus through an external reactance of 0.3 per unit. The synchronous reactance of the generator is 0.2 per unit and the voltage magnitude of the constant voltage bus is 1.0 per unit with its angle being 0°. The generator delivers 0.9 per unit power to the constant voltage bus when the angle of its terminal voltage is 15°. We have to determine the magnitude and angle of its internal emf.

Let us denote the angle of the terminal voltage by δ_t, while its magnitude by V_t. Since the generator delivers 0.9 per unit power to the constant voltage bus of magnitude 1.0 per unit through a 0.3 per unit reactance, we can write from (9.4)

$$0.9 = \frac{V_t \times 1}{0.3}\sin\delta_t = \frac{V_t}{0.3}\sin(15°)$$

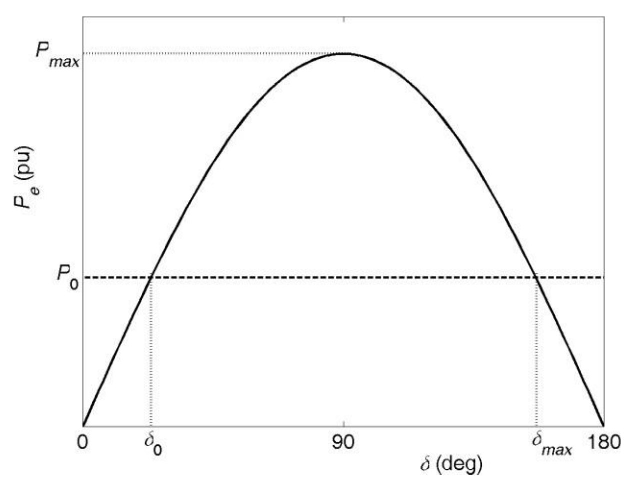


Fig. 9.2 A typical power-angle curve.

Solving the above equation we get V_t = 1.0432 per unit. Therefore the current flowing through the line is

$$I_s = \frac{1.0432\angle 15° - 1}{j0.3} = 0.9 - j0.0255 \text{ per unit}$$

The internal emf of the generator is then given by

$$E_g = V_t\angle\delta_t + j0.2I_s = 1.0128 + j0.45 = 1.1082\angle 23.96°$$

Then the magnitude of the internal emf is 1.1082 per unit angle its angle δ is 23.96°. It can be readily verified that the power delivered is

$$\frac{1.1082 \times 1}{0.3+0.2}\sin(23.96°) = 0.9 \text{ per unit}$$

6. Consider the system of Example 9.1. Let us assume that the system is operating with $P_m = P_e = 0.9$ per unit when a circuit breaker opens inadvertently isolating the generator from the infinite bus. During this period the real power transferred becomes zero. From Example 9.1 we have calculated $\delta_0 = 23.96° = 0.4182$ rad and the maximum power transferred as

$$P_{\max} = \frac{1.1082 \times 1}{0.5} = 2.2164 \text{ per unit}$$

We have to find the critical clearing angle.

From (9.15) the accelerating area is computed as by note that $P_e = 0$ during this time. This is then given by

$$A_1 = \int_{0.4182}^{\delta_{cr}} 0.9\, d\delta = 0.9\delta_{cr} - 0.9 \times 0.4182 = 0.9\delta_{cr} - 0.3764$$

To calculate the decelerating area we note that $\delta_m = \pi - 0.4182 = 2.7234$ rad. This area is computed by noting that $P_e = 2.2164\sin(\delta)$ during this time. Therefore

$$\begin{aligned} A_2 &= \int_{\delta_{cr}}^{2.7234} (2.2164\sin\delta - 0.9)\, d\delta \\ &= -2.2164 \times \cos(2.7234) + 2.2164\cos(\delta_{cr}) - 0.9 \times 2.7234 + 0.9\delta_{cr} \\ &= 2.2164\cos(\delta_{cr}) + 0.9\delta_{cr} - 0.4257 \end{aligned}$$

Equating $A_1 = A_2$ and rearranging we get

$$\delta_{cr} = \cos^{-1}\left(\frac{0.0493}{2.2164}\right) = 1.5486 \text{ rad} = 88.73°$$

***7.* Consider the system in which a generator is connected to an infinite bus through a double circuit transmission line as shown in Fig. 9.5. The per unit system reactances that are converted in a common base, are also shown in this figure. Let us assume that the infinite bus voltage is $1\angle 0°$. The generator is delivering 1.0 per unit real power at a lagging power factor of 0.9839 to the infinite bus. While the generator is operating in steady state, a three-phase bolted short circuit occurs in the transmission line connecting buses 2 and 4 – very near to bus 4. The fault is cleared by opening the circuit breakers at the two ends of this line. We have to find the critical clearing time for various values of *H*.**

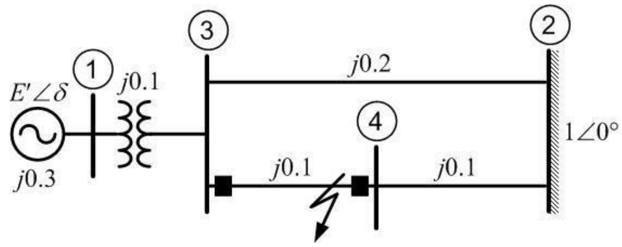


Fig. 9.5 Schematic diagram of the power system of Example 9.4.

Let the current flowing through the line be denoted by *I*. Then the power delivered to the infinite bus is

$$P_e = 1 = \mathrm{Re}\left[1.0 \times I^*\right] = 0.9839\,|I|$$

From the above equation we get

$$I = 1.0164\angle -\cos^{-1}(0.9839) = 1.0164\angle -10.3^\circ$$

The total impedance during the time when both the lines are operational, the impedance between the generator and the infinite bus is *j*(0.3+0.1+0.1) = *j*0.5 per unit. Then the generator internal voltage is

$$E'\angle\delta = 1.0 + j0.5\times 1.0164\angle -10.3^\circ = 1.2\angle 24.625^\circ$$

Therefore the machine internal voltage is E' = 1.2 per unit its angle is 24.625° or 0.4298 rad.

The pre-fault equivalent circuit is shown in Fig. 9.6 (a). From this figure we can write the power transfer equation as

$$\text{Pre - fault :} \quad P_e = \frac{1.2}{0.5}\sin\delta = 2.4\sin\delta$$

Once the fault is cleared by opening of the breakers connected near buses 2 and 4, only of one the two lines will be operational. Therefore during the post fault period, the impedance between the generator and the infinite bus is *j*(0.3+0.2+0.6) = *j*0.5 per unit as shown in Fig. 9.6 (b). Then the post-fault power transfer equation is given by

$$\text{Post - fault :} \quad P_e = \frac{1.2}{0.6}\sin\delta = 2.0\sin\delta$$

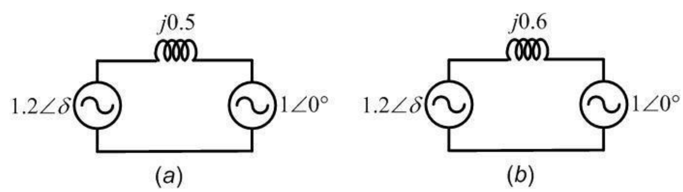


Fig. 9.6 Equivalent circuit: (a) pre-fault and (b) post-fault.

It is to be noted that since one of the two lines is functional during the fault, the power transfer during the fault will not be zero. The equivalent circuit during the fault is shown in Fig. 9.7. Since the fault has occurred very near to bus-4, we can assume that this bus has been short circuited. We shall find the Thevenin equivalent of the portion of the circuit to the right of the points *A* and *B*.

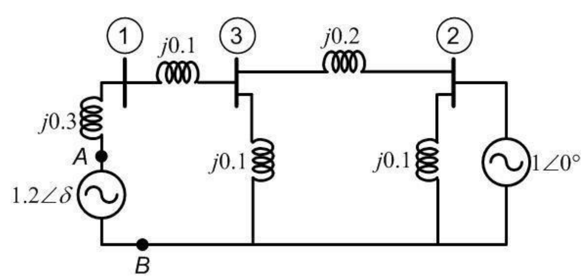


Fig. 9.7 Faulted equivalent circuit.

The circuit between buses 2 and 3 has been converted into an equivalent Y using Δ-Y transformation. This is shown in Fig. 9.8 (a). From this figure we find the Thevenin impedance as

$$X_{th} = 0.45 + \frac{0.05\times 0.025}{0.075} = 0.4667 \text{ per unit}$$

Also the Thevenin voltage is then given by

$$V_{th} = 1\angle 0^\circ \times \frac{j0.025}{j0.075} = 0.3333\angle 0^\circ \text{ per unit}$$

The reduced circuit is shown in Fig. 9.8 (b). From this circuit we can write the following power transfer equation during the fault

$$\text{During} - \text{fault}: \quad P_e = \frac{1.2 \times 0.3333}{0.4667} \sin\delta = 0.857\sin\delta$$

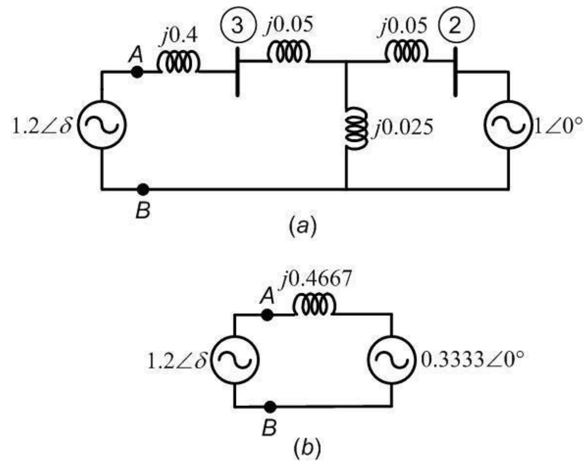


Fig. 9.8 Equivalent circuit during the fault: (a) Δ-Y transformed and (b) Thevenin equivalent.

Three power-angle curves are shown in Fig. 9.9. From this figure we find that

$$\delta_{\max} = \pi - \sin^{-1}\left(\frac{1}{2}\right) = 2.618 \text{ rad}$$

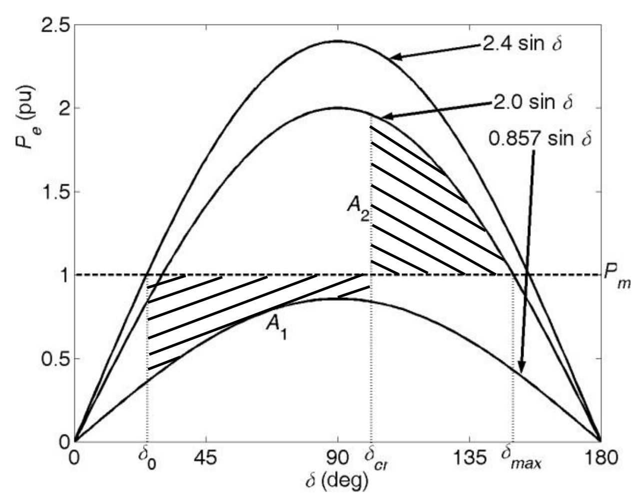


Fig. 9.9 Power-angles curves for the three modes of operation of the system of Example 9.4.

The accelerating area is given by

$$\begin{aligned} A_1 &= \int_{0.4298}^{\delta_{cr}} (1 - 0.857\sin\delta)\, d\delta \\ &= \delta_{cr} - 0.4298 + 0.857\cos\delta_{cr} - 0.857\cos(0.4298) \\ &= \delta_{cr} + 0.857\cos\delta_{cr} - 1.2089 \end{aligned}$$

and the decelerating area is

$$A_2 = \int_{\delta_{cr}}^{2.618} (2\sin\delta - 1)\,d\delta$$

$$= -2\cos(2.618) + 2\cos\delta_{cr} - 2.618 + \delta_{cr}$$

$$= \delta_{cr} + 2\cos\delta_{cr} - 0.8859$$

Equating the two areas we get

$$\delta_{cr} = \cos^{-1}\left(\frac{-0.323}{1.1429}\right) = 1.8573 \text{ rad} = 106.41^\circ$$

As mentioned earlier, the critical clearing angle depends on the system network configuration. The critical clearing time, however, is dependent on *H* and will vary as this parameter varies. Usually numerical methods are employed for finding out the clearing time. We shall however demonstrate the effect of *H* through a MATLAB program. The program uses the built-in ordinary differential equation solver though which the swing equations are solved. The results obtained are listed in Table 9.1. It can be seen that as the value of *H* is increases, the clearing time is also increases, even though the clearing angle remains the same. This is obvious as the value of *H* increases, the response of the system becomes more sluggish due to larger inertia. Hence, the rotor takes more time to accelerate.

Table 9.1 Effect of *H* on critical clearing time

H (MJ/MVA)	Approximate Critical Clearing Time (s)
2	0.2783
4	0.3936
6	0.4821
8	0.5566
10	0.6223

8. Explain the modified Euler's method to solve swing equation.

Euler method

Consider plotting our function f(x(t)) as a function of t. What we want to do, based on (11), is to obtain the area under the curve of f(x(t)) from t=kT-T to t=kT, as illustrated in Fig. 1.

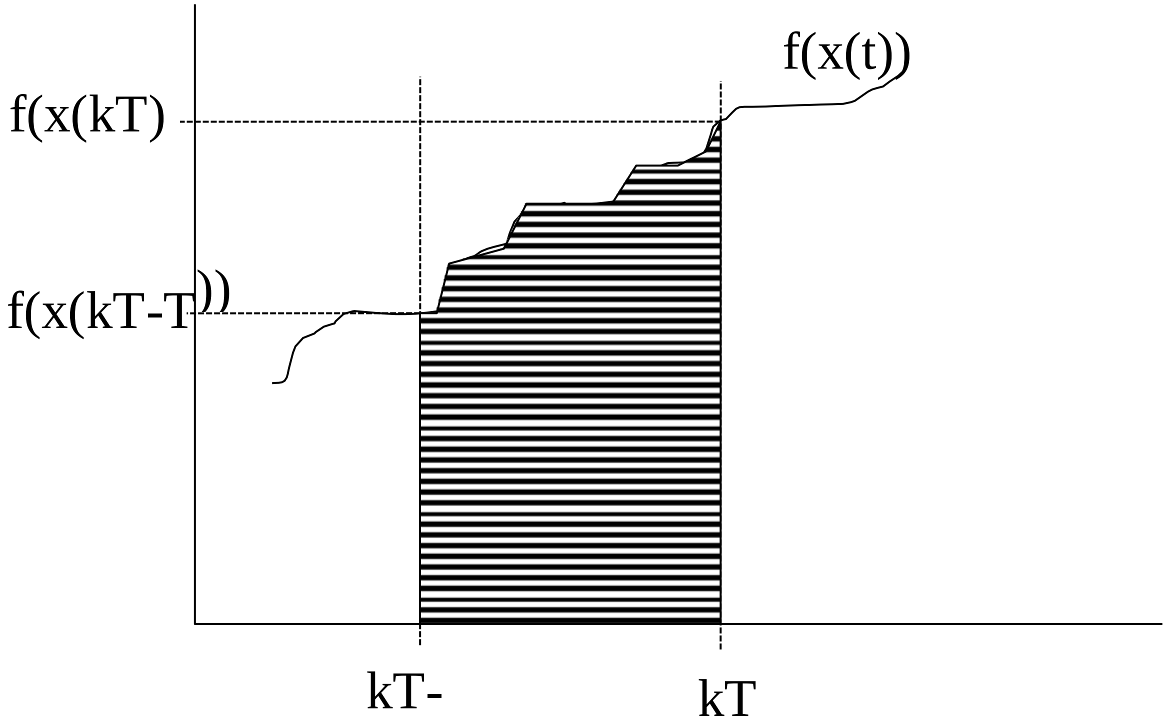


Fig. 1

Here is a proposed approach: Assume that f is constant throughout the interval at f(kT-T). This assumption is clearly not good if kT is large, but it might be reasonable if kT is made small enough.

This approach approximates the integral as the area shaded by the vertical lines in Fig. 2. One observes that it misses the area shaded by the horizontal lines in Fig. 2.

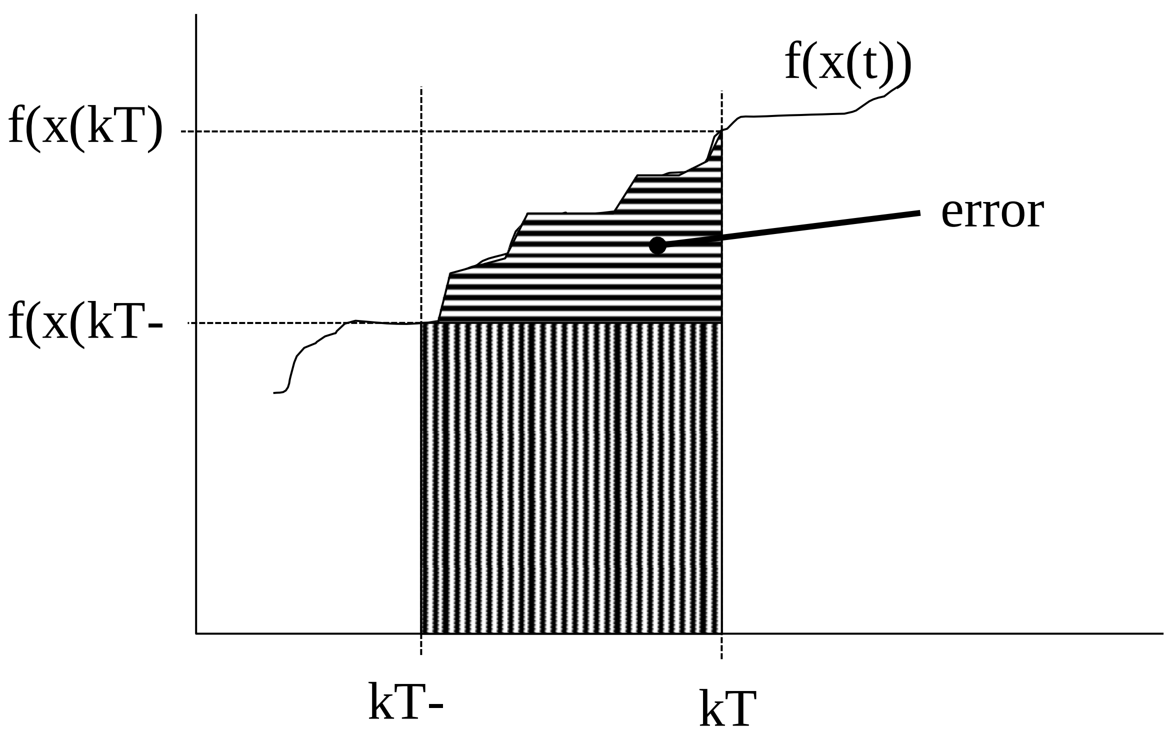


Fig. 2

Analytically, (10) becomes

$$x(kT) = x(kT - T) + Tf(x(kT - T)) \tag{12}$$

where the last term is Δx. This approach is also called the "forward rule" because we assume a value for f and hold it constant as we move forward in time.

<u>2.1 Alternative development of forward rule</u>

We may also develop the forward rule in another way….

Assume that we know x(kT-T) and that T is chosen small enough so that x(kT) is close to x(kT-T). Then by Taylor series,

$$\begin{aligned} x(kT) &= x(kT - T) + T\dot{x}\big|_{t=kT-T} + \frac{T^2}{2}\ddot{x}\big|_{t=kT-T} + \frac{T^2}{3!}\dddot{x}\big|_{t=kT-T} + \ldots. \\ &= x(kT - T) + T\dot{x}\big|_{t=kT-T} + \mathrm{O}(T^2) \end{aligned} \tag{13}$$

where $O(T^2)$ is the remainder of the Taylor series, and its argument T^2 indicates that the lowest power of T present in the remainder is T^2.

If T is small enough, $O(T^2)$ is negligible and

$$x(kT) = x(kT - T) + T\dot{x}\big|_{t=kT-T} \tag{14}$$

where the last term is Δx. This is illustrated in Fig. 3.

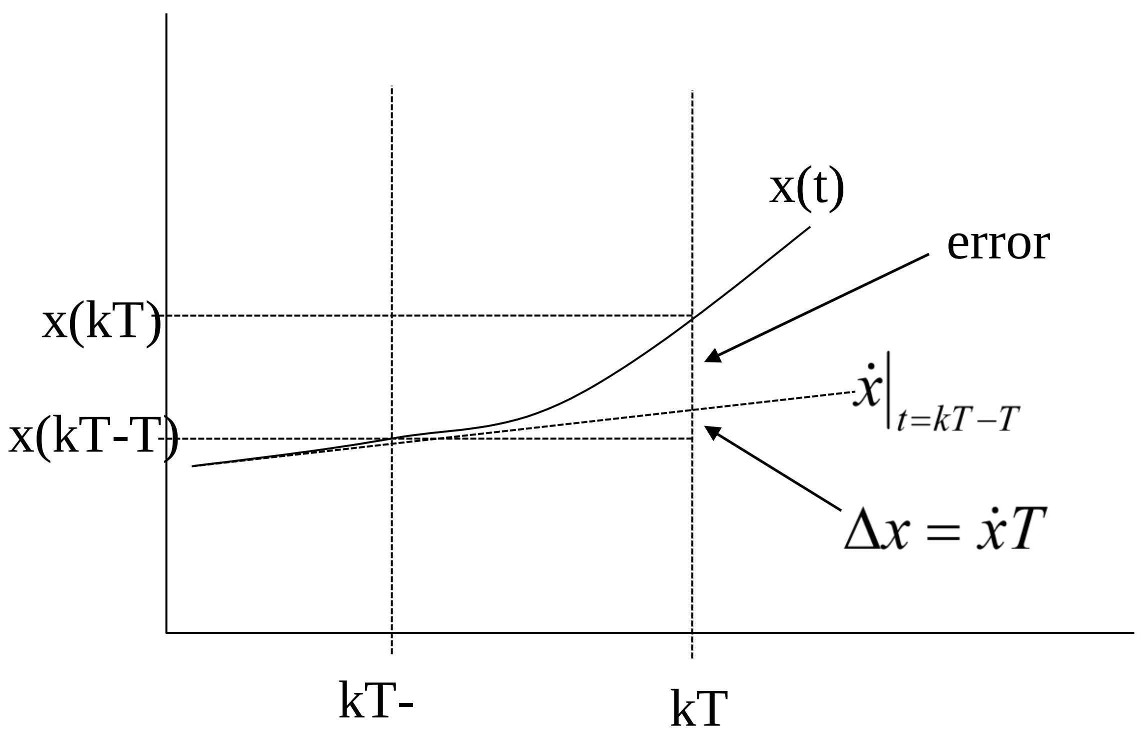


Fig. 3

Recalling that $\dot{x} = f(x)$, (14) becomes

$$x(kT) = x(kT - T) + Tf(x(kT - T)) \tag{15}$$

which is the same as (12), our forward rule.

From both Figs. 2 and 3, we can observe that the forward method will incur some error, and this error will increase with larger values of T.

One major problem with this method is that if T is too large, the error will *propagate* from one time step to another.
This means that error incurred at one time step kT will add to the error incurred at later time steps. Solution methods where this can happen are said to be numerically unstable.
However, occurrence of this phenomenon does depend on our choice of T. I will provide you with a way to consider this issue.

Reducing the error

Two algorithms that improve on the forward (Euler) method are predictor-corrector and Runge-Kutta. We will look at both briefly.
<u>3.1 Predictor-corrector method</u>
This method is called the modified Euler in your text. The idea here is that we will take a step to compute x(kT) (a predictor) and then we will use that calculation to recalculate that same step (the corrector).
Step 1: Predict x(kT) using Euler to get x(kT):

$$x^p(kT) = x(kT-T) + T\,\underbrace{f(x(kT-T))}_{\dot{x}(kT-T)} \tag{32}$$

Step 2: Use $x^p(kT)$ to obtain a corrected value $x^c(kT)$:

a. Get a corrected derivative f^c as the average of the derivatives at x(kT-T) and $x^p(kT)$:

$$f^c = \frac{1}{2}\left[f(x(kT-T)) + f(x^p(kT))\right] \tag{33}$$

b. Then apply the forward rule:

$$\begin{aligned} x^c(kT) &= x(kT-T) + Tf^c \\ &= x(kT-T) + \frac{T}{2}\left[f(x(kT-T)) + f(x^p(kT))\right] \end{aligned} \tag{34}$$

This method is illustrated in Fig. 5.

$$\dot{x}\Big|_{t=kT}$$

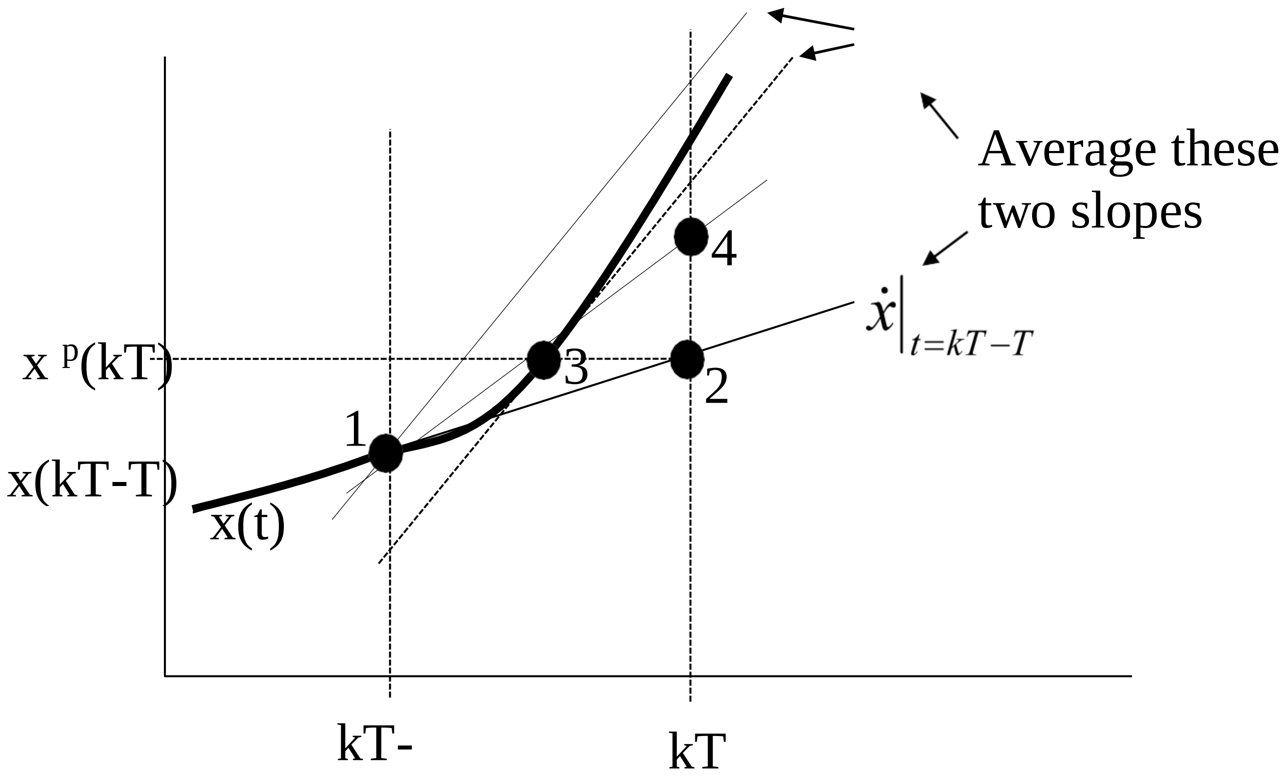


Fig. 5

Understanding the method is facilitated by observing the sequence of points in Fig. 5. The derivative f(x(kT-T)) is obtained at point 1. The predicted point $x^p(kT)$ is obtained at point 2. The derivative of the predicted point $f(x^p(kT))$ is obtained at point 3. The final point, point 4, is obtained from averaging the two obtained derivatives f(x(kT-T)) and $f(x^p(kT))$.

One can observe intuitively from Fig. 5 that the error will be reduced. This method can be shown to be equivalent to considering up to the second derivative term in the Taylor series, therefore the error is of $O(T^3)$.

This is a significant improvement over the Euler method, but it is still a numerically unstable algorithm. In other words, for the predictor-corrector method, for a given minimum eigenvalue, T can be larger than it can be in the Euler method, but it is still true that the algorithm may be unstable if T is too large.

9. Explain the Runge-kutta method to solve the swing equation.

Runge-Kutta method
(pronounced Run-gah Kut-tah)
This algorithm was developed in 1895, and it also applies the idea of averaging, similar to predictor-corrector, but in a slightly different way.
There are different R-K algorithms of different order. We will only study one of them, the 4th order R-K.
The 4th order R-K method requires, at each successive time step, computing 4 different increments Δx_j, as follows:

Increment Δx_j	Derivative used
$\Delta x_1 = K_1 = Tf(x(kT-T))$	Start-point derivative only
$\Delta x_2 = K_2 = Tf\left(x(kT-T)+\frac{K_1}{2}\right)$	First interior derivative
$\Delta x_3 = K_3 = Tf\left(x(kT-T)+\frac{K_2}{2}\right)$	Second interior derivative
$\Delta x_4 = K_4 = Tf\left(x(kT-T)+K_3\right)$	Approx. end-point derivative

Note the following about the K_i"s.

1. K_i is always used to compute K_{i+1}.

2. Each K_i is *not* a derivative but rather an increment in the integration variable, i.e.,

$$\Delta x_i = K_i = x_i(kT) - x(kT-T) \tag{35}$$

   Any of the K_i"s *could* be used to obtain the new value x(kT).

3. Use of K_1 to obtain the new value x(kT) is equivalent to the Euler method.

4. The derivatives are computed at three different locations within the interval:

   - The beginning of the time step x(kT-T)
   - The first interior derivative x(kT-T)+K_1/2
   - The second interior derivative x(kT-T)+K_2/2
   - The approximate end-point derivative x(kT-T)+K_3

Figure 6 below illustrates the sequence of calculations, which can be understood by following the single arrows from point 1 to point 2 to point 3, and then the double arrows from point 3 to point 4 to point 5, and then the triple arrows from point 5 to point 6 to point 7 to point 8.

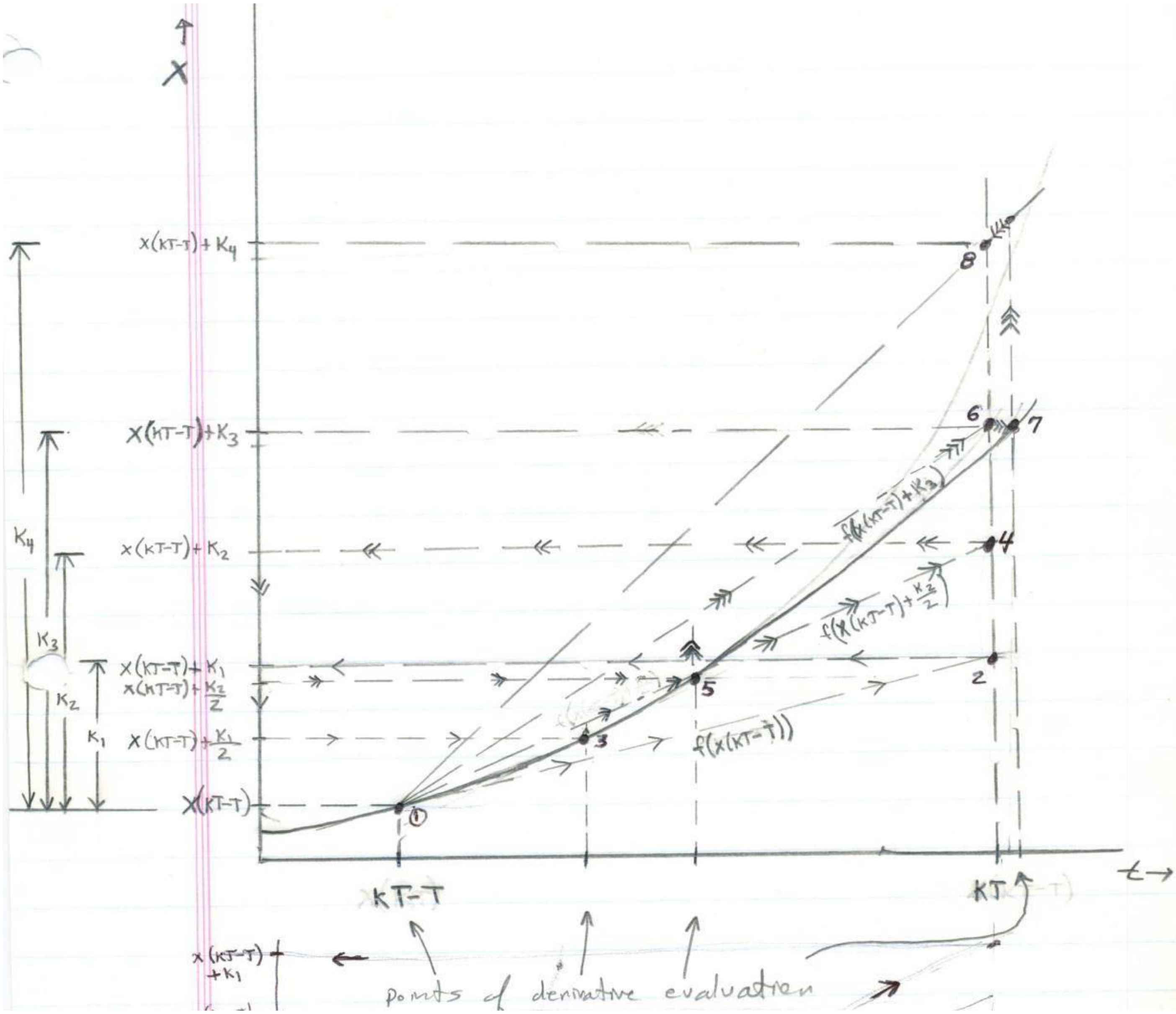


Fig. 6

Once each of the increments K_1-K_4 are computed, then the final increment is obtained by taking a weighted average of the four increments, where the middle increments are weighted heaviest, according to (36).

$$\Delta x = \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right) \tag{36}$$

The middle increments (K_2 and K_3) are weighted heaviest as they are computed based on slopes that will be more representative of the slope in the interval than the beginning (K_1) and final (K_4) increments.

The R-K method can be shown to be equivalent to considering up to the fourth derivative term in the Taylor series, therefore the error is $O(T^5)$. Although this is a significant improvement over the Euler or the P/C method, R-K is also a numerically unstable algorithm therefore the stability domain.